Comparative analysis of the cryptocurrency and the stock markets using the Random Matrix Theory

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Abstract. This article demonstrates the comparative possibility of constructing indicators of critical and crash phenomena in the volatile market of cryptocurrency and developed stock market. Then, combining the empirical cross-correlation matrix with the Random Matrix Theory, we mainly examine the statistical properties of cross-correlation coefficients, the evolution of the distribution of eigenvalues and corresponding eigenvectors in both markets using the daily returns of price time series. The result has indicated that the largest eigenvalue reflects a collective effect of the whole market, and is very sensitive to the crash phenomena. It has been shown that introduced the largest eigenvalue of the matrix of correlations can act like indicators-predictors of falls in both markets.

Keywords: stock market, cryptocurrency, Bitcoin, complex system, measures of complexity, crash, Random Matrix Theory, indicator-precursor.

1 Introduction

The instability of global financial systems with regard to normal and natural disturbances of the modern market and the presence of poorly foreseeable financial crashes indicate, first of all, the crisis of the methodology of modeling, forecasting and interpretation of modern socio-economic realities. The modern paradigm of synergetic is a complex paradigm associated with the possibility of direct numerical simulation of the processes of complex systems evolution [1; 11; 20; 19; 28].

Complex systems are systems consisting of a plurality of interacting agents possessing the ability to generate new qualities at the level of macroscopic collective behavior, the manifestation of which is the spontaneous formation of noticeable temporal, spatial, or functional structures. As simulation processes, the application of quantitative methods involves measurement procedures, where importance is given to complexity measures. I. Prigogine notes that the concepts of simplicity and complexity are relativized in the pluralism of the descriptions of languages, which also determines the plurality of approaches to the quantitative description of the complexity phenomenon [21]. Therefore, we will continue to study Prigogine's manifestations of the system complexity, using the current methods of quantitative analysis to determine the appropriate measures of complexity.

The key idea here is the hypothesis that the complexity of the system before the crashes and the actual periods of crashes must change. This should signal the corresponding degree of complexity if they are able to quantify certain patterns of a complex system. Significant advantage of the introduced measures is their dynamism, that is, the ability to monitor the change in time of the chosen measure and compare it with the corresponding dynamics of the output time series. This allowed us to compare the critical changes in the dynamics of the system, which is described by the time series, with the characteristic changes of concrete measures of complexity. It turned out that quantitative measures of complexity respond to critical changes in the dynamics of a complex system, which allows them to be used in the diagnostic process and prediction of future changes.

Cryptocurrency market is a complex, self-organized system, which in most cases can be considered either as a complex network of market agents, or as an integrated output signal of such a network – a time series, for example, prices of individual cryptocurrency. Thus the cryptocurrency prices exhibit such complex volatility characteristics as nonlinearity and uncertainty, which are difficult to forecast and any results obtained are uncertain. Therefore, cryptocurrency price prediction remains a huge challenge.

The stock market is one of the more developed economic segments of the financial market, highly capitalized and globalized with well-studied trends. Therefore, a comparative analysis of fragments of these markets is of obvious scientific and applied interest.

Unfortunately, the existing nowadays classical econometric [5; 8; 34] and modern methods of prediction of crisis phenomena based on machine learning methods [2; 3; 7; 10; 13; 14; 15; 25; 36] do not have sufficient accuracy and reliability of prediction.

Thus, lack of reliable models of prediction of time series for the time being will update the construction of at least indicators which warn against possible critical phenomena or trade changes etc. In our previous works, we constructed some indicators of crisis phenomena using the methods of nonlinear dynamics [276; 27] and the theory of complex networks [31]. Similar approaches, like the Random Matrix Theory, are developed in the framework of interdisciplinary science, called econophysics [17; 24]. This work is dedicated to the construction of such indicators – precursors based on the Random Matrix Theory.

