Complexity Theory and Dynamic Characteristics of Cognitive Processes

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Abstract. The features of modeling of the cognitive component of social and humanitarian systems have been considered. An example of using entropy multiscale, multifractal, recurrence and network complexity measures has shown that these and other synergetic models and methods allow us to correctly describe the quantitative differences of cognitive systems. The cognitive process is proposed to be regarded as a separate implementation of an individual cognitive trajectory, which can be represented as a time series and to investigate its static and dynamic features by the methods of complexity theory. Prognostic possibilities of the complex systems theory will allow to correct the corresponding pedagogical technologies. It has been proposed to track and quantitatively describe the cognitive trajectory using specially transformed computer games which can be used to test the processual characteristics of thinking.

Keywords: Cognitive systems · Complexity · Complex networks · Entropy · Recurrence plot · Computer games · New pedagogical technologies

1 Introduction

It has become recently clear that pedagogical science operates the transmission of a kind of structured information that is knowledge. Information, as the main concept of cybernetics, is characterized by a metric function and, thus, the search for optimal management of educational processes is translated into a plane of mathematical modeling [1–3].

In science strict conditional constructions have been predominant for a long time. Initially, these views were developed in science and mathematics, and then moved into the humanitarian field, in particular, in pedagogy. As a result, many attempts have been made to organize education as a perfectly functioning machine. According to the dominant ideas then, for the education of a person only need to learn how to manage such a "machine", that is to turn education into a kind of production and technological process.

For many complex systems, the phenomenon of self-organization is characteristic [4]. It leads to the fact that very often a few variables, the so-called order parameters, are detected very often for the description of an object, which is described by a

V. Ermolayev et al. (Eds.): ICTERI 2019, CCIS 1175, pp. 231–253, 2020.
https://doi.org/10.1007/978-3-030-39459-2_11

large or even infinite number of variables [5]. These parameters "subordinate" other variables, defining their values. The researchers are aware of the mechanisms of self-organization, which lead to the allocation of parameters of order, methods of their description and the corresponding mathematical models. However, it is likely, our brain has a brilliant ability to find these parameters, to "simplify reality", has more effective algorithms for their selection. The process of learning, education allows one to find successful combinations that can be the order parameter in certain situations or the mechanisms of searching for such parameters ("learn to study", "learn to solve non-standard tasks").

It is also advisable to use the ideas of a soft (or fuzzy) simulation. As it was said by V.I. Arnold, in the case of hard and soft models [6], has a place in pedagogical science. Since in humanitarian systems the results of their interaction and development can not be predicted in detail, by analogy with complex quantum systems one can speak the principle of uncertainty for humanitarian systems. In the process of learning always occur unplanned small changes, fluctuations in the various pedagogical systems (and the individual, and the team of students, and knowledge systems). Therefore, the basis of modern educational models should lie in the principle of uncertainty in a number of managerial and educational parameters.

Network education refers to a new educational paradigm [7], which it calls "networking". Its distinctive features are learning based on the synthesis of the objective world and virtual reality by activating both the sphere of rational consciousness and the sphere of intuitive, unconscious. Unlike the traditional, network education strategy is focused not on the systematization of knowledge and the assimilation of the next main core of information, but on the development of abilities and motivation to generate their own ideas [8].

Within the framework of recent research in the Davos forum, 10 skills were identified, most demanded by 2022 [9]: (1) Analytical thinking and innovation; (2) Active learning and learning strategies; (3) Creativity, originality and initiative; (4) Technology design and programming; (5) Critical thinking and analysis; (6) Complex problem-solving; (7) Leadership and social influence; (8) Emotional intelligence; (9) Reasoning, problem-solving and ideation; (10) Systems analysis and evaluation. Obviously, the cognitive component in the transformation processes of Industry 4.0 is dominant, which actualizes attention to the study of cognitive processes.

The complexities here are reduced to the fact that cognitive processes are poorly formalized. Therefore, the field of theoretical works until recently was virtually empty. The picture has fundamentally changed with the use of recent synergetic studies [4, 5]. The fact is that the doctrine of the unity of the scientific method asserts: for the study of events in the social-humanitarian systems, the same methods and criteria apply to the study of natural phenomena.

The process of intellection is a cognitive process characterized by an individual cognitive trajectory whose complexity is an integro-differential characteristic of an individual. The task is to quantify cognitive trajectories and present them in the form of a time series that can be analyzed quantitatively. The theory of complexity introducing various measures of complexity, allows us to classify cognitive trajectories by complexity and choose more complex, as more efficient ones. The analysis procedure can be done dynamically, by correcting the trajectories by means of progressive pedagogical technologies.

Previously, we introduced various quantitative measures of complexity for particular time series, in particular: algorithmic, fractal, chaos-dynamic, recurrent, nonreversible, network, and others. The calculation of these measures is implemented as a set of toolboxes in the MATLAB environment with a user-friendly interface [10]. Significant advantage of the introduced measures is their dynamism, that is, the ability to monitor the time of change in the chosen measure and compare with the corresponding dynamics of the output time series. This allowed us to compare the changes in the dynamics of the system, which is described by the time series, with characteristic changes in concrete measures of complexity and draw conclusions about the properties of the cognitive trajectory.

Objects of research are cognitive processes that control neurophysiological and other cognitive characteristics of a person:

– the length of the full step of all ages children [11], a healthy young person and the elderly, or those with neurodegeneration (Alzheimer's, Parkinson's, Huntington's, etc. [12]);
– human recalls of words [13];
– objects of cognitive linguistics – the works of various authors, different genres, written in different languages [14];
– discretized multi-genre musical compositions [15];
– processual characteristics of cognitive games [8].

The corresponding databases in the form of time series are in open access [16].

In this paper, we consider some of the informative measures of complexity and adapt them in order to study the cognitive processes. The paper is structured as follows. Section 2 describes previous studies in these fields. Section 3 presents information mono- and multiscale measures of complexity. Section 4 describes the technique of fractal and multifractal. Section 5 shows the possibilities of a recurrence analysis of some cognitive processes. Network measures of complexity and their effectiveness in the study of cognitive processes are presented in Sect. 6. Research methods for the processual characteristics of thinking using gaming technologies are described in Sect. 7.

2 Analysis of Previous Studies

Researchers interested in human cognitive processes have long used computer simulations to try to identify the principles of cognition [17]. Existing theoretical developments in this scientific field describe complex, dynamic, and emergent processes that shape intra- (e.g., cognition, motivation and emotion) and inter- (e.g., teacher–student, student–student, parent–child interactions, collaborative teams) person phenomena at multiple levels. These processes are fundamental characteristics of complex systems but the research methods that are used sometimes do not match the complexity of processes that need to be described.

From the set of methods of the theory of complex systems we consider only those relating to information, recurrent, fractal, and network complexity measures.

