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The early history of computer-assisted mathematics instruction for engineering students in the United States: 1965-1989

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Abstract. The article discusses ICT development issues in teaching mathematics to engineering students in the United States. The nature of trends in the convergence of information systems in higher technical education and other tendencies in the United States are described in the article. The primary historical stages of computer-assisted mathematics training for engineering students in the United States are defined. The study of historical sources has allowed six stages to be recognized. The use of ICT for teaching mathematics is examined at each stage. It demonstrates the inconsistencies and key elements of using ICT to teach mathematics to engineering students. This article covers the first three stages (1965-1989) of computer-assisted mathematics training for engineering students in the United States.

Keywords: American experience, computer-assisted mathematics instruction, United States, engineering education, ICT in teaching mathematics

1. Introduction

At the current stage of the information society's evolution, the use of information and communication technologies (ICT) promotes the internationalization of the labor market, the expansion of various forms of personal mobility, and the globalization of education. The increased movement of students, entrants, and university graduates is an essential result of globalization. Increasing academic mobility, as well as the adoption of international norms and standards that allow academic qualifications from other nations to be compared and acknowledged, increases competition among universities and promotes higher education quality.

Education investment is an essential condition for any country's social and economic development. Globalization of education, in this context, contributes to the personal and professional growth of professionals involved in the creation and implementation of new technologies, namely engineers.

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The United States' higher engineering education institutions have made substantial pedagogical advances and have a well-established system for developing engineering professionals based on the methodical application of ICT tools. The challenge of raising the standard of training in universities in the globalized higher education sector should be resolved by incorporating the greatest innovations in global pedagogy and making inventive use of the expertise of top engineering universities.

The use of ICT in the process of teaching mathematics to engineering students fosters student self-realization, which aids in increasing cognitive activity, developing critical thinking, developing students' skills of independent work, developing creative abilities, increasing responsibility for their work, and improving the teaching process.

Therefore, in order to modernize higher engineering education and make it capable of advancing scientific and technological progress quickly and effectively, it is necessary to study the history and current state of development of ICT tools for teaching mathematics engineering students in engineering universities in the United States, which hold the top places in the ranking of the world's best universities [67].

There are various contradictions, particularly between the modern criteria for an engineer and the real level of their higher education training, as well as between the desire to improve the skills of mathematics teachers and their degree of awareness in ICT teaching mathematics. In the context of their evolution and convergence, the general trends in the development of ICT tools for teaching mathematics to engineering students in the United States remain unexplored.

With this in mind, the *purpose* of the article is to analyze the process of development of ICT tools for teaching mathematics engineering students of higher technical educational institutions in the United States and highlight the stages of development of computer-assisted mathematics training for engineering students in the United States.

2. Results

It has been long enough since enough software has been created to be useful in math instruction. Some of them are already organically integrated into the pedagogical process, and for some the development of methods of use is in its infancy.

Sinclair and Jackiw [51, p. 235-253] proposes classifying the use of ICT for educational purposes by the technique of their application rather than by the content of mathematics.

The following stages can be identified according to the historical and pedagogical examination of the literature on the use of ICT in mathematics instruction for engineering students in the United States:

- The first stage lasted from 1965 until 1972. The lower limit of the stage is related with the PDP-8 minicomputer, the cost of which was significantly cheaper than that of its predecessors, leading to the purchase of the PDP-8 for educational purposes. The major trend of the stage is the development of a sufficient number of computers equipped with high-level languages at U.S. academic institutions, as well as the characteristics of ICT hardware (usage of mainframes with limited network access).
- The second stage lasted from 1973 until 1980. The distribution of UNIX Version 5 to a group of educational institutions, which demonstrated the potential of UNIX for the

educational process, is related to the lower limit of the stage. The trend of the stage is connected to the adoption of mini- and microcomputer systems, as well as the UNIX network operating system, in U.S. universities.

