Multifractal Properties of the Ukraine Stock Market

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Abstract

Recently the statistical characterizations of financial markets based on physics concepts and methods attract considerable attentions. We used two possible procedures of analyzing multifractal properties of a time series. The first one uses the continuous wavelet transform and extracts scaling exponents from the wavelet transform amplitudes over all scales. The second method is the multifractal version of the detrended fluctuation analysis method (MF-DFA). The multifractality of a time series we analysed by means of the difference of values singularity stregth $\alpha_{\max}$ and $\alpha_{\min}$ as a suitable way to characterise multifractality. Singularity spectrum calculated from daily returns using a sliding 1000 day time window in discrete steps of 1…10 days. We discovered that changes in the multifractal spectrum display distinctive pattern around significant "drawdowns". Finally, we discuss applications to the construction of crushes precursors at the financial markets.

Key words: Multifractal, stock market, singularity spectrum

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1. Introduction

Multivariate time series are detected and recorded both in experiments and in the monitoring of a wide number physical, biological and economic systems. A first instrument in the investigation of multivariate time series is the correlation matrix. The study of the properties of the correlation matrix has a direct relevance in the investigation of mesoscopic physical systems, high energy physics, investigation of microarray data in biological systems and econophysics [1].

Quantifying correlations between different stocks is a topic of interest not only for scientific reasons of understanding the economy as a complex dynamical system, but also for practical reasons such as asset allocation and portfolio risk estimation [2–5]. Unlike most physical systems, where one relates correlations between subunits to basic interactions, the underlying "interactions" for the stock market problem are not known.

Recent empirical and theoretical analysis have shown that this information can be detected by using a variety of methods. In this paper we used some of these methods based on Random Matrix Theory (RMT) [6], correlation based clustering, topological properties of correlation based graph and multifractal analyses [7] .

In this paper the different aspects of multiscale properties Ukraine stock market are discussed. The so-called financial stylized facts comprising, among others, the non-negligible fat tails of log-return distributions, volatility clustering and its long-time correlations, anomalous diffusion etc. counter that the financial dynamics is more complex than it is commonly assumed can also be inferred from a number of recently-published papers discovering and exploring the multifractal characteristics of data from the stock markets.

The concept of multifractality was developed in order to describe the scaling properties of singular measures and functions which exhibit the presence of various distinct scaling exponents in their different parts. Soon the related formalism was successfully applied to characterize empirical data in many distant fields like turbulence, earth science, genetics, physiology and, as already mentioned, in finance [1].

In the present paper we analyze data from the Ukraine stock market focusing on their fractal properties. We apply both on the multifractal detrended fluctuation analysis and on the which are a well-established methods of detecting scaling behaviour of signals.

2. Methods and data

Our analysis was performed on the time series of tick-by-tick recordings for daily returns of all stocks extracted from database time series of prices the First Stock Trade System (FSTS) index (www.kinto.com) for the ten-year period 1997-2006. For comparison similar analyses was conducted for of Russian stock market (Russian Trade System (RTS) – www.rts.com). The daily indices of the FSTS and RTS is the largest markets in Ukraine and Russia consisting of stocks from various sectors. Indices are basically an average of actively traded stocks, which are weighted according to their market value.

There are two possible procedures of analyzing multifractal properties of a time series. The first one uses the continuous wavelet transform and extracts scaling exponents from the wavelet transform amplitudes over all scales. This wavelet transform modulus maxima (WTMM) method [8] has been proposed as a mean field generalized multifractal formalism for fractal signals. We first obtain the wavelet coefficient at time t0 from the continuous wavelet transform defined as:

$$W_a(t_0) \equiv a^{-1} \sum_{t=1}^{N} p(t)\psi((t-t_0)/a)$$

where $p(t)$ is the analyzed time series, ψ is the analyzing wavelet function, a is the wavelet scale (i.e., time scale of the analysis), and N is the number of data points in the time series. For ψ we use the third derivative of the Gaussian, thus filtering out up to second order polynomial trends in the data. We then choose the modulus of the wavelet coefficients at each point t in the time series for a fixed wavelet scale a.