The paper is structured as follows. Section 2 describes previous studies in these fields. Section 3 presents classification of crashes and critical events on the example of a key cryptocurrency Bitcoin during the entire period (16.07.2010 – 10.01.2019) and stock market by the example of the index S&P 500 during the entire period (17.03.1980

– 10.01.2019). In Section 4, new indicators of critical and crash phenomena are introduced using the Random Matrix Theory.

2 Analysis of previous studies

Random Matrix Theory (RMT) developed in this context the energy levels of complex nuclei, which the existing models failed to explain [9; 16; 18; 37]. Deviations from the universal predictions of RMT identify system specific, nonrandom properties of the system under consideration, providing clues about the underlying interactions.

Unlike most physical systems, where one relates correlations between subunits to basic interactions, the underlying “interactions” for the financial systems problem are not known. Here, we analyze cross correlations between financial agents (stocks, cryptocurrencies) by applying concepts and methods of RMT, developed in the context of complex quantum systems. Wherein the precise nature of the interactions between subunits are not known.

RMT has been applied extensively in studying multiple financial time series among which stock markets are central [12; 22; 23; 26; 35]. The first fundamental work in the field of modelling self-organization processes in the US stock market after the S&P 500 index using the RMT method was the study of [23]. Using extensive databases (every minute, hourly, daily), an analysis of their correlation properties is carried out. It is shown that there is a small part of the eigenvalues and eigenvectors containing important information about the structural and dynamic properties of the market. In particular, the authors of [23] found that the largest eigenvalue corresponds to an influence common to all stocks. Analysis of the remaining deviating eigenvectors shows distinct groups, whose identities correspond to conventionally identified business sectors. Finally, the authors discuss applications to the construction of portfolios of stocks that have a stable ratio of risk to return. Further studies, for example, [12; 22; 26; 35] developed the work of [23] and adapted the methodology to other financial objects.

As for the cryptocurrency market, the work here has just begun [3332; 33]. In the work [33], the classic scheme [23] was used for crypto assets with similar conclusions. The authors [32] analyzed the structure of the cryptocurrency market based on the correlation-based agglomerative hierarchical clustering and minimum spanning tree and examined the market structures. As a result, the authors demonstrated the leadership of the Bitcoin and Ethereum in the market, six homogeneous clusters composed of relatively less-traded cryptocurrencies, and transformation of the market structure after the announcement of regulations from various countries.

We will calculate the correlation properties of stock and crypto markets and compare the calculation results.

3 Data

At the moment, there are various research works on what crises and crashes are and how to classify such interruptions in the stock markets and market of cryptocurrencies.

We have created our classification of such leaps and falls, relying on Bitcoin time series during the entire period (16.07.2010 – 10.01.2019) of verifiable fixed daily values of the Bitcoin price (BTC) (https://finance.yahoo.com/cryptocurrencies). Critical US stock market events considered over time period 17.03.1980 – 10.01.2019 (https://finance.yahoo.com/quote /^GSPC?p=^GSPC).

Critical events are those falls that could go on for a long period of time, and at the same time, they were not caused by a bubble. The bubble is an increasing in the price of the cryptocurrencies that could be caused by certain speculative moments. Therefore, according to our classification of the event with number (1, 3–6, 9–11, 14, 15) are the crashes that are preceded by the bubbles, all the rest – critical events. More detailed information about crises, crashes and their classification in accordance with these definitions is given in the Table 1.

Table 1. List of Bitcoin major corrections ≥ 20% since June 2011

No	Name	Days in correction
1	07.06.2011 – 10.06.2011	4
2	15.01.2012 – 16.02.2012	33
3	15.08.2012 –18.08.2012	4
4	08.04.2013 –15.04.2013	8
5	04.12.2013 –18.12.2013	15
6	05.02.2014 – 25.02.2014	21
7	12.11.2014 – 14.01.2015	64
8	11.07.2015 – 23.08.2015	44
9	09.11.2015 – 11.11.2015	3
10	18.06.2016 – 21.06.2016	4
11	04.01.2017 – 11.01.2017	8
12	03.03.2017 – 24.03.2017	22
13	10.06.2017 – 15.07.2017	36
14	16.12.2017 – 22.12.2017	7
15	13.11.2018 – 26.11.2018	14

