Enhancing mathematics education with GeoGebra and augmented reality

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Abstract

This article explores the potential of integrating GeoGebra software with augmented reality (AR) technology to enhance mathematics education. Recent studies have shown promising results in using GeoGebra AR to promote spatial skills, conceptual understanding, and engagement in fields like geometry, algebra, and calculus. Examples of GeoGebra AR applications in educational settings are provided, spanning both secondary and higher education. Recommendations for further research and implementation are discussed. The integration of GeoGebra AR into mathematics curricula, coupled with appropriate teacher training and support, has the potential to revolutionize how students encounter and engage with mathematical concepts.

Keywords

augmented reality, GeoGebra 3D, mathematics education

1. Introduction

Mathematics education constantly evolves to incorporate new technologies that can enhance teaching and learning [1]. GeoGebra, a powerful dynamic mathematics software, has gained widespread adoption in recent years [2]. GeoGebra allows for interactive exploration of mathematical concepts through dynamic visualizations. More recently, GeoGebra has ventured into the realm of augmented reality (AR) with its GeoGebra AR applications. AR overlays virtual information onto the real world, creating immersive experiences that merge the physical and digital [3, 4, 5]. The combination of GeoGebra and AR presents exciting opportunities for engaging students with mathematical ideas in new ways.

The purpose of this article is to examine the current state of research on GeoGebra AR in mathematics education and to provide examples of its applications across various mathematical domains and educational levels. We also offer examples and recommendations for successful integration into mathematics curricula.

2. GeoGebra AR in secondary mathematics education

Several studies have investigated the impact of GeoGebra AR on mathematics learning at the secondary level. Del Cerro Velázquez and Méndez [6] found that integrating GeoGebra AR into a secondary geometry curriculum improved students' spatial intelligence and academic performance compared to traditional instruction. In a quasi-experimental study, Guntur and Setyaningrum [7] reported that secondary students who used an AR module with GeoGebra had significantly higher spatial and problem-solving skills than those in a control group.

GeoGebra AR has been applied to various mathematical topics in secondary education:

- In **geometry**, GeoGebra AR allows students to visualize and manipulate 3D shapes, explore cross-sections, and discover geometric properties [8, 9]. Lainufar et al. [10] described a project-based

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geometry unit where secondary students used GeoGebra AR, reporting high levels of acceptance and engagement.

- For **algebra**, graphing functions and surfaces in AR provides an intuitive way to understand their behavior and characteristics [11].
- GeoGebra AR can support the learning of **trigonometric functions** by allowing students to explore graphs and transformations in an immersive 3D environment [12].

Mailizar and Johar [13] used the Technology Acceptance Model (TAM) to examine factors affecting secondary students' intention to use GeoGebra AR in a project-based geometry unit. They found perceived usefulness to be the strongest predictor of behavioral intention, highlighting the importance of demonstrating the practical value of AR tools.

These studies provide evidence for the effectiveness of GeoGebra AR in promoting spatial abilities, conceptual understanding, and engagement in secondary mathematics. However, more research is needed to understand its long-term effects on achievement and attitudes.

3. GeoGebra AR in higher education and STEAM

At the tertiary level, GeoGebra AR has been explored in various mathematics courses and in interdisciplinary STEAM (Science, Technology, Engineering, Arts, and Mathematics) contexts [14].

In a calculus course, Caridade [15] used GeoGebra AR to visualize 3D graphs and solids of revolution, reporting increased student engagement and understanding. Similarly, Cheong et al. [16] found GeoGebra AR to be an effective tool for teaching multivariable calculus, particularly in enhancing visualization of 3D surfaces.

GeoGebra AR has also been applied in engineering mathematics. Iparraguirre-Villanueva et al. [17] integrated GeoGebra AR into a spatial geometry course for engineering students, finding improved performance and positive attitudes compared to a control group.

In STEAM education, GeoGebra AR allows for the integration of mathematical modeling with arts, culture, and history. El Bedewy et al. [18, 19, 20] describe STEAM practices where participants model historical architecture using GeoGebra AR and 3D printing, connecting mathematical concepts to cultural heritage. These interdisciplinary approaches can make mathematics more engaging and meaningful for students.

Cahyono and Lavicza [21] explored how STEAM projects involving GeoGebra AR and 3D printing can be designed for cross-country math trails. Pre-service teachers created AR models of local landmarks and linked them to school mathematics topics, allowing users in different countries to engage with the math trail.