Entropic measures in general are relevant for a wide variety of linguistic and computational subfields. In the context of quantitative linguistics, entropic measures are used to understand laws in natural languages, such as the relationship between word frequency, predictability and the length of words, or the trade-off between word structure and sentence structure [18]. Together with Shannon's entropy, more complex versions are used: Approximate entropy, Sample entropy [19]. In order to demonstrate the scale-invariant properties of cognitive processes, these types of entropy were used in a multiscale version in the study of cognitive processes of cerebral activity [20], human locomotion functions [21], in linguistics [19].

Cognitive processes like most complex systems [22] exhibit fractal properties [23, 24]. Thus, extensive studies of the fractal properties of a wide range of neurological, physiological, and cognitive processes showed [23] that fluctuations in the time series describing these processes are not an exception, but a natural indicator of the complexity of the processes.

Recurrence quantification analysis (RQA) is a nonlinear approach to assessing patterns in time series data. It essentially identifies the degree to which a measured time series repeats itself, and the nature of those repetitions, whether they reflect deterministic or predictable dynamics or are incidental due to random fluctuation [25, 26]. An example is the work [27] in which the authors demonstrates the feasibility of using RQA as a tool to compare speech variability across speakers and groups. RQA offers promise as a technique to assess effects of potential stressors (e.g., linguistic or cognitive factors) on the speech production system.

In recent years, the complex networks methods [28] have become widespread. They not only allow the construction and exploration of networks with obvious (as in linguistics) nodes and links [29], but also those reproduced from the time series by actively developing methods [30, 31]. Moreover, networks of networks or multiplex networks [32] are being actively studied; their application has also come to cognitive science [33, 34], teaching practices [27, 35].

In our recent works, we have used some of the modern methods of the theory of complex systems for the analysis of such a complex system as crypto and stock markets [36–38]. Some of the results discussed below are summarized in [39]. In this paper, modern methods of the theory of complexity, complex networks, and cognitive games are used for the quantitative description of cognitive trajectories.

Gaming technologies are an undoubted trend of modern pedagogical technologies [40], but are rarely used as a tool for cognitive [39, 41–43]. In Sect. 7, we show the capabilities of a certain category of games to act as a non-invasive, procedural tool for studying cognitive personality trajectories.

3 Information Mono- and Multiscale Measures of Complexity

Based on the different nature of the methods laid down in the basis of the formation of the measure of complexity, they pay particular demands to the time series that serve the input. For example, information requires stationarity of input data. At the same time, they have different sensitivity to such characteristics as determinism, stochasticity, causality and correlation. Obviously, the classic indicators of algorithmic complexity are unacceptable and lead to erroneous conclusions. To overcome such difficulties, multiscale methods are used.

The idea of this group of methods includes two consecutive procedures: (1) coarse graining ("granulation") of the initial time series – the averaging of data on non-intersecting segments, the size of which (the window of averaging) increased by one when switching to the next largest scale; (2) computing at each of the scales a definite (still mono scale) complexity indicator. The process of "rough splitting" consists in the averaging of series sequences in a series of non-intersecting windows, and the size of which – increases in the transition from scale to scale [25]. Each element of the "granular" time series is in accordance with the expression:

$$y_j^\tau = 1/\tau \sum_{i=(j-1)\tau+1}^{j\tau} x_i, \quad 1 \le j \le N/\tau, \tag{1}$$

where τ characterizes the scale factor. The length of each "granular" row depends on the size of the window and is even N/τ. For a scale equal to 1, the "granular" series is exactly identical to the original one.

We demonstrate the work of multi-scale measures of complexity on examples of Approximate and Sample Entropy [20]. Approximate Entropy (ApEn) is a "regularity statistic", which determines the possibility of predicting fluctuations in time series.

When calculating ApEn for a given time series S_N consisting of N values $t(1), t(2), t(3), \ldots, t(N)$ two parameters are chosen, m and r. The first of these parameters, m, indicates the length of the template, and the second – r – defines the similarity criterion. The sequences of time series elements S_N consisting of m numbers taken starting from the number i are called, and are called vectors $p_m(i)$. The two vectors (patterns), $p_m(i)$ and $p_m(j)$, will be similar if all the difference pairs of their respective coordinates are less than the values of r, that is, if $|t(i+k) - t(j+k)| < r, \quad 0 \le k < m$. For the considered set of all vectors p_m of the length m of the time series S_N, values $C_{im}(\mathrm{r}) = \mathrm{n_{im}(r)/(N - m + 1)}$ can be calculated. Where $n_{im}(r)$ – the number of vectors in p_m, similar to the vector $p_m(i)$ (taking into account the chosen similarity criterion r). The value $C_{im}(r)$ is the fraction of vectors of length m, which are similar to the vector of the same length, whose elements begin with the number i. For a given time series, the values $C_{im}(r)$ for each vector in p_m are calculated, after which there is an average value $C_m(r)$ that expresses the prevalence of similar vectors of length m in a row S_N. Directly the ApEn for the time series S_N using the vectors of length m and the similarity criterion r is determined by the formula $ApEn(S_N, m, r) = \ln(C_m(r)/C_{m+1}(r))$ that is, as a natural logarithm of the ratio of the repetition of vectors of length m to the repetition of vectors in length $m + 1$.

Thus, if there are similar vectors in the time series, ApEn will estimate the logarithmic probability that the subsequent intervals after each of the vectors will be different. The smaller ApEn values correspond to the greater likelihood that vectors follow similar ones. If the time series is very irregular, the presence of such vectors can not be predictable and the value of ApEn is relatively large.

Sample Entropy (SampEn) is similar to the ApEn, but when calculating the SampEn:

– does not take into account the similarity of the vector to itself;

– when calculating the probabilistic values of SampEn, the length of the vectors is not used.

In Fig. 1a shows the ApEn dependence of the scale to the test signals – flicker $(1/f)$, white noise (wnoise) and electrocardiogram signal (ECG) compared to the shuffled signal (ECG shuffled).

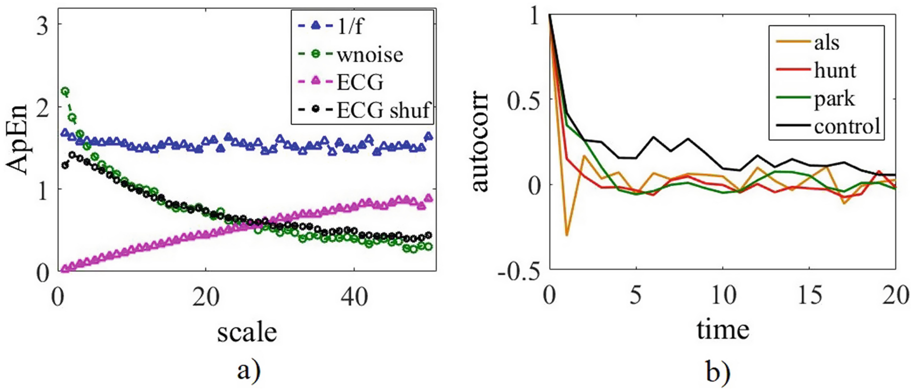

Fig. 1. (a) ApEn of artificial and natural signals, depending on the scale; (b) autocorrelation functions for control the stride intervals fluctuations and Alzheimer's disease (als), Huntington's (hunt) and Parkinson's (park)

From Fig. 1a it is evident that, as expected, a flicker signal was a scale-invariant one. The ECG signal is complex on a large scale. Its complexity is lost when shuffled and becomes very close to a random signal.