Next, we estimate the partition function

$$Z_q(a) \equiv \sum_i |W_a(t)|^q$$

where the sum is only over the maxima values of $|W_a(t)|$, and the powers q take on real values. By not summing over the entire set of wavelet transform coefficients along the time series at a given scale a but only over the wavelet transform modulus maxima, we focus on the fractal structure of the temporal organization of the singularities in the signal. We repeat the procedure for different values of the wavelet scale a to estimate the scaling behavior

$$Z_q(a) \propto a^{\tau(q)}.$$

In analogy with what occurs in scale-free physical systems, in which phenomena controlled by the same mechanism over multiple time scales are characterized by scale-independent measures, we assume that the scale-independent measures, $\tau(q)$, depend only on the underlying mechanism controlling the system.

Altrnatively procedure is the multifractal version of the detrended fluctuation analysis method (MF-DFA) [9]. Given the time series of price values $p_s(t_s(i)), i = 1, ..., N_s$ of a stock ss recorded at the discrete transaction moments $t_s(i)$, one may consider logarithmic price increments (or returns) $g_s(i) = \ln(p_s(i+1)) - \ln(p_s(i))$. For the time series of the log-price increments $G_s \equiv \{g_s(i)\}$ one needs to estimate the signal profile

$$Y(i) = \sum_{k=1}^{i} (g_s(k) - < g_s >), i = 1, ..., N_s \tag{1}$$

where ¡. . . ¿ denotes the mean of G_s. $Y(i)$ is divided into M_s disjoint segments of length n starting from the beginning of G_s. For each segment $\nu, \nu = 1, ..., M_s$, the local trend is to be calculated by least-squares fitting the polynomial $P_\nu^{(l)}$ of order l to the data, and then the variance

$$F^2(\nu, n) = \frac{1}{n} \sum_{j=1}^{n} \left\{ Y\left[(\nu - 1)n + j\right] - P_\nu^{(l)}(j) \right\}^2. \tag{2}$$

In order to avoid neglecting data points at the end of G_s which do not fall into any of the segments, the same as above is repeated for M_s segments starting from the end of G_s. The polynomial order l can be equal to 1 (DFA1), 2 (DFA2), etc. The variances (2) have to be averaged over all the segments ν and finally one gets the qth order fluctuation function

$$F_q(n) = \left\{ \frac{1}{2M_s} \sum_{\nu=1}^{2M_s} \left[F^2(\nu, n) \right]^{q/2} \right\}^{1/q}, q \in R. \tag{3}$$

In order to determine the dependence of F_q on n, the function $F_q(n)$ has to be calculated for many different segments of lengths n.

If the analyzed signal develops fractal properties, the fluctuation function reveals power-law scaling

$$F_q(n) \propto n^{\tau(q)} \tag{4}$$

for large n. The family of the scaling exponents $\tau(q)$ can be then obtained by observing the slope of log-log plots of F_q vs. n. $\tau(q)$ can be considered as a generalization of the Hurst exponent H with the equivalence $H \equiv \tau(2)$. Now the distinction between monofractal and multifractal signals can be performed: if $\tau(q) = H$ for all q, then the signal under study is monofractal; it is multifractal otherwise. By the procedure, $\tau(q), q < 0$ describe the scaling properties of small fluctuations in the time series, while the large ones correspond to $\tau(q), q > 0$. It also holds that $\tau(q)$ is a decreasing function of q.

By knowing the spectrum of the generalized Hurst exponents, one can calculate the singularity strength α and the singularity spectrum $f(\alpha)$ using the following relations:

$$\alpha = \tau(q) + q\tau'(q), \quad f(\alpha) = q\,[\alpha - \tau(q)] + 1, \tag{5}$$

where $\tau'(q)$ stands for the derivative of $\tau(q)$ with respect to q.