These examples demonstrate the versatility of GeoGebra AR in higher education, from traditional mathematics courses to innovative STEAM projects. More research is needed on the affordances and challenges of implementing GeoGebra AR in these diverse contexts.

4. Augmented reality and mathematical thinking

Augmented reality technology has the potential to support and enhance key aspects of mathematical thinking. Nam et al. [22] argue that GeoGebra AR can serve as a tool to connect abstract mathematical knowledge to real-world situations, using the example of learning about parallel lines and planes in space. By manipulating AR models, students can test hypotheses, refine their ideas, and construct new knowledge.

Bagossi et al. [23] developed an analytical tool called the Timeline to investigate the relationship between student-teacher-artifact interactions and meaning-making when using GeoGebra AR. Their analysis revealed how different phases of AR-supported activities contributed to students' mathematical development.

In a study on second-order covariation, Bagossi and Swidan [24] compared students' reasoning in GeoGebra and AR environments. While both environments supported covariation, the AR group showed fuller emergence of the concept after physical experimentation, highlighting the role of embodied interactions.

Walkington et al. [25] examined the new kinds of embodied interactions that arise in an AR-based version of GeoGebra, using the Microsoft HoloLens 2. They found that the immersive 3D environment afforded novel interactions related to perspective, scale, and depth, with implications for the design of future AR math tools.

These studies provide initial insights into how AR technology, and GeoGebra AR specifically, can shape students' mathematical thinking and reasoning. More research is needed to unpack the cognitive processes involved and to design AR-based tasks that optimize learning.

5. Case study: stereometry teaching

5.1. Tasks on combinations of polyhedron and solids of revolution

Consider the way it is possible to inscribe a sphere into the right rectangular pyramid via the use of 3D Geometry. In order to construct the base of the pyramid, it is necessary to use the Right Polygon tool, by pointing two points on the 3D canvas – adjacent vertexes of the base, and indicating that the right polygon has 4 vertexes. Then one should construct the diagonals of the square (the Segment tool) and define the center (Intersection point). Then through the center of the square, which is also the center of the circle inscribed in the square, one draws a straight line perpendicular to the plane of the square. On this straight line, one chooses an arbitrary point (Point on the object) and constructs a polyhedron (Pyramid). The perpendicular to the plane of the square straight line is the geometric location of points, equidistant from the sides of the base of the right pyramid.

To determine the position of the center inscribed sphere in the pyramid, one constructs a geometric location of points that are equidistant from the edges of the dihedral angle at the base of the pyramid. Since there is no construction of the bisector plane in the GeoGebra tools, it is necessary to construct a linear angle of the dihedral angle at the base and then bisector of the very angle. The plane passing through the vertex of the pyramid perpendicular to the edge of the base is built (Plane through the point perpendicular to the straight line; Intersection point). Instead of a plane, it is possible to draw a straight line from the vertex of the pyramid perpendicular to the edge of the base (straight, perpendicular to straight). Next, one should find the intersection point of the constructed plane / perpendicular with the edge of the base (Intersection point of the straight line and the plane / Intersection point of two straight lines). Then one builds the bisector of the obtained linear angle.

The point of its intersection with the perpendicular to the base of the pyramid, drawn from the top of the pyramid, will determine the center of the inscribed sphere (Point of intersection). Finally, one constructs the inscribed sphere (Sphere outside the center and radius), specifying in sequence the center of the sphere and the point of intersection of the diagonals of the square [26].

For better understanding and mastering of the algorithm the construction of the sphere inscribed around the pyramid the students setting of the canvas are adjusted to be able to show the step-by-step procedure of the construction.

With AR, the students can understand the basic concepts of 3D geometrical shapes, their relationships and ways to construct the 3D shapes and the objects in 3D space. Importantly, AR can provide a dynamic visualization of 3D structures of geometrical shapes. This feature helps the students to understand a comprehensive background of 3D geometrical shapes and improve the abilities of geometrical structures. Moreover, the hand gesture based interactions furnish an intuitive and convenient way for the students to directly control and interact with geometrical shapes in 3D space.

GeoGebra Augmented Reality application allows you to transfer the constructed figure into the space of the room (figure 1. Having built a figure, we press the "AR" button. Next, you need to use the camera to select the environment in which we plan to move the object. For example, on the table. By tapping on the screen, the figure will be transferred to the real world [27] where it can be explored. The phone camera will serve our eyes. Immersing the phone in a virtual figure we will see it from the inside, we can bypass it, also the application allows you to resize, color [28].