Cognitive signals differ in the functions of autocorrelation: the more complex ones have a longer "memory", which is manifested in the slowdown of the function of autocorrelation with the lag (Fig. 1b). Accordingly, the signal of a healthy person and a multiscale entropy measure is more complicated (Fig. 2a). We also investigated multiscale complexity measures for time series of stride intervals in children age from 40 to 163 months (Fig. 2b). Obviously, the complexity of the signal for an older child is increasing.

The next study cognitive signal is the time series of time intervals between human-to-word responses (human recalls of words). Recall in memory refers to the mental

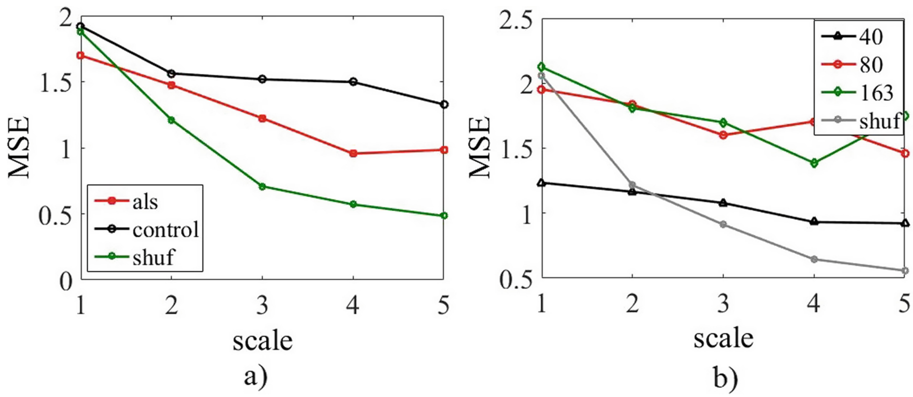

Fig. 2. Multiscale entropy of stride intervals time series templates for healthy and Alzheimer's disease (a) and children of all ages (b).

process of retrieval of information from the past and is a way to study the memory processes of humans and animals [13]. Recall describes the process in which a person is given a list of items to remember and then is tested by being asked to recall. For the autocorrelation function and the multiscale SampEn we obtain the results presented in Fig. 3.

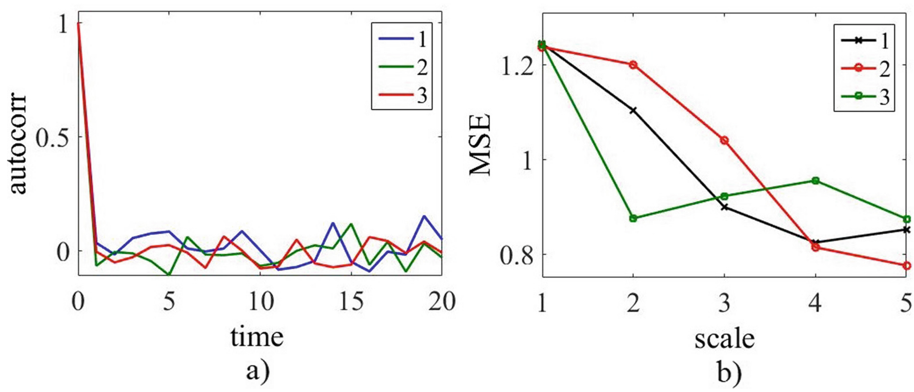

Fig. 3. Autocorrelation functions (a) and MSE measures of complexity (b) signals 1–3 series of recalls

Unfortunately, due to insufficient statistics and short-term time series, identification of differences in these methods is not possible.

Similar studies for musical works of various genres (aria, blues, brazilian samba) and literary works by famous authors (L. Tolstoy's "War and Peace", L. Carroll "Alice in Wonderland", C. Dickens "Cricket on the Hearth") gave the results shown in Fig. 4.

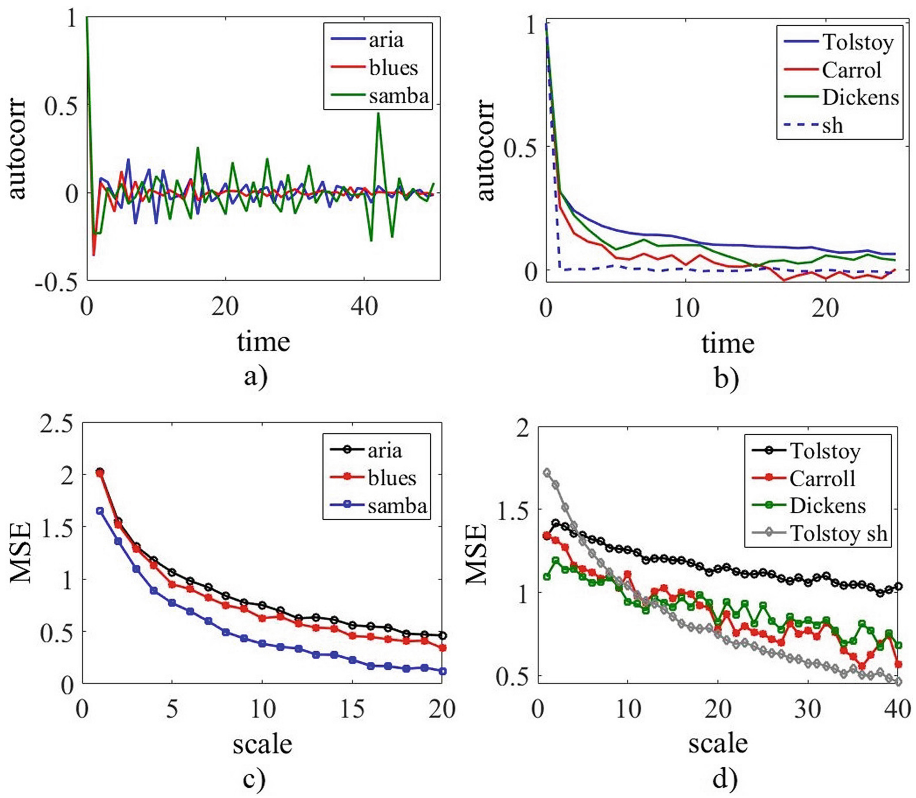

Fig. 4. Autocorrelation of musical (a) and literary (b) works; (c) – (d) are the corresponding multiscale entropy measures of complexity

Obviously, MSE allows classification of the analyzed cognitive signals in terms of complexity.