- The third stage lasted from 1981 until 1989. The introduction of MS DOS and the widespread adoption of the IBM PC coincide with the stage's lower limit. The trend of the stage is connected to the adoption of personal computers and associated software at U.S. universities for the instruction of mathematics.
- The fourth stage lasted from 1990 until 1997. The development of the World Wide Web is related to the stage's lower limit. The primary trend of this stage is the use of Web 1.0 technologies in American university mathematics curricula.
- The fifth stage lasted from 1998 until 2003. The emergence of LMS and the U.S. government initiative on using technology to improve training opportunities through the Internet and multimedia represent the lower limit of the stage, respectively. The stage's tendency is integrating learning management systems into the delivery of advanced mathematics lessons.
- The sixth stage, which began in 2004 and continues now. The lower limit of the stage is connected to the formal launch of the MIT OpenCourseWare website as well as the recognition of massive open online courses by the majority of U.S. academic institutions. The development of cloud learning technologies and the migration of mathematics assistance tools to the Web environment are the current trends.

We suggest starting the description with the early stages (the first three).

2.1. 1965-1972: At the beginning

1965–1972 was the first stage. Despite successful but isolated attempts, the use of ICT in the teaching of mathematics was not systemic until 1965. The official appointment for the position of lecturer (assistant lecturer) in the Mathematics Department of the University College London (UK) in the second volume of the New Scientist magazine (July 1957) serves as an example. One of the qualification requirements for the position was knowledge of programming languages (primarily Fortran) and computer technology.

By the way, this programming language's name, which highlights the applied component of engineering mathematics instruction, is derived from FORmula TRANslation (translation of formulas in a language understandable by the computer). However, the straight transfer of programming in this language to the engineering school instructional process has encountered challenges due to:

1) the state of development of ICT tools: the prevalence of mainframes – "big" computers of high cost – whose maintenance was aimed at reducing the cost of downtime, so the leading mode of operation of such computers was batch (non-interactive mode of execution of a certain sequence of programs and analysis of their results), while for learning the leading mode of operation should be dialogic;
2) limited financial resources of educational institutions, which resulted primarily in the use of outdated microcomputers, which often did not have Fortran development tools;

3) lack of psychological and pedagogical bases for the use of ICT tools in education.

The Skinner's work on the programmed learning (programmed instruction) was aimed at solving the third challenge [52]. The concept of programmed learning envisaged such an organization of the process of acquiring knowledge, skills and abilities that at each stage of the learning process clearly defined the knowledge, skills and abilities to be acquired and controlled the process of their acquisition [75]. The main idea of this concept is the management of learning, educational and cognitive activities of students through the curriculum.

The second challenge was solved in 1964 by Kemeny and Kurtz [30]. The BASIC (Beginner's All purpose Symbolic Instruction Code) language they created was perhaps the first attempt to implement instructional learning at the level of a programming language [39].

It should not be assumed that just computers in the narrow sense were the focus of informatization in the U.S. educational system. For instance, at Hope College, discussions were underway in 1964 to provide electronic desk calculators and a high speed computer for instructional purposes [62].

Electronic calculators (in fact, specialized microcomputers) in mathematics teaching at that time were the leading, but not the only tools – for example, Suppes [54] provides a list of ICT tools for teaching mathematical logic: a computer terminal with the possibility of visual and audio presentation of educational materials, a keyboard for entering written answers, a microphone for audio answers and a light pen for selecting objects.

Thus, at the beginning of 1965, the U.S. education system had a sufficient number of computers of various levels, equipped with high-level languages, which makes it possible to consider this year as a conditional lower boundary of the first stage of the development of the computer-assisted mathematics instruction for engineering students in the United States.

In 1965, DEC (Digital Equipment Corporation) released the first commercially successful minicomputer, the PDP-8.

As mentioned in [16, 37], the PDP-8 computer was used in mathematics classes at mathematics faculties. Harvey [16] shows how two mathematics teachers proposed the creation of a separate computer department, partly to attract kids who didn't think of themselves as mathematically inclined, and partly because they couldn't both give the computer facility the attention it needed and also do the rest of their jobs.

Due to their wide use, the PDP series of minicomputers is frequently mentioned in academic studies of the history of information and communication technology. The proliferation of this series in academic and research organizations was aided by the best ICT tool combination available at the time at affordable pricing. REDUCE [19], a general-purpose computer algebra system, was one of the research attempts to develop a dialogue mathematical system [18].

Piaget's research on early childhood psychology provided an opportunity for his student Papert [45] in 1967 at the Massachusetts Institute of Technology, without departing entirely from constructivism, to develop a new learning tool – the Logo programming language [39], based on a constructionist approach to learning [1]. In Logo, its user, a programmer, acted as a "teacher" for the main object of the LOGO microworld, the turtle, "teaching" it to perform certain actions through programming.