Accordingly, during this period in the Bitcoin market, many crashes and critical events shook it. Thus, considering them, we emphasize 15 periods on Bitcoin time series, whose falling we predict by our indicators, relying on normalized returns and volatility, where normalized returns are calculated as

$$g(t) = \ln X(t+\Delta t) - \ln X(t) \cong [X(t+\Delta t) - X(t)] / X(t), \tag{1}$$

and volatility as

$$V_T(t) = \frac{1}{n} \sum_{t'=t}^{t+n-1} |g(t')| \tag{2}$$

Besides, considering that $g(t)$ should be more than the $\pm 3\sigma$, where σ is a mean square deviation.

A similar procedure makes it possible to present a classification of crashes, crises and critical events for index S&P 500 with Table 2.

Table 2. List of S&P 500 index historical corrections ≥ 20% since October 1987

No	Name	Days in correction
1	02.10.1987 – 19.10.1987	12
2	17.07.1990 – 23.08.1990	28
3	01.10.1997 – 21.10.1997	15
4	17.08.1998 – 31.08.1998	11
5	14.08.2002 – 01.10.2002	34
6	16.10.2008 – 15.12.2008	42
7	09.08.2011 – 22.09.2011	32
8	18.08.2015 – 25.08.2015	6
9	29.12.2015 – 20.01.2016	16
10	03.12.2018 – 24.12.2018	15

Calculations were carried out within the framework of the algorithm of a moving window. For this purpose, the part of the time series (window), for which there were calculated measures of complexity, was selected, then the window was displaced along the time series in a one-day increment and the procedure repeated until all the studied series had exhausted. Further, comparing the dynamics of the actual time series and the corresponding measures of complexity, we can judge the characteristic changes in the dynamics of the behavior of complexity with changes in the time series. If this or that measure of complexity behaves in a definite way for all periods of crashes, for example, decreases or increases during the pre-crashes period, then it can serve as an indicator or precursor of such a crashes phenomenon.

Calculations of complexity measures were carried out both for the entire time series, and for a fragment of the time series localizing the crash. In the latter case, fragments of time series of the same length with fixed points of the onset of crashes or critical events were selected and the results of calculations of complexity measures were compared to verify the universality of the indicators.

In the Figure 1 output Bitcoin time series, normalized returns $g(t)$, and volatility $V_T(t)$ calculated for the window size 100 are presented.

From Figure 1 we can see that during periods of crashes and critical events normalized profitability g increases considerably in some cases beyond the limits $\pm 3\sigma$. This indicates about deviation from the normal law of distribution, the presence of the "heavy tails" in the distribution g, characteristic of abnormal phenomena in the market. At the same time volatility also grows.

We observe a similar picture for the index S&P 500 (Fig. 2). These characteristics serve as indicators of critical and collapse phenomena as they react only at the moment of the above mentioned phenomena and don't give an opportunity to identify the corresponding abnormal phenomena in advance. In contrast, the indicators described below respond to critical and crash phenomena in advance. It enables them to be used as indicators-precursors of such phenomena and in order to prevent them.