4 Fractal Measures of Complexity

The fractal nature of cognitive processes attracts the close attention of researchers (see, for example, [23]). Therefore, many fractal measures of complexity have been developed, among which the leading width of the spectrum of the singularity obtained from the analysis of detrended fluctuation analysis leading.

In the general case, the procedure of popular multifractal detrended fluctuation analysis (MF–DFA) is implemented by the following algorithm [44]. Let be a sequence of length N. Then determine the accumulation $Y(i) \equiv \sum_{k-1}^{i} (x_k - \bar{x}),\ i = \overline{1..N}$. We divide it into $N_S = \mathrm{int}(N/s)$ segments of the same length S that do not overlap. For each of the N_s sequences, we calculate the local trend by the least squares method, determine the deviations $F^2(v,s) = 1/s \sum_{k-1}^{s} \left(Y((v-1)s+i) - y_v(i)\right)^2$ for each segment v, $v = \overline{1..N}$ and for each $v = \overline{N_s + 1..2N_s}$. There $y_v(i)$ is an interpolating polynomial on the segment v. Find the mean for all subsequences to obtain the function of q-order fluctuations $F_q(s) = \left(\frac{1}{2N}\sum_{v=1}^{2N_s}\left(F^2(s,v)\right)^{q/2}\right)^{1/q}$. The standard DFA method corresponds to the case $q = 2$. Determine the scaling behavior of the fluctuation function by analyzing the double logarithmic scale of the dependence $F_q(s)$ on q.

To characterize the multifractal set, the so-called multifractal spectrum function $f(\alpha)$ (the spectrum of singularities of a multifractal) is used, which is actually equal to the Hausdorff dimension of a homogeneous fractal subset of the initial set, giving a dominant contribution to the statistical sum for a given q value. The value $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$ – the width of the spectrum of the multifractality – characterizes the degree of complexity of the system.

In Fig. 5 shows the spectrum of multifractality of some of the cognitive systems described above. We see that more complex signals have wider spectrum of multifractality. Consequently, the multifractal measure of complexity can be used to analyze cognitive signals. We have implemented a dynamic procedure for calculating multifractal parameters, which allows you to follow the change in the complexity of the signal in time.

5 Recurrence Analysis of Cognitive Processes

The recurrent properties of cognitive processes can be investigated using an effective method of nonlinear dynamics – recurrence analysis. Its essence is reduced to the construction of the so-called recurrence plot from the initial time series, and then its quantitative analysis. Recurrence plots (RPs) have been introduced to study dynamics and recurrence states of complex systems. A phase space trajectory can be transformed from a time series $U_i = \{u_1, \ldots, u_n\}$ ($t = i\Delta t$, where Δt is the sampling time) into time-delay structures

$$X_i = (U_i, U_{i+1}, \ldots, U_{i+(m-1)\tau}), \tag{2}$$

where m stands for the embedding dimension and τ for the time delay. Both of them can be calculated from the original data using false nearest neighbors and mutual information [26].

A RP is a plot representation of those states which are recurrent. The recurrence matrix and the states are considered to be recurrent if the distance between them within the ε-radius. In this case, the recurrence plot is defined as:

$$R_{ij} = \Theta(\varepsilon - \|x_i - x_j\|), i,j = 1, \ldots, N, \tag{3}$$

and $\|\ \|$ is a norm (representing the spatial distance between the states at times i and j), ε is a predefined recurrence threshold, and Θ is the Heaviside function (ensuring a binary $\mathbf{R}$).

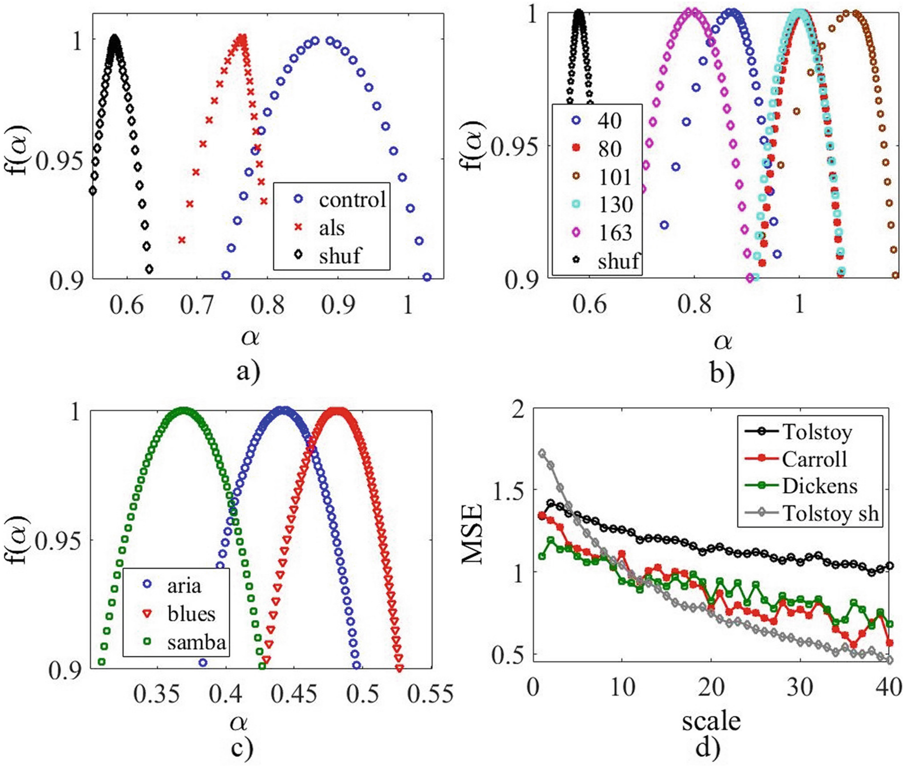

Fig. 5. Spectrum of multifractality (a) reaching the stage of healthy and sick patients; (b) children of all ages; (c) musical works of different genres; (d) literary works

As an example, the figure shows phase portraits of Alzheimer's patient (blue curve in Fig. 6a), a healthy patient (red curve) and calculated for a mixed (random) time series (brown curve) of the stride intervals.

Comparison of both phase portraits and RP's indicates that the distribution of black and light dots is significantly different, which allows us to quantify these differences. For the quantitative description of the system, the small-scale clusters such as diagonal and vertical lines can be used. The histograms of the lengths of these lines are the base of the recurrence quantification analysis [25, 26, 45].

6 Complex Network Methods for Studying Cognitive Processes

6.1 Cognitive Linguistics and Complex System Theory

One of the most important areas of cognitive science is cognitive linguistics. Cognitive linguistics is a trend in linguistics that studies and describes the language in terms of cognitive mechanisms that underlie human mental activity.