As Molnar [39] noted, the Logo programming language is designed to encourage rigorous thinking in mathematics. The design of the LOGO environment has had a significant impact on the further development of teaching aids and learning concepts. For example, one of Papert's collaborators, Kay [29], proposed Dynabook [64] in 1968, "a personal computer for children of all ages", equipped with the Smalltalk programming environment. It was the first high-level language that supported experimentation with a wide range of mathematical objects, from numbers to geometric objects.

Thus, with the release of Logo and Smalltak-72, programmed learning ceased to be the dominant concept, which led to the choice of the upper limit of the first stage.

Turning again to the history of the use of ICT in Hope College, we can trace their evolution during the first stage:

1967 – a calculator laboratory was set up in 1967 with IME Electronic Calculators [25];

1968 – IBM 1130 was used to generate pseudo-random numbers [2];

1969 – the NSF project on "Instructional Use of Computers in Statistics" was part of a larger project headed by the University of North Carolina at Chapel Hill [63];

1970 – the two semester sequence "Applied Statistics and Computer Programming I and II" was offered; the programming language was Fortran [62];

1972 – Tanis [55] published a "Laboratory Manual for Mathematical Probability and Statistics".

Leading academic and reesearch institutions have historically not shied away from innovation. For instance, in 1968, MIT created the Macsyma computer algebra system [33] as part of Project MAC (the Project on Mathematics and Computation), and in 1971, IBM created the Scratchpad system [3, 27] under the leadership of Jenks [26].

The invention of the floppy disk in 1971 helped to personalize the use of mainframes and minicomputers: 81.6 KB of data stored on an 8-inch disk was enough to store text documents (articles, programs, etc.), the most common in the academic environment [12, 66].

To support the teaching of mathematics during this period, a number of specialized devices were developed:

- KENBAK-1 (1971) was the first non-professional educational computer. As noted Blankenbaker [6], his "criteria for the computer were low cost, educational, and able to give the user satisfaction with simple programs" [5];
- HP-35 (1972) was the first HP calculators that contained both integrated circuits and LEDs [24]. This contributed to its compactness and convenience in teaching mathematics. Horn [22], who used HP calculators as a mathematics teacher [23], is the author of many articles on their use [49]. As a specialized microcomputer, the HP-75 (further development of the HP-35) provided the possibility of programming in BASIC [20], and its successor – in the Reverse Polish Lisp [21].

The HP-35 had clearly been designed for use by engineers and engineering student [17, p. 140]. According to Harvey [17], in the teaching mathematics, calculators should be used, in particular, for calculator-based testing. On the one hand, there was a problem with ineffective calculator use in teaching mathematics, and on the other, there was a problem with ineffective calculator-based teaching techniques.

The academia and industry cooperation has led to a significant increase in the provision of educational institutions with ICT facilities: for example, if in 1965 less than 5% of all educational institutions were provided with computers for educational needs [41, p. 51], in 1972 data transmission through the network became relevant due to a significant increase in the provision of computer equipment and communication facilities [72, p. 11].

The analysis makes it possible to determine the main features of the use of ICT tools in teaching mathematics to engineering students at the first stage of their development:

1) dialog mode of work with educational software;
2) emergence of the first systems to support mathematical activities without programming in general purpose languages;
3) variety of hardware and software;
4) dominance of behaviorism in the justification of ICT use and curriculum development;
5) introduction of programming in mathematics courses.

These features gave rise to a number of contradictions:

- between the need to strengthen the applied aspect of teaching mathematics to engineering students and the lack of training time for simultaneous mastering of mathematics and programming;
- between the need to strengthen the applied aspect of teaching mathematics to future engineers and the lack of training time for simultaneous mastery of mathematics and programming;
- between the diversity of ICT tools and the need for standardization of teaching aids.

A partial solution of these contradictions was achieved at the first stage – there were new, more adequate approaches to modeling the learning process, the first systems of computer mathematics and in 1969 – the UNIX operating system [31], aimed at combining various ICT tools in a single network environment.

2.2. 1973-1980: Age of UNIX

The second stage – 1973-1980 – is associated with the spread of the UNIX network operating system in U.S. universities, the use of mini- and microcomputer systems.