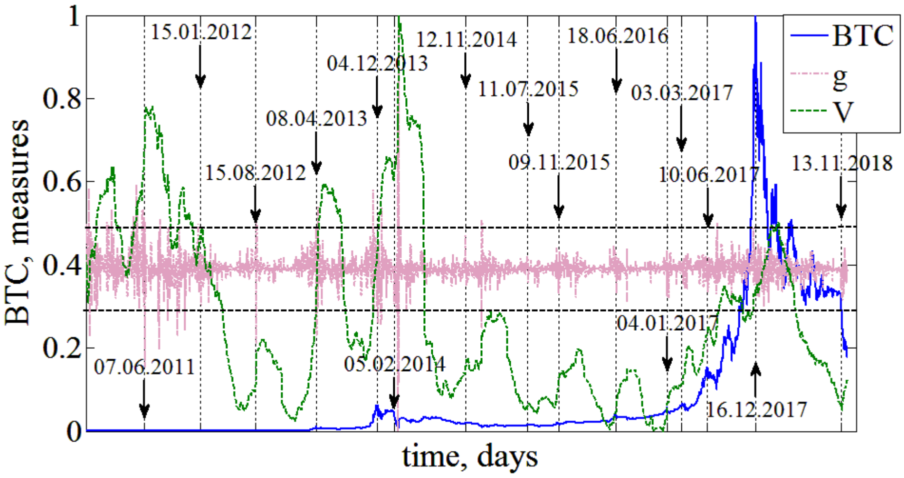


Fig. 1. The standardized dynamics, returns $g(t)$, and volatility $V_T(t)$ of BTC/USD daily values. Horizontal dotted lines indicate the $\pm 3\sigma$ borders. The arrows indicate the beginning of one of the crashes or the critical events.

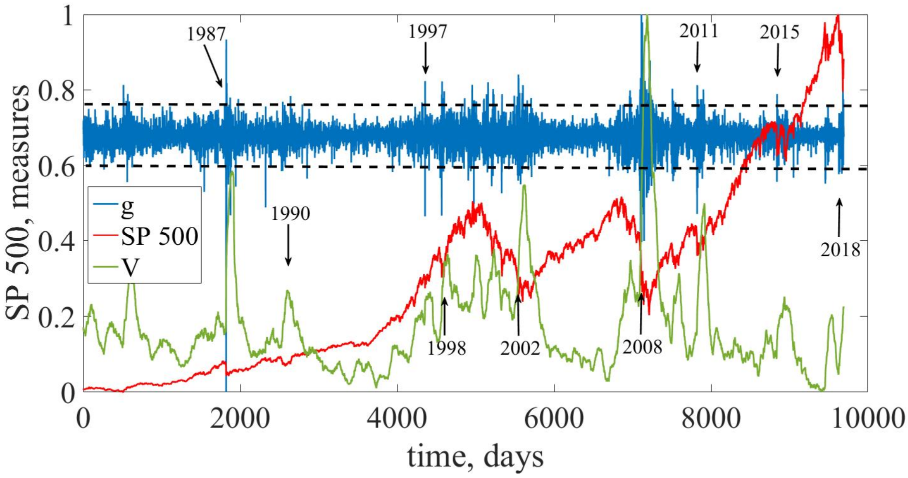


Fig. 2. The standardized dynamics, returns $g(t)$, and volatility $V_T(t)$ of S&P 500 daily values. Horizontal dotted lines indicate the $\pm 3\sigma$ borders. The arrows indicate the beginning of one of the crashes or the critical events.

4 Random Matrix Theory

Special databases have been prepared, consisting of cryptocurrency and S&P 500 index components time series for a certain period of time. The largest number of cryptocurrencies 1047 contained a base of 456 days from 31.12.2017 to 10.01.2019, and the smallest (24 cryptocurrencies) contained a base of 1567 days, respectively, from

04.08.2013 to 10.01.2019. For the logarithmic return (1) of the i cryptocurrencies or stock price we calculate the pairwise cross-correlation coefficients between any two returns time series. For the largest databases, a graphical representation of the pair correlation field is shown in the Figure 3a, c. For comparison, a map of correlations of randomly mixed time series of the same length is shown in Figure 3b, d.