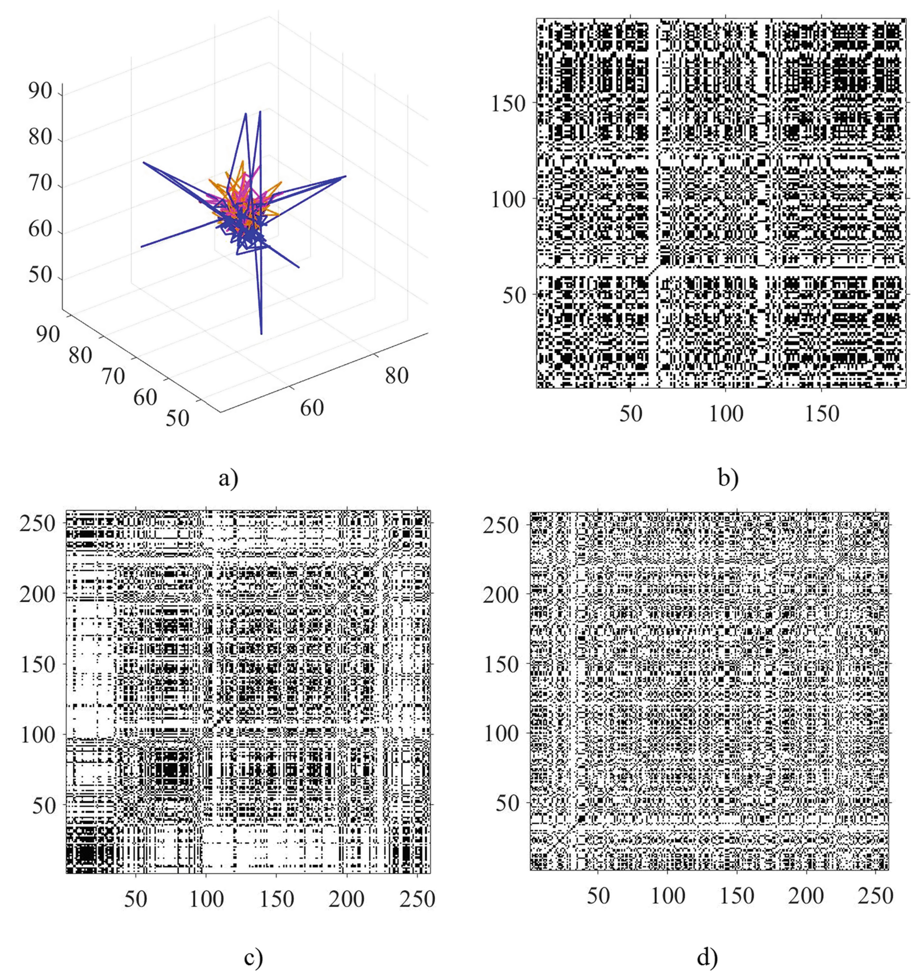

Fig. 6. (a) Phase portraits Alzheimer's patient (blue curve), a healthy patient (red curve) and calculated for a mixed (random) time series (brown curve) of the stride intervals; RP for Alzheimer's patient (b), healthy person (c) and a random (d) stride intervals time series (Color figure online)

The significance of language for cognition is extremely great, because it is through language that one can objectivize the mental activity, that is, verbalize it. On the other hand, learning a language is an indirect way of studying cognition, because cognitive and language structures are in certain proportions. One of the tools of the study of cognitive linguistics is the theory of complex networks. The nodes in such networks are elements of these complex systems, and the links between nodes – the interaction between the elements.

In the last decade, the structural properties of language, the texts of literary works and texts related to religious consciousness, as well as the organization of musical works and painting began to study and analyze from the point of view of the application of methods of the complex networks theory [28]. Relevant networks form a special, little-known category, which is called cognitive networks [29].

The term “cognitive networks” was proposed in studies on the research of the network structure of the natural language. Of particular interest is the study of cognitive networks for understanding the principles of brain function [30]. To date, research using the theory of complex networks in the study of the brain contributed to a deeper understanding of the general patterns of interconnection of different levels of its structural organization, and the involvement in these studies of the concept of cognitive networks will take into account some of the features of the human creative functions.

Let us consider the peculiarities of the application of the theory of complex systems in the problems of cognitive linguistics. The first step in applying the theory of complex networks to the analysis of the text is the presentation of this text as a set of nodes and links, the construction of a language network. There are different ways of interpreting nodes and connections, which leads, accordingly, to different representations of the network of languages. Along with the sequential, “linear” analysis of texts, the construction of networks, whose nodes are their elements – words or phrases, fragments of natural language, can reveal structural elements of the text, without which he loses his connectivity. In this case, the task of determining which of the important structural elements are also important information, such as determining the information structure of the text, is relevant. Such elements can also be used to identify still poorly defined components of the text, such as collocation, paraphrase unity, for example, when searching for similar snippets in different texts.

We know several approaches to constructing networks of texts, so-called word networks, and different ways of interpreting nodes and relationships, which leads, accordingly, to different types of representation of such networks. The nodes can be connected to each other, if the corresponding words are next to the text, belong to the same sentence or paragraph, connected syntactically or semantically. Preserving syntactic relationships between words leads to the image of a text in the form of a directed network, where the direction of link corresponds to the subordination of the word [31].

Thus, if a complex cognitive network (or another linguistic unit) (for example, a semantic graph) is created by a certain algorithm from a certain text, then it is expedient to use the topological and spectral measures [10] of such complex cognitive networks and even trace their dynamics within the framework of the algorithm of the moving window [36].

We will conduct a preliminary study of the frequency distribution of words in some well-known English-language literary works. The software developed by us analyzes the text of the work, creates a dictionary, each word of which has a unique code, in the process of analysis, the number of each of the words of the work is calculated, and also exports a text file of word codes that can be subjected to further analysis (frequency analysis, graph construction, its analysis).

At the same time, we will show only the fact that the constructed cognitive warnings convey the individual properties of complex networks in general.

In Fig. 7 on the logarithmic scale, the frequencies of words in the works are given. The linear trend corresponds to the distribution of α from the Zipf law [25]: $P(k) \propto ck^{-\alpha}$.