3. Results

Figure 1 shows the First Stock Trade System index and Russia Trade System index, from 1997 to 2006.

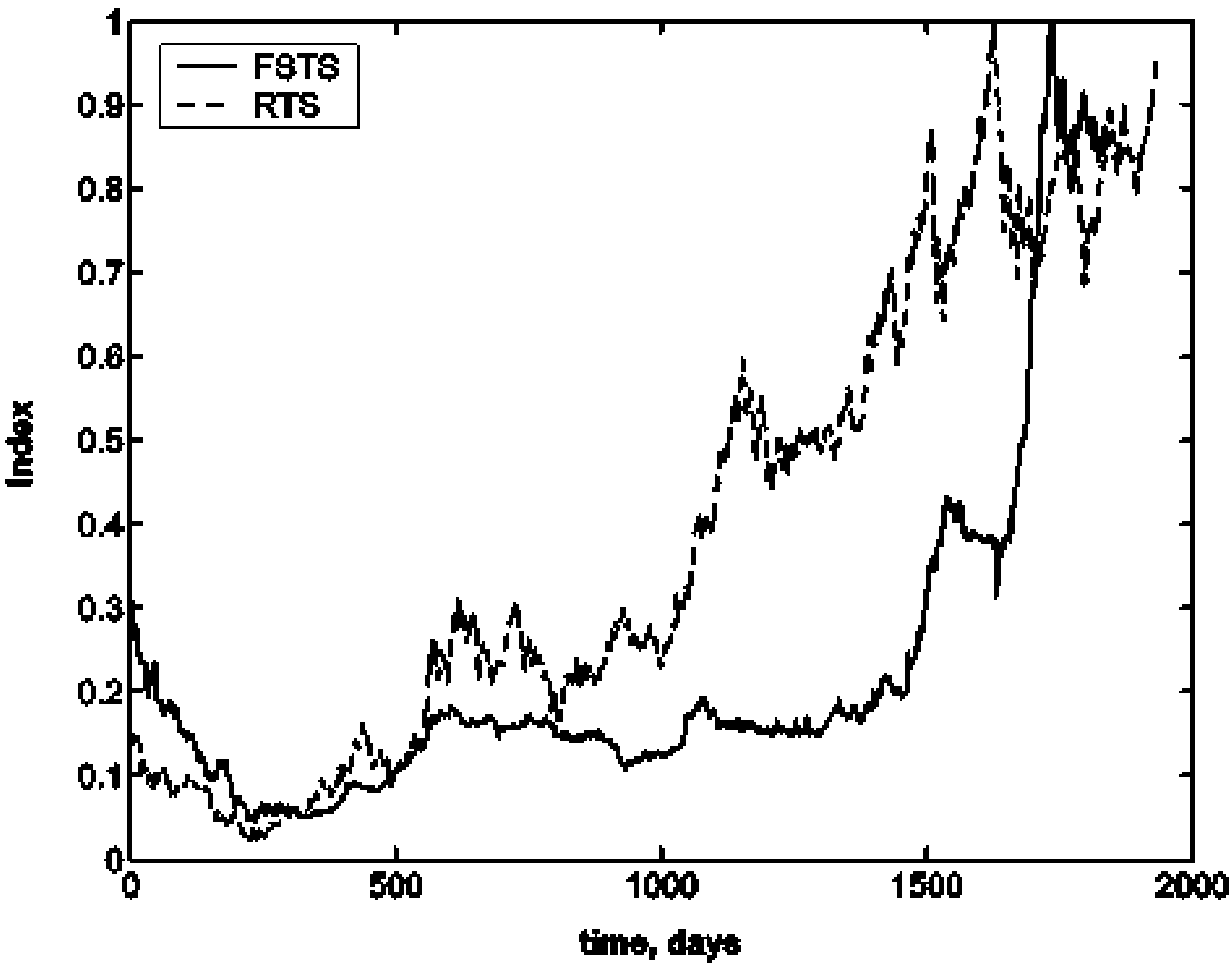

Fig. 1. FSTS and RTS indexes plotted for all the days reported in the period 1997 to 2006