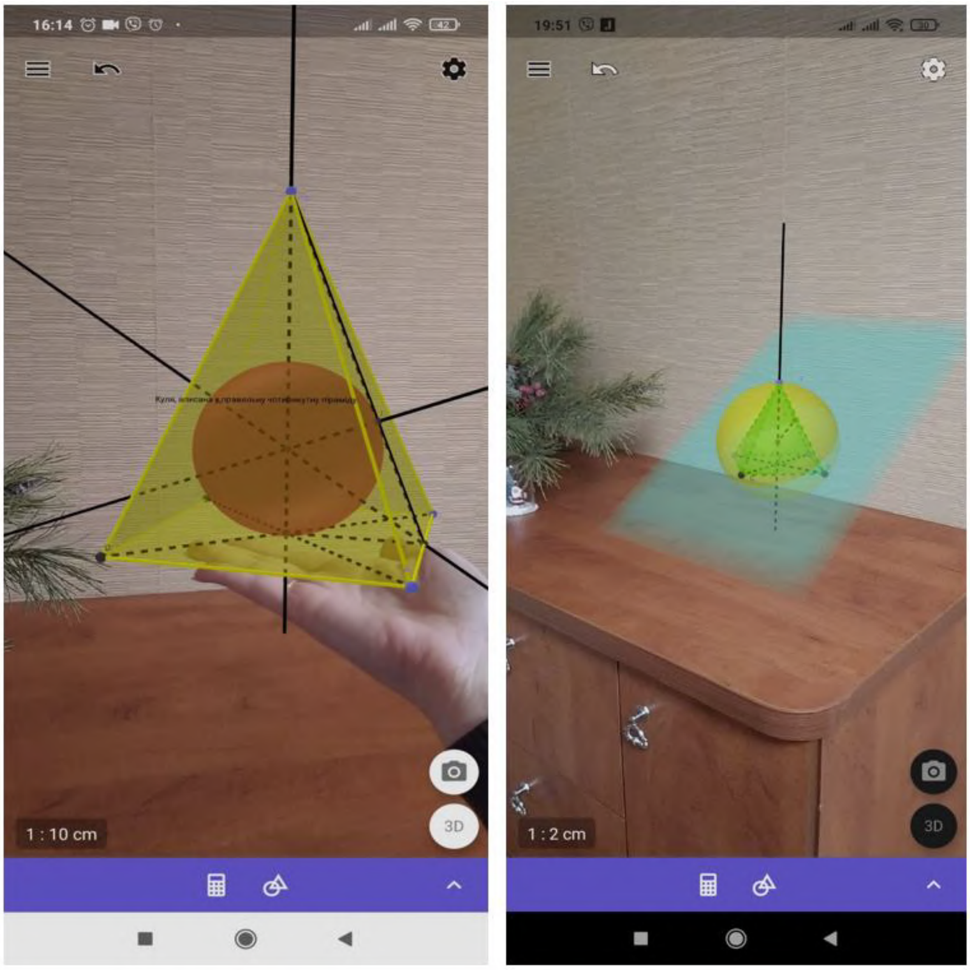

Figure 1: GeoGebra AR demos.

With the experiences of interacting with the 3D shapes using their own hand gestures, the students can improve their own awareness of the relationships of the 3D shapes and easily remember or retain the knowledge about the 3D shapes.

5.2. Stereometric problems of applied content

Geometry is an abstract science, often taught without proper implementation of its applied orientation. This leads to the fact that a significant part of students do not feel the need to study this subject, because they do not see the possibility of using the acquired geometric knowledge, in particular in stereometry, in the future. And so there is a need to connect stereometric problems with life. We propose to consider two problems of applied direction, for the solution of which we consider it expedient to involve the GeoGebra 3D application. We offered these tasks to students of the State University of Economics and Technologies.

Problem 1. What percentage of wood goes to waste when made of wooden logs, 5 m long and 20 cm and 15 cm in diameter, beams with a rectangular cross-section of the maximum cross-sectional area?

Problem 2. Calculate the volume of the largest beam with a base in the shape of a rectangle, which can be carved from a log of cylindrical shape. The length of the log is 5 m and the thickness is 20 cm. What percentage of wood will go to waste?

Using these tasks, we conducted research on the basis of two parallel groups majoring in “Finance and Credit”. 18 students of the experimental group (EG) and 17 students of the control group (CG) took part in the study. In the experimental group, the task was to solve problems based on a dynamic figure, the control group solved the same problems, but with the help of static.

The proposed questionnaire consisted of several questions that students answered while solving problems.