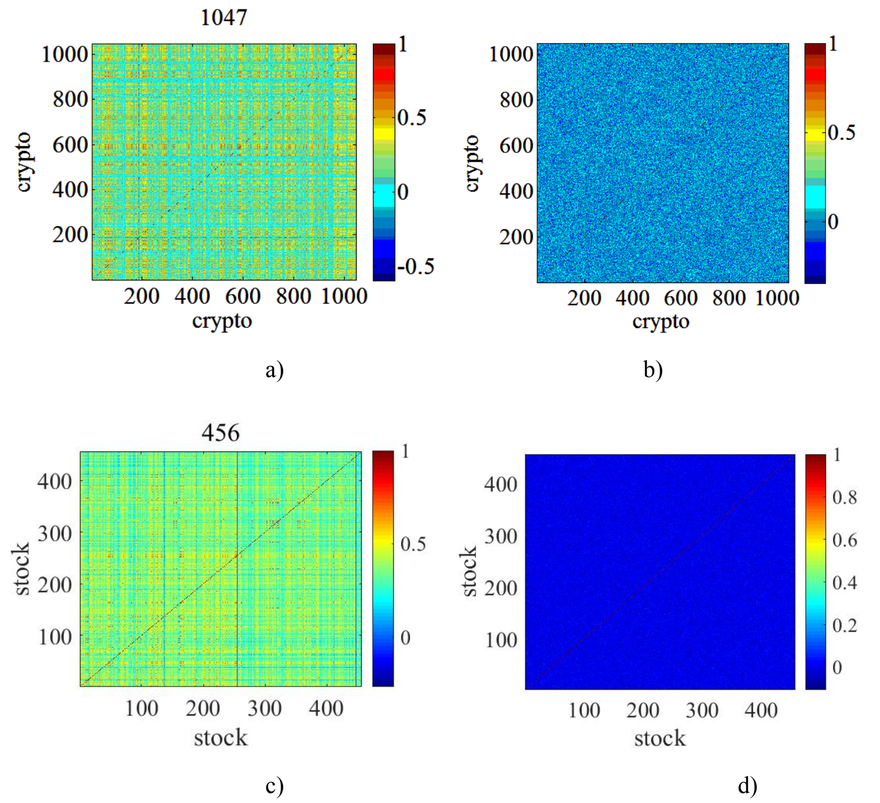


Fig. 3. Visualization of the field of correlations for the initial (a, c) and mixed (b, d) matrix cryptocurrency and S&P 500 index respectively. The largest number S&P 500 index components is 456.

For the correlation matrix C we can calculate its eigenvalues, $C = U\Lambda U^T$, where U denotes the eigenvectors, Λ is the eigenvalues of the correlation matrix, whose density $f_c(\lambda)$ is defined as follows, $f_c(\lambda) = (1/N)dn(\lambda)/d\lambda$. $n(\lambda)$ is the number of eigenvalues of C that are less than λ. In the limit $N \to \infty$, $T \to \infty$ and $Q = T/N \geq 1$ fixed, the probability density function $f_c(\lambda)$ of eigenvalues λ of the random correlation matrix M has a close form [18]:

$$f_c(\lambda) = \frac{Q}{2\pi\sigma^2} \frac{\sqrt{(\lambda_{\max} - \lambda)(\lambda - \lambda_{\min})}}{\lambda} \quad (3)$$

with $\lambda \in [\lambda_{\min}, \lambda_{\max}]$, where $\lambda_{\min}^{\max}$ is given by $\lambda_{\min}^{\max} = \sigma^2(1 + 1/Q \pm 2\sqrt{1/Q})$ and σ^2 is equal to the variance of the elements of matrix M [18].

We compute the eigenvalues of the correlation matrix C, $\lambda_{\max} = \lambda_1 > \lambda_2 > \cdots > \lambda_{15} = \lambda_{\min}$. The probability density functions (pdf) of paired correlation coefficients c_{ij} and eigenvalues λ_i for matrices of 132, 312, 458 cryptocurrencies and 163, 312, 456 S&P 500 index components are presented in Figure 4.