Our calculations indicate that the time series of price increments for all companies can be of the multifractal nature (see fig.2). Consistently with the log-log plots, the highest nonlinearity of the spectrum and the strongest multifractality are attributes of RTS (Russia), and the smallest nonlinearity and the weakest multifractal character correspond to FSTS (Ukraine). The multifractal nature of the data can also be expressed in a different manner, i.e. by plotting the singularity spectra $f(\alpha)$(Eq. 5). It is a more plausible method because here one can easily assess the variety of scaling behaviour in the data. The evolution of $f(\alpha)$ is analyzed by using a moving window of length 1000 data point shifted by 1 point. Such a window ensures that we obtain statistically reliable results.
The maxima of $f(\alpha)$are typically placed in a close vicinity of $\alpha = 0.5$ indicating no significant autocorrelations exist. The multifractal character of price fluctuations can originate from the existence of the long-range correlations in the price increments (via volatility) as well as from their non-Gaussian distributions.
The richest multifractality (the widest $f(\alpha)$curve $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$) is visible for RTS, the poorest one for FSTS (fig. 3).

4. Conclusions

We study the multifractal properties of Ukraine Stock Market. We show that the signals for the price increments exhibit the characteristics that can be interpreted in terms of multifractality. Its degree expressed by the widths $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$) of the singularity spectra $f(\alpha)$different for the Russian and Ukrainian stock markets. Greater value for the Russian market related to more effective functioning. In this case the width of singularity spectrum can serve as the measure of efficiency of functioning of the complex system.

References

[1] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez , D.-U. Hwang, Physics Reports 424 (2006) 175 – 308