1. What figures will we work with? Positive answer: CG – 6 students (35%), EG – 7 students (39%).
2. How are the figures relative to each other? Positive answer: CG – 7 students (41%), EG – 7 students (39%).
3. What shape should be the cross section of the beam to maximize its size? The volume of the beam will be the largest if the cross section of the beam is square. It is not necessary to compose a function and study it to the extreme, it is enough to use the formula to calculate the area of a quadrilateral inscribed in a circle. Positive answer: CG – 4 students (24%), EG – 3 students (17%). In the second stage, the CG group was shown a figure for the problem on paper, the EG group was shown a figure in GeoGebra (figure 2) and considered in dynamics.
4. After that, the groups were asked the last question about the cross-section again, the statistics of positive answers improved: CG – 6 students (35%), EG – 9 students (50%).
5. What is meant by waste from the manufacture of logs? The positive answer that this is the difference between the volume of the truncated cone and the volume of the parallelepiped was given by: CG – 9 students (53%), EG – 12 students 67%.

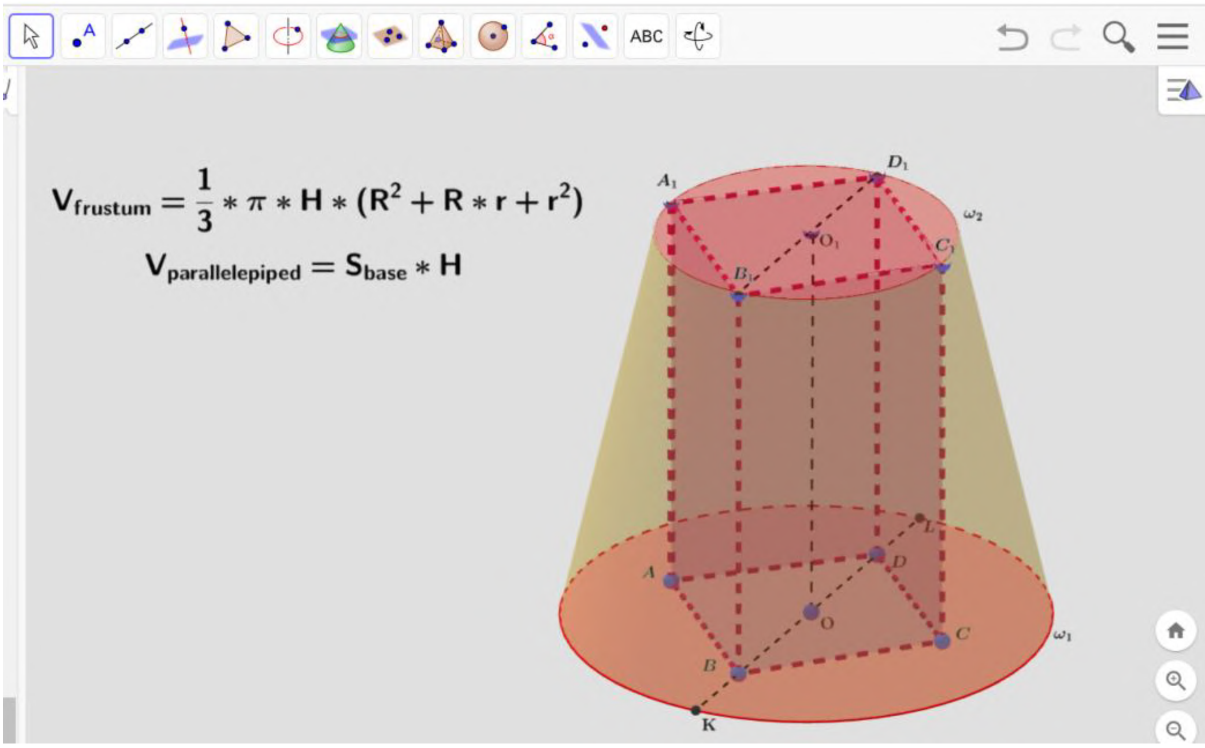

Figure 2: A parallelepiped inscribed in a truncated cone.

The dynamic image in GeoGebra helped the EG group to improve the statistics of responses, after the demonstration of the figure on paper this effect could not be achieved. The results of the survey showed that the highest efficiency is achieved when demonstrating dynamic models.