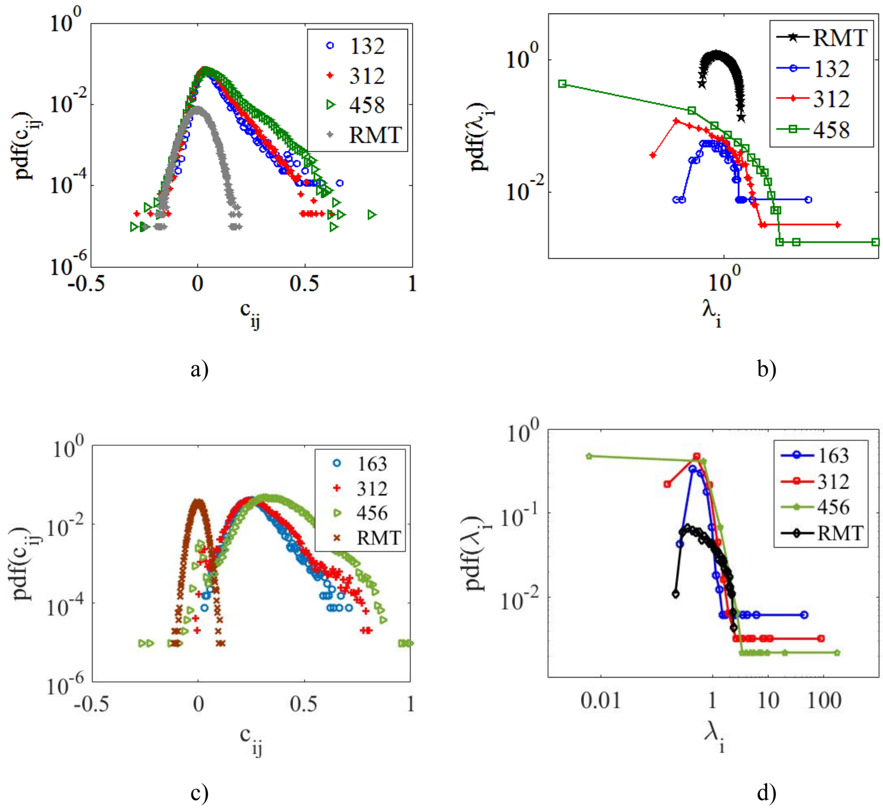


Fig. 4. Comparison of distributions of the pair correlation coefficients (a, c) and eigenvalues of the correlation matrix (b, d) with those for RMT for cryptocurrency market (a, b) and stock market (c, d).

Accordingly, for correlation matrices in the case of S&P 500 index, the dimensions of the matrices are as follows: 163, 312 and 456 (Fig. 4c, d). From Figures 4, it can be seen that the distribution functions for the paired correlation coefficients of the selected matrices differ significantly from the distribution function described by the RMT. It can be seen that the crypto market has a significantly correlated, self-organized system (Fig. 4a) and the difference from the RMT of the case, the correlation coefficients exceed the value of 0.6-0.8 on "thick tails". The distribution of the eigenvalues of the correlation matrix also differs markedly from the case of RMT. In our case, only one-third of its own values refer to the RMT region. However, the stock market is even more correlated. On it, the difference with RMT data is even more obvious.

The picture of correlations changes with changing market trends. This is clearly demonstrated by Figure 5, which shows the window distribution functions of pair correlation coefficients.

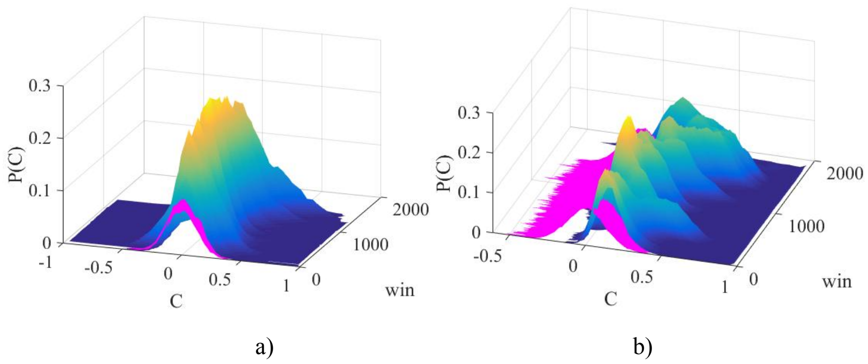


Fig. 5. Comparison of the window distributions of the pair correlation coefficients for cryptocurrencies (a) and S&P 500 index components (b).