[2] B.B. Mandelbrot and J.W. van Ness, SIAM Review 10 (1968) 422-437

[3] V. Plerou, P. Gopikrishnan, L.A.N. Amaral, M. Meyer and H.E. Stanley,

Phys. Rev. E 60 (1999) 6519-6529

[4] P. Gopikrishnan, V. Plerou, L.A.N. Amaral, M. Meyer and H.E. Stanley,

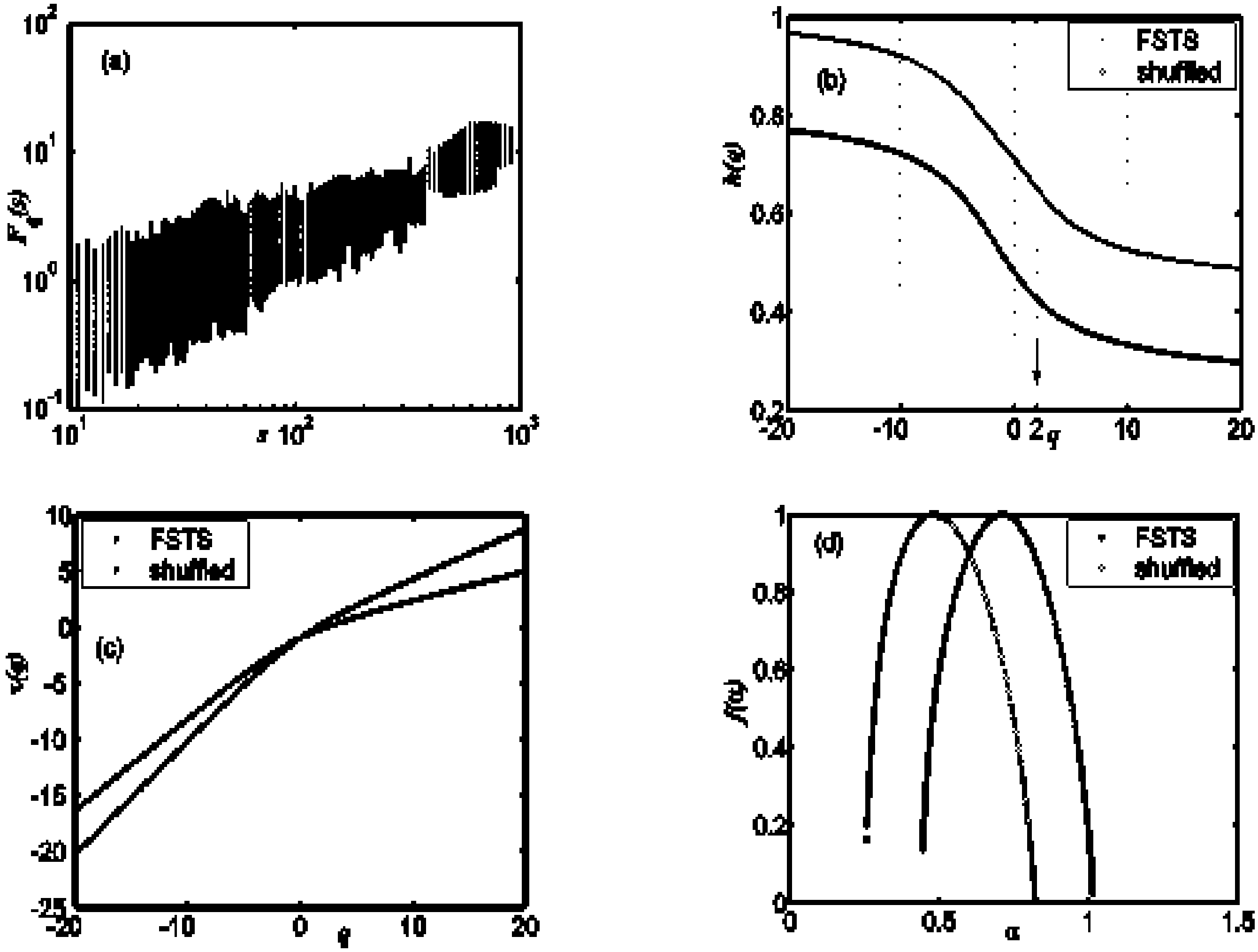

Fig. 2. (a) Log-log plots of the q-th order fluctuation F_q for time series of price increments as a function of segment size n for different values of q between -20 (bottom line) and 20 (top line). (b)Scaling regions allow one to estimate $h(q)$ according to Eq. 4. (c) Multifractal spectra for price increments; a nonlinear behaviour of $\tau(q)$ can be considered a manifestation of multiscaling. (d) singularity spectra $f(\alpha)$ according to Eq. 5. Open circles corresponds to shuffled data