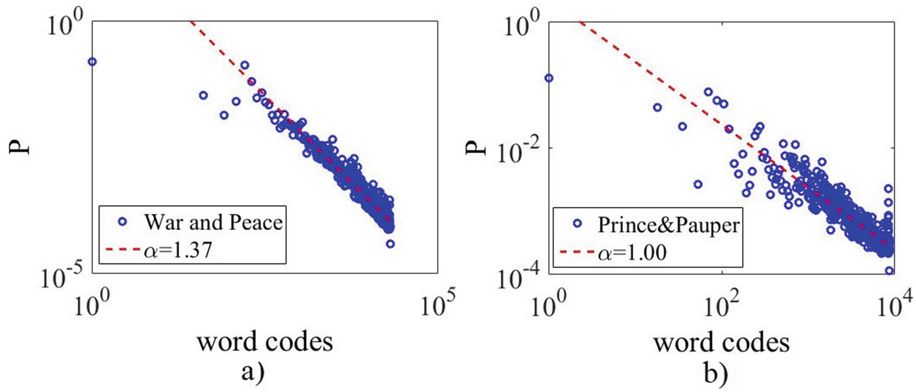

Fig. 7. The distribution of the frequency of words in the in novels of L. Tolstoy's "War and Peace", α = 1.37 and of Mark Twain "The Prince and the Pauper" by, α = 1.00

It is known that in order to comply with the Zipf law, the index should be approximately equal to units. Execution on this linguistic unit (text) of a rank distribution of the type of the Zipf law may be a sign of "correctness" (a good organization) of this text taken as a whole. In this case, Twain's novel is better organized, although this usually requires a wider and more profound analysis.

6.2 Transformation of a Time Series into a Network

In recent years, interesting algorithms for the transformation of time series into a network have been developed, which allows to extend the range of known characteristics of time series even to network ones. Recently, several approaches have been proposed to transform time sequences into complex network-like mappings. Three main classes can be distinguished. The first is based on the study of the convexity of successive values of the time series and is called visibility graph (VG) [46]. The second analyzes the mutual approximation of different segments of the time sequence and uses the technique of recurrent analysis [25, 26]. Finally, if the basis of forming the links of the elements of the graph is to put correlation relations between them, we obtain a correlation graph [45].

Recurrent networks are built from recurrence plots introduced above, the transition from which to the adjacency matrix is obvious [25, 26].

The algorithm of the VG is realized as follows. Take a time series $Y(t) = [y_1, y_2 ... y_n]$ of length N. Each point in the time series data can be considered as a vertex in an associative network, and the edge connects two vertices if two corresponding data points can "see" each other from the corresponding point of the time series (see Fig. 1).

Formally, two values y_a, of the series (at the time of time t_a) and y_b (at the time of time t_b) are connected, if for any other value (y_c, t_c), which is placed between them (that is, $t_a < t_c < t_b$), the condition is satisfied $y_c < y_a + (y_b - y_a)((t_c - t_a)/(t_b - t_a))$. Note that the visibility graph is always connected by definition and also is invariant under affine transformations, due to the mapping method.

An alternative (and much simpler) algorithm is the horizontal visibility graph (HVG) [45], in which a connection can be established between two data points a and b, if one can draw a horizontal line in the time series joining them that does not intersect any intermediate data by the following geometrical criterion: $y_a, y_b > y_c$ for all c such that $t_a < t_c < t_b$ (Fig. 8).

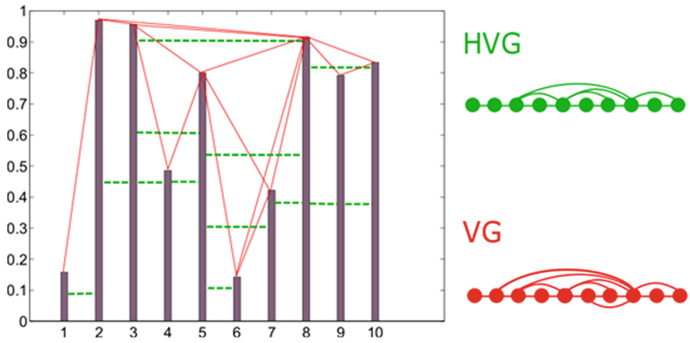

Fig. 8 The Illustration of constructing the visibility graph (red lines) and the horizontal visibility graph (green lines) (Color figure online)

6.3 Spectral and Topological Characteristics of Cognitive Networks

Spectral theory of graphs is based on algebraic invariants of a graph – its spectra. The spectrum of graph G is the set of eigenvalues of a matrix $S_p(G)$ corresponding to a given graph. The eigenvalues of the adjacency matrix A (the zeros of the polynomial $|\lambda I - A|$) and the spectrum of the matrix A (the set of eigenvalues) are called respectively their eigenvalues and the spectrum of graph G.

Another common type of graph spectrum is the spectrum of the Laplace matrix L. The Laplace matrix is used to calculate the tree graphs, as well as to obtain some important spectral characteristics of the graph. In particular, the positive eigenvalues λ_2 is called the index of algebraic connectivity of the graph. This value represents the "force" of the connectivity of the graph component and is used in the analysis of reliability and synchronization of the graph.

Important derivative characteristics are spectral gap, graph energy, spectral moments and spectral radius. The spectral gap is the difference between the largest and the next eigenvalues of the adjacency matrix and characterizes the rate of return of the system to the equilibrium state. The graph energy is the sum of the modules of the eigenvalues of the graph adjacency matrix. The spectral radius is the largest modulus of the eigenvalue of the adjacency matrix.

Among the topological measures one of the most important is the node degree k – the number of links attached to this node. For non-directed networks, the node's degree k_i is determined by the sum $k_i = \sum_j a_{ij}$, where the elements a_{ij} of the adjacency matrix.

To characterize the "linear size" of the network, useful concepts of mean $<l>$ and maximum $l_{\max}$ shortest paths. For a connected network of N nodes, the average path length is equal to $<l> \; = 2/(n(N-1))\sum_{i>j} l_{ij}$, where l_{ij} – the length of the shortest path between the nodes. The diameter of the connected graph is the maximum possible distance between its two vertices, while the minimum possible is the radius of the graph.

If the average length of the shortest path gives an idea of the whole network and is a global characteristic, the next parameter – the clustering coefficient – is a local value and characterizes a separate node. For a given node m, the clustering coefficient C_m is defined as the ratio of the existing number of links between its closest neighbors to the maximum possible number of such relationships $C_m = 2E_m/(k_m(k_m-1))$. Here $k_m(k_m-1)/2$ is the maximum number of links between the closest neighbors. The clustering coefficient of the entire network is defined as the average value C_m of all its nodes. The clustering coefficient shows how many of the nearest neighbors of the given node are also the closest neighbors to each other. He characterizes the tendency to form groups of interconnected nodes – clusters. For real-life networks, the high values of the clustering coefficient are high.

The important measure is the link density in the graph, which is defined as the number of links n_e, divided by the expression $n_n(n_n-1)/2$, where n_n is the number of nodes of the graph.

Figure 9 shows the results of calculations of two of the many spectral measures of complexity – spectral gap and maximum node degree $k_{\max}$. Results were obtained within the framework of the algorithm of a moving window. For this purpose, the part of the time series (window), for example, 200 words of a specified text fragment or which there were calculated measures of complexity, was selected, then, the window was displaced along the time series in a 25 day increment and the procedure repeated until all the studied series had exhausted.