And in this case, the stock market is more responsive to changes in market dynamics.

Eigenvectors correspond to the participation ratio PR and its inverse participation ratio IPR

$$I^k = \sum_{l=1}^{N} [u_l^k]^4, \tag{4}$$

where u_l^k, $l = 1, \ldots, N$ are the components of the eigenvector u^k (Fig. 6a). So PR indicates the number of eigenvector components that contribute significantly to that eigenvector. More specifically, a low IPR indicates that they contribute more equally. In contrast, a large IPR would imply that the factor is driven by the dynamics of a small number of assets. The irregularity of the influence of the eigenvalues of the correlation matrix is determined by the absorption ratio (AR), which is a cumulative risk measure

$$AR_n = \sum_{k=1}^{n} \lambda_k / \sum_{k=1}^{N} \lambda_k, \tag{5}$$

and indicates which part of the overall variation is described from the total number N of eigenvalues.

Figure 6 shows the results of IPR (a, b) calculations for both sets of matrices, as well as the results in the framework of the algorithm of a moving window, comparative calculations of the distribution function of eigenvalues (c, d) and IPR (e, f).

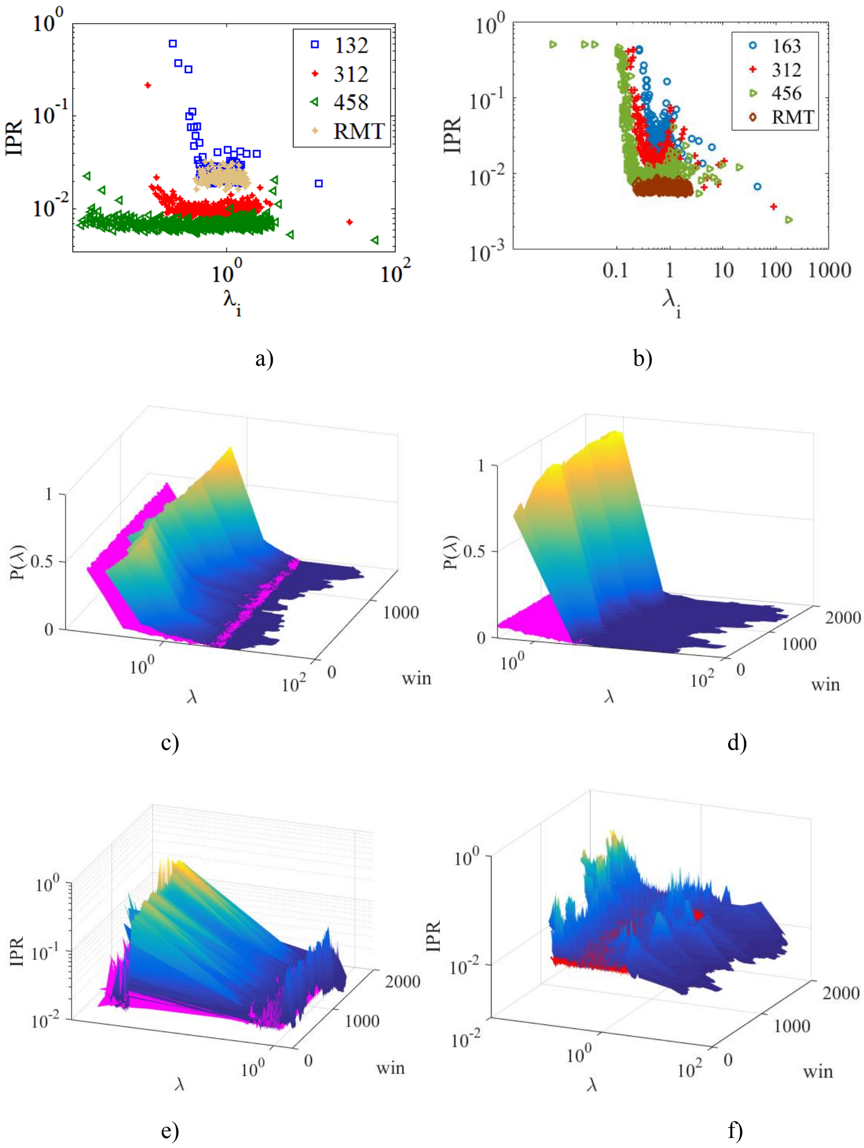
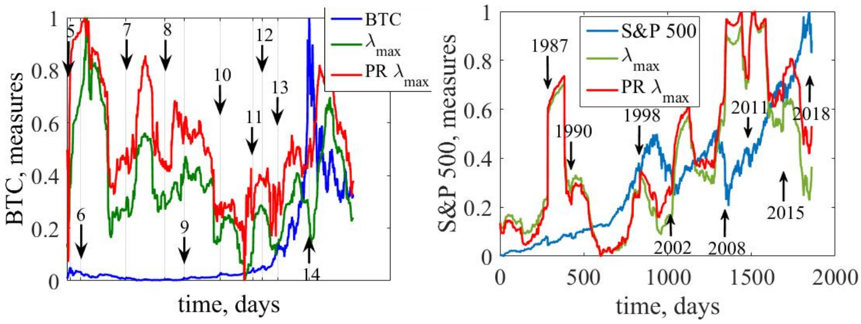