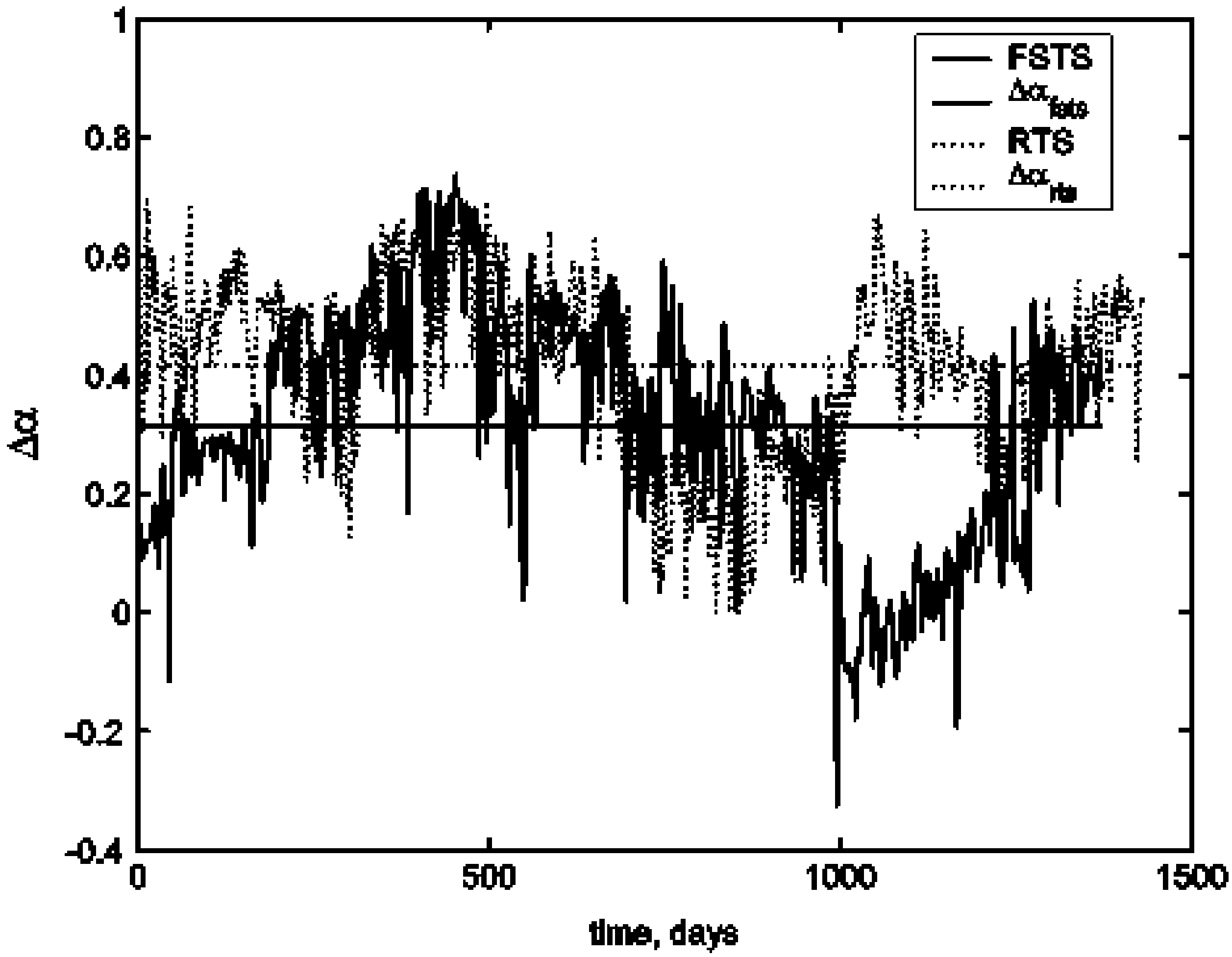

Fig. 3. Comparison of the widths of the $f(\alpha)$ spectra Russia and Ukraine stock markets

Phys. Rev. E 60 (1999) 5305-5316

[5] V. Plerou, P. Gopikrishnan, L.A.N. Amaral, X. Gabaix and H.E. Stanley,

Phys. Rev. E 62 (2000) R3023-R3026

[6] V. Plerou, P. Gopikrishnan, B.Rosenow, L. A. N. Amaral, T.Guhr, H. E. Stanley Phys.Rev E, 65 (2002) 066126

[7] S. Drozdz, J. Kwapien, F. Gruemmer, F. Ruf and J. Speth, Acta Phys. Pol. B 34 (2003) 4293-4305

[8] J. F.Muzy, E. Bacry, and A. Arneodo, Phys. Rev. Lett. **67** (1991), 3515

[9] J.W. Kantelhardt, S.A. Zschiegner, E. Koscielny-Bunde, A. Bunde, Sh. Havlin and H.E. Stanley, Physica A 316 (2002) 87-114