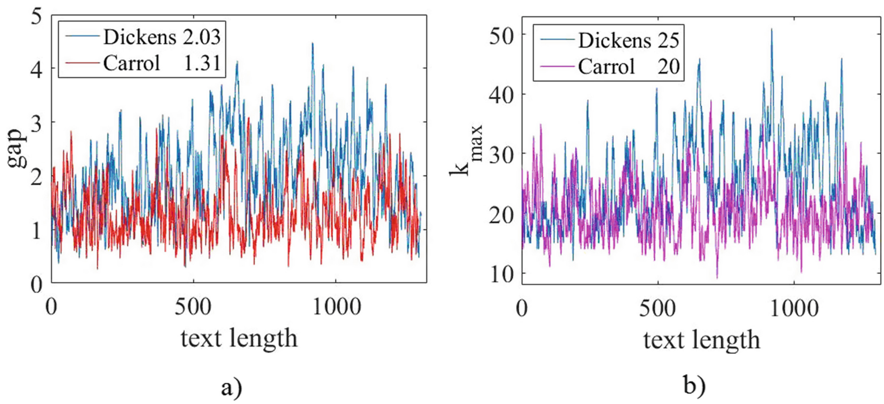

Fig. 9. Spectral measures of complexity: spectral gap (a) and maximum node degree (b)

We see that during the movement along the text, the measures of complexity change, although in general their average values indicated in the legend clearly separate the more complex text from the less complex. Moreover, the results are consistent with those obtained by other methods above.

7 Gamified Methods for Testing Cognitive Trajectories

Interaction with the virtual world during the game includes a wide range of cognitive processes and can affect the gamer's cognitive characteristics, such as attention, memory, spatial perception, thinking control and planning [47].

In this context, testing of cognitive functions during the gameplay seems promising. The player does not suspect that he is being tested, so his behavior and all functions remain natural.

Method of investigation thinking in computer game process was introduced by A.E. Kiv and others [48, 49]. A model of thinking space, where thinking processes considered as an accumulation of steps of thinking or elements of thinking was proposed. Thinking space (TS) contains discrete thinking elements, and each of them corresponds to given thinking step of a person in the process of his moving to the problem solution. Thinking steps may be divided in three groups: effective steps (ES), wrong steps (WS) and intermediate steps (IS). In this case three differential equations may be written, which describe the "kinetic" of each kind of steps. For example, for N_1 the equation is:

$$\dot{N}_1 = I_1 + a_1 N_1 + b_1 N_2^k - c_1 N_3, \tag{4}$$

where N_1, N_2 and N_3 is ES, IS and WS accordingly; t – time; I_1 – parameter of intuition; a_1 – coefficient the quality of information processing (parameter of logic); b_1 – coefficient of strategy thinking; k – "degree of psychological reaction"; c_1 – coefficient of critical thinking.

Described model TS has to be modified in each case accordingly to different fields of human activities taking into account the character of problems which must be solved. In this case cognitive thinking of human, namely memory, attention, coefficient the quality of information processing, coefficient of critical thinking are investigated.

Based on a mathematical model of thinking processes, two new computer games-tests have been developed and each game measuring some cognitive function. For example, game "Memory" as it measures indicators of memory and attention. Game "Path" measures indicators of logical and strategic thinking. Coefficient the quality of information processing measured in game "Path" by the formula: $a_1 = \dot{N}_1 / N_1$. Coefficient of critical thinking and memory measure in game "Memory" by the formula: $c_1 = \dot{N}_1 - a_1 N_3$.

Registration of indicators in game "Memory" is carried out in this way. After the player starts the game and moves from the main menu to the direct game starts the countdown, which will continue until the end of the game. The second indicator, measured during the game is quantity of wrong steps. Wrong step considered situation, when previous and next card do not match. In this case quantity of wrong steps increase by two.

The next indicator that is measured is quantity of effective steps. The step in this game is considered effective if the same cards are found in pairs.

For game “Path” indicators more complicated way. The basis for registering effective and false steps was the shortest path search algorithm. After building the game level, the shortest path to the key and from the key to the door is calculated. After the player starts the game and moves from the main menu to the direct game, a countdown starts, which will last until the end of the game. For the convenience of accounting for effective and wrong steps, the playing field was programmatically divided into cells. The move in this game is considered effective if the main character moves along the shortest path to the key, and then to the door.

Registration of false moves follows the same principle. If a player deviates from the shortest path by one grid cell, the wrong step is counted.

According to the task, it is necessary to distinguish several age groups, in this case, the study was conducted for the following age groups: 6–12 years, 13–18 years, 19–35 years, 36–50 years. For each of the age groups, 30 people were tested using both computer games. The results are presented in the Table 1.

Table 1. Experimental coefficient the quality of information processing (a_1) and coefficient of critical thinking (c_1) measurement results for age groups: 6–12 years, 13–18 years, 19–35 years, 36–50 years.

Age, years	t, sec	ES	WS	a_1	c_1
“Memory”					
6–12	78,23	18	20,96	0,0107	0,0058
13–18	50,7	18	12,9	0,0221	0,0131
19–35	55,5	18	13,03	0,024	0,0116
36–50	58,1	18	13,6	0,0227	0,0110
“Path”					
6–12	28,3	45,2	7,1	0,2249	0,0067
13–18	22,8	51,1	3,2	0,7003	0,0203
19–35	23,7	52,7	4	0,5559	0,0141
36–50	25	51,9	4,9	0,4236	0,0112

As can be seen copes with tasks most quickly age group 13–18 years. Further by a small margin is the group 19–35 years then 36–50 years group. The group 6–12 years takes the longest to complete the tasks. This is due to the fact that at this age only the cognitive abilities are formed and formed. Critical thinking criterion is most important in the age group of 13–18 years, then 19–35 years, 36–50 years and 6–12 years.

During the game “Memory” the coefficient of information processing is greatest in the age group of 19–35 years, then 36–50 years, 13–18 years and 6–12 years. Such results can be explained by the fact that the cognitive ability responsible for the quality of information processing reaches its greatest development at the age of 19 to 35, after which it begins to slow down.

Cognitive activity, as you know, includes the processes of learning and thinking. Both learning and thinking cannot be observed directly. Therefore, researchers use indirect evidence of the results of these processes. One of the simplest methods for fixing cognitive activity is through a maze.

The goal of this research was using the maze as a model for the quantitative description of cognitive trajectories. To reach the goals we created the computer application. The mazes solution process analysis is performed on the data collected by the developed application. The application allows users to solve predefined mazes or mazes that are generated using various algorithms (Kruskal's, Prim's, Wilson's, Eller's, backtracking generator, binary tree maze and others). Our computer program created the ideal maze.

In this study, we analyzed pre-defined mazes. Our goal of creating and solving by the subjects of predefined mazes was to have a sample of data that allows a more accurate and objective comparison of individual user approaches in solving mazes.

Mazes can be created or solved both automatically and non-automatically. The most interesting combination is an automated (computer) approach against a human solution and vice versa, since these two combinations show differences in the behavior of a person and a computer and their perception of a problem (generation or solution).