Optimization tasks using Geogebra were proposed by us in the textbook [27]. We supplemented the sets of tasks using Geogebra with visual aids for specialized teaching of mathematics, realization of interdisciplinary connections of the beginnings of mathematical analysis and stereometry. In this case, you can use the expressions to calculate the volume of the body to track the change in this value and find the optimal size of the beam. It is also advisable to use the "Function Inspector" tool in GeoGebra to find the extreme values of the function and visualize the abstractions.

It is convenient to write the formulas on the canvas at once, and then open them step by step during the discussion. To make such a blank in the application GeoGebra 3D, you must first build a truncated cone (by crossing the cone plane), then through the center of a smaller circle and a point on it build a line. Draw a perpendicular line to the obtained line, choosing the center of a smaller circle as a point. Mark the points of intersection of the lines with the circle and through the obtained 4 points build a square (using the Polygon tool), connecting the points in series. From the vertices of the square we lower the perpendiculars to the lower base of the cone (larger circle) and mark the points of intersection of the perpendiculars with the plane of the base of the cone, through the obtained 4 points we build a square, connecting the points in series. Using the Prism tool, build a prism by selecting a polygon of the base (square) and the vertex at one of the points of the smaller circle.

During the in-depth study of mathematics at the Kryvyi Rih Pokrovsky Lyceum, we offered students the problem of stereometry for optimization according to the textbook by Skanavi [29]. After calculating the optimal dimensions of the prism / pyramid, the polygon scan was drawn and glued. Models in dynamics created by means of system of dynamic mathematics were offered for demonstrations. Here are examples of mathematical problems that students had to reformulate as problems of applied content.

1. (15.194) What are the dimensions of the base radius and the height of the open cylindrical tank, so that at a given volume V for its manufacture was spent the least amount of sheet metal?
2. (15.195) The side face of a regular quadrangular pyramid has a constant given area and is inclined to the plane of the base at an angle α. At what value of α is the volume of the pyramid the largest?
3. (15.196) In a regular quadrangular pyramid with the edge of the base a and the height H, a regular quadrangular prism is inscribed so that its lower base is located at the base of the pyramid, and the vertices of the upper base are placed on the side edges. Find the edge of the base and the height of the prism that has the largest side surface.
4. (15.197) The side edge of a right triangular pyramid has a constant given length and forms an angle α with the plane of the base. At what value of α will the volume of the pyramid be the largest?
5. (15.198) In a regular triangular pyramid, the side face has a constant given constant area and forms an angle α with the plane of the base. At what value of α is the distance from the center of the base of the pyramid to its side face the largest?
6. (15.199) A pyramid is inscribed in a cone with a given constant volume, which is based on an isosceles triangle with an angle at the vertex equal to α. At what value of α is the volume of the pyramid the largest?
7. (15.200) The generating cone has a constant length and forms an angle α with the height of the cone. A regular hexagonal prism with equal edges is inscribed in the cone (the base of the prism is located in the plane of the base of the cone). At what value of α is the side surface of the prism the largest?

Solving problems of applied content will provide an opportunity to motivate, intensify the educational and cognitive activities of students and promote the practical application of acquired knowledge.

5.3. Project work in GeoGebra 3D

One of the effective means of developing students' cognitive activity is the project method. After all, the project method includes a set of research, search, problem, creative approaches, promotes the creative development of students, prepares them to solve problem situations in everyday life. Therefore, it is advisable to offer students to perform mini-projects while studying the section of stereometry.

The task of the project will be to build a playground in the GeoGebra 3D application, using the maximum number of studied geometric shapes: prisms, pyramids, spheres, cones, cylinders, etc. (figure 3). Performance appraisal is a mandatory element of the organization of project work. The effectiveness of the project lies in the ratio of planned expectations with the final results. Created designs can be designed in the yard with an augmented reality application.