Fig. 6. Inverse participation ratio (a) and moving window dynamics of the eigenvalues distribution (b), IPR for the initial and mixed (or random) matrices (c).

The difference in dynamics is due to the peculiarities of non-random correlations between the time series of individual assets. Under the framework of RMT, if the eigenvalues of the real time series differ from the prediction of RMT, there must exists hidden economic information in those deviating eigenvalues. For cryptocurrencies markets, there are several deviating eigenvalues in which the largest eigenvalue λ_{max} reflects a collective effect of the whole market. As for PR the differences from RMT appear at large and small λ values and are similar to the Anderson quantum effect of localization [4]. Under crashes conditions, the states at the edges of the distributions of eigenvalues are delocalized, thus identifying the beginning of the crash. This is evidenced by the results presented in Figure 7.



a) b)

Fig. 7. Measures of complexity λ_{max} and its participation ratio. The numerics in the figure indicate the numbers of crashes and critical events in accordance with the Tables 1, 2.

We find that both λ_{max} and PR λ_{max} have large values for periods containing the market crashes and critical events. At the same time, their growth begins in the pre-crashes periods. At the same time, the stock market is more responsive to crisis phenomena.

5 Conclusions

Consequently, in this paper, we have shown that monitoring and prediction of possible critical changes on both the stock and cryptocurrency markets is of paramount importance. As it has been shown by us, the theory of complex systems has a powerful toolkit of methods and models for creating effective indicators-precursors of crashes and critical phenomena. In this paper, we have explored the possibility of using the Random Matrix Theory measures of complexity to detect dynamical changes in a complex time series. We have shown that the measures that have been used can indeed be effectively used to detect abnormal phenomena for the used time series data.

As it has been shown by us, the econophysics has a powerful toolkit of methods and models for creating effective indicators-precursors of crisis phenomena. We have shown that the largest eigenvalue λ_{max} may be effectively used to detect crisis phenomena for the cryptocurrencies time series. We have concluded though by emphasizing that the most attractive features of the λ_{max} and PR λ_{max} namely its conceptual simplicity and computational efficiency make it an excellent candidate for a fast, robust, and useful screener and detector of unusual patterns in complex time series.

Thus, the results of this study confirm the main provisions of the concept of early diagnosis of crisis phenomena by calculating various measures of complexity of financial systems [6; 27; 29; 30; 31].

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