Solving path length is the number of nodes in the found path to the exit. Nodes completed is the total number of nodes visited (including the solution path). We estimated the maze complexity by the path length, which includes all the nodes visited.

The program generates mazes according to various algorithms. There is an option to select a mode randomly defined or determined in advance. There is a possibility of choosing the solution maze: random mouse, solution according to the algorithm, passing with a computer mouse. It is possible to build a maze graph and calculate the maze complexity measures.

Information about each attempt to solve the maze is recorded for further analysis. The following tools were used to visualize the data and present the statistical sample: line charts (for example, the time travel function visualization), mazes screenshots, maze density maps, the maze complexity calculation.

For testing we defined testing groups as in the previous case: 6–12 years old, 13–18 years old, 19–35 years old, 36–50 years old. For each age group, 30 people were tested using mazes (40×40) built on the basis algorithm: Prim's, Kruskal's or backtracking generator. The results are presented in Table 2.

After experiment, we came to the following conclusions. The results of our research agree with [50] where shown that the number deadlocks in non-Depth-first search (DFS) algorithms (Kruskal's and Prim's) is much higher than in DFS-based algorithms. Although DFS-based algorithms have relatively fewer dead ends, they tend to be longer (thus increasing the maze river factor). Our comparison of the mazes complexity built on the basis of various algorithms proves this statement. We found out – as expected – that the solution to mazes non-DFS based (Kruskal and Prim) took the shortest time (Fig. 10). Short dead ends are easier to detect and avoid, which speeds up the solution.

The maze complexity also depends on the location initial and final cell. Both cells may visually appear close to each other, but the path between them can be complex and branched. Random mazes in this program have initial and final cells located randomly, but always opposite to each other. Our results support the conclusions [50] - the maze

Table 2. The path length (L) and the complexity of cognitive trajectory (C_L) measurement results for age groups: 6–12 years, 13–18 years, 19–35 years, 36–50 years.

Age, years	t, sec	ES	WS	L	C_L
Maze 40 × 40 "Prim's algorithm"					
6–12	96	68	7	75	10^{-2}
13–18	80	68	2	70	10^{-2}
19–35	91	68	4	72	10^{-2}
36–50	93	68	5	73	10^{-2}
Short. Way	68	68	0	68	$20 \cdot 10^{-2}$
Rand. Way	25739	68	25671	25739	$4 \cdot 10^{-5}$
Maze 40 × 40 "Kruskal's algorithm"					
6–12	1003	187	345	532	$2 \cdot 10^{-3}$
13–18	404	187	16	203	$5 \cdot 10^{-3}$
19–35	406	187	18	205	$5 \cdot 10^{-3}$
36–50	688	187	100	387	$3 \cdot 10^{-3}$
Short. Way	187	187	0	187	$5 \cdot 10^{-3}$
Rand. Way	327382	187	327195	327382	$3 \cdot 10^{-6}$
Maze 40 × 40 "Backtracking generator"					
6–12	1500	545	865	1410	$7 \cdot 10^{-4}$
13–18	776	545	60	605	$2 \cdot 10^{-3}$
19–35	854	545	238	783	10^{-3}
36–50	1006	545	674	1219	10^{-3}
Short. Way	545	545	0	545	$2 \cdot 10^{-3}$
Rand. Way	2244111	545	2243566	2244111	$5 \cdot 10^{-7}$

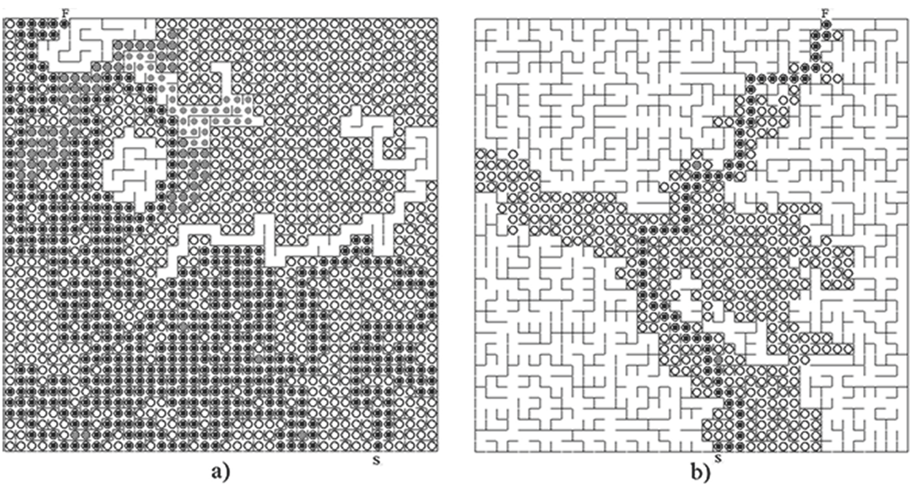

Fig. 10. Mazes solutions (maze generation according to various algorithms: a – backtracking generator (DFS-based)), b – Prim's, where gray circle – user trajectory, black rectangle – the shortest path, white circle – random mouse.

complexity is primarily affected by its size. We agree with [51] where defined the complexity and the maze difficulty while using the term “maze” to actually mean “graph of a maze”. Based on the constructed labyrinth graph, we can calculation the maze complexity measures.

Metrics that allow to evaluate the solution by a person is the completion time (t) and the path length (L). We calculate the complexity of the cognitive trajectory as $C_L \sim 1/L$.

8 Conclusions

The modeling of social and humanitarian systems, the core of which is a cognitive component, can be carried out within the framework of a synergetic paradigm, the modern point of which is the theory of complex networks. The considered separate methods of the theory of complex systems demonstrate the possibility of quantitative analysis of cognitive functions. In particular, the results obtained in this paper suggest that informational (mono and multiscale), recurrence, fractal and multifractal, as well as network measures of complexity can be used to quantify cognitive processes. Basic methods of converting a time series into a complex network, constructing adjacency matrix and Laplacian matrix and then solving eigenvalue problems are considered. The spectral measures of complexity calculated within the framework of the moving window algorithm make it possible to describe various faces of dynamic network complexity. This allows us to classify normal and anomalous phenomena, to offer a method for analyzing the cognitive trajectory over time, to model possible methods for its correction, taking into account external conditions.

The possibility of using computer games to analyze many cognitive trajectories is shown. One of the research options is modelling the steps of thinking in the process of solving a specific problem. An alternative way is reduced to a quantitative description of the actual trajectory – the processual characteristics of thinking.

In addition to the fundamental scientific significance – the understanding of the work of the human brain – work in this direction aims to overcome the general crisis of the educational system, the essence of which is the inadequacy of the goals, content, forms and methods of education new conditions.

Our developments may be applicable in the study of the cognitive activity of a group or one person. In future studies, we will continue to search for a solution to the problem of transforming cognitive trajectories into time series for a deep study of the cognitive processes mechanisms.

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