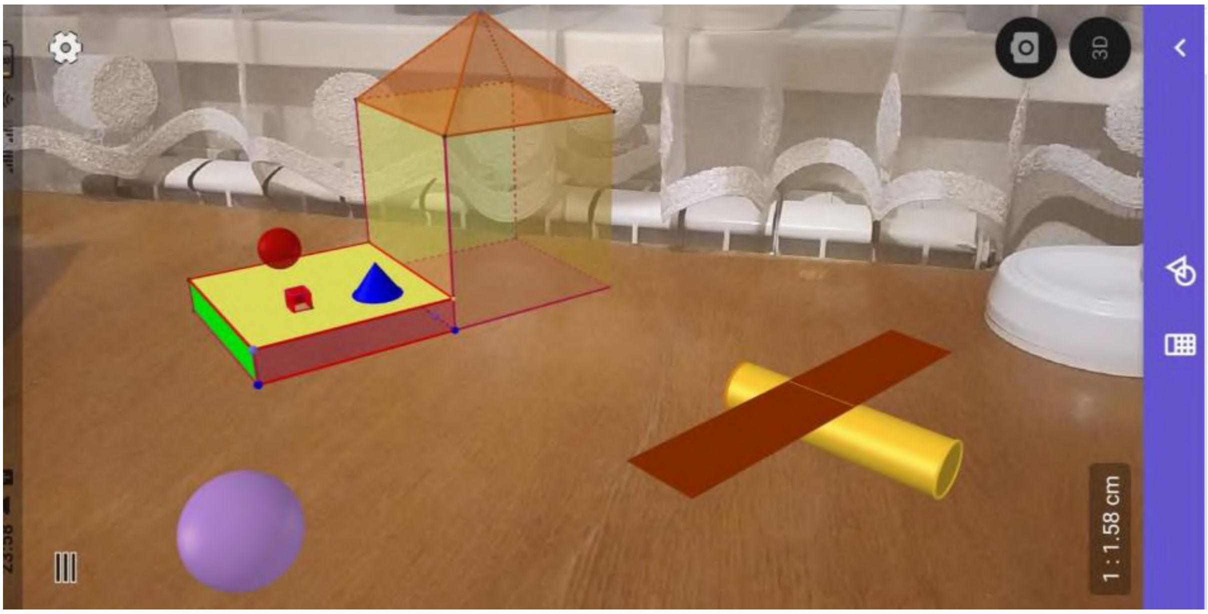

Figure 3: Sample implementation of the project "Playground".

There are three stages of self-regulated, namely the Planning Phase, at this stage students set steps for learning, namely (1) Analyzing learning tasks, (2) Determining learning objectives, and (3) Planning learning strategies. In the analyzing stage, students implement a plan that is constantly monitored to ensure it leads to learning goals. In the determining stage, students determine how well the learning strategy is chosen and how to achieve these learning goals [30].

Students were also asked to develop a project "Artist's Room", in which students will model a room from improvised means, and before that it is advisable to offer to make a layout in GeoGebra. In this way, students will already know where to start, what sizes of objects to take, what colors will impress, what shapes are needed to create a room, they will learn to break an object into simple geometric bodies and shapes.

Project work interests students in the subject, increases mental activity and creative thinking, helps to mobilize knowledge in practice and quickly adapt to unusual situations. During the construction of a playground or an artist's room, students use innovative abilities, invention, STEM competencies are formed, such as critical thinking, creativity, organizational skills, teamwork, emotional intelligence, ability to interact effectively, cognitive flexibility.

6. Integrating GeoGebra AR into mathematics curricula

To effectively incorporate GeoGebra AR into mathematics curricula, educators should consider the following guidelines [31]:

1. Choose AR activities that align with learning objectives and promote active engagement.
2. Provide students with clear instructions and scaffold their exploration in the AR environment.
3. Encourage collaborative learning and discussion to solidify conceptual understanding.
4. Assess learning outcomes and gather feedback to refine implementation.

Professional development is crucial for teachers to learn best practices for using GeoGebra AR [32]. Teacher education programs should expose pre-service teachers to the capabilities of GeoGebra AR and how to integrate it purposefully into their future classrooms.

Successful integration also requires attention to technological infrastructure and equity. Schools need access to AR-capable devices and reliable internet, and teachers need ongoing technical support. Ensuring equal access to AR learning experiences for all students is an important consideration.

7. Conclusion and future work

GeoGebra AR represents a promising frontier in mathematics education, offering dynamic, interactive experiences that bridge the gap between abstract concepts and the real world. Research has begun to demonstrate its potential to enhance spatial reasoning, conceptual understanding, and engagement in mathematics at both the secondary and tertiary levels. Thoughtful integration of GeoGebra AR into mathematics curricula, coupled with teacher training and ongoing support, can enrich students' learning experiences.

As GeoGebra AR continues to evolve, several areas merit further research and development:

- More work is needed to design and evaluate professional development models that prepare teachers to effectively integrate GeoGebra AR into their practice.
- The potential of GeoGebra AR to bridge mathematics with other STEAM fields deserves further exploration, building on initial work in architecture, arts, and culture [18, 19, 20].
- More research is needed to understand how the embodied interactions afforded by GeoGebra AR shape students' mathematical thinking and problem solving [25].
- The role of GeoGebra AR in supporting mathematics learning across different instructional modes, including remote and hybrid settings, should be investigated.

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