From the memoirs of Harvey [16]: "My own learning about computers took place mainly at the Artificial Intelligence laboratories of MIT and Stanford. I decided to create an environment at the high school that would be as similar as possible to those labs. ... I installed a PDP-11/70 running version 7 Unix. ... we were an alpha test site for 2.9BSD, the PDP-11 version of Berkeley Unix. The installation, testing, and debugging of this new system was carried out entirely by students [of the Lincoln-Sudbury Regional High School]".

This excerpt mentions PDP-11 [4], a further development of PDP-8 used in the first stage, and shows the interest of the manufacturer (DEC) in providing its own ICT resources to universities which used UNIX. BSD, the name of the UNIX version, honors the university's development contributions (Berkeley Software Distribution) [74].

The primary characteristic of UNIX, mobility (mainly of software portability), has made it possible for this system's software to be distributed across a range of ICT devices, from mainframes to mini- (and later micro-)computers.

UNIX was used to develop a number of well-known computer mathematics systems, including MATLAB (late 1970s [38]), Maple (1980 [71]), and others, which were created as programming languages for instructing students in mathematics using a variety of mathematical libraries without having to learn Fortran.

Due to the fact that UNIX OS and its software were mobile, it became possible to combine not only the computing resources of different computers, but also users, their programs and data in a network environment: "The resulting Unix system provided users with interactive remote terminal computing and a shared file system. Source code was provided with the system, and the community of users could share ideas and programs directly and informally. Because Unix ran on a relatively inexpensive minicomputer, small research groups could experiment with it without dealing with computation center bureaucracies" [69].

The beginning of the second stage was marked by the generalization of the experience of using ICT teaching aids in mathematics; in particular, the NATO Advanced Study Institute on Computer-Based Science Instruction [57] (1976), the Eighth Conference on Computers in the Undergraduate Curricula [56] (1977) and others. *Michigan Association for Computer Users in Learning Journal* established in 1977, and in 1978 it published article by Tanis [58] on utilizing computers to learn probability and statistics

The personalization of ICT tools started after the invention of the floppy disks was continued by the developments of "1977 trinity" – Apple II, TRS-80 Model I and Commodore PET computers [40].

New Apple II tools for common users — a graphical display and a mouse manipulator — facilitated the use of a constructive approach to teaching mathematics. This made it possible to develop dynamic geometry systems, computer graphics, and instructional video games. Additionally, graphic commands were added to the Apple II's native BASIC language. The design was so successful that it served as the inspiration for both the Apple Macintosh and the Agat, an educational and home computer developed in the Soviet Union – Agat [7].

As noted by Confrey et al. [11, p. 3], the instructional goal of the Apple Classrooms of Tomorrow Research project was to create a constructivist-based learning environment that promotes student construction of mathematical concepts through repeated cycles of developing a problematic, acting to resolve the problematic, and reflecting on these actions.

Navarro [43] mentions the use of TRS-80 computers as classroom teaching tools. Teachers used a program in Level II BASIC for a TRS-80 computer that simulates a Turing machine and discusses the nature of the device. The program is run interactively and is designed to be used as an educational tool by computer science or mathematics students studying computational or automata theory.

The proliferation of personal computers with a graphical interface focused on gaming activities (Atari 800 and others) also contributed to the development of game-based mathematics learning [13]. The educational software industry at the second stage was focused mainly on Apple II [50].

The 1978 publication of "Laboratory Manual for Probability and Statistical Inference" [59] had a profound impact on the theory and practices of computer-assisted mathematics instruction. After the initial attempts to educate engineering students mathematics using computers 15 years before, there has been a substantial shift: from teaching mathematics alongside programming to the appropriate use of software in the learning process.

Metcalfe et al. [34] from Xerox PARC (the same laboratory that developed the computer mouse, graphical user interface, and Smalltalk) obtained a U.S. patent for Ethernet technology, and the first open online information service Compuserve debuted in 1979. Although UNIX systems were at its core, it also brought together personal computers.

Thus, we can identify the following characteristics of the second stage of development of the theory and methods of using ICT in teaching mathematics to engineering students in the United States:

1) transition from the use of programming languages in teaching to the use of mathematical libraries, computer mathematics systems and high-level languages;
2) use of computer graphics in educational software;
3) emergence of new classes of educational software – educational video games, dynamic geometry systems and spreadsheets;
4) emergence and spread of computer networks that facilitated active communication between teachers and students;
5) development of ICT tools for teaching mathematics – graphing and symbolic calculators.

A number of contradictions can be identified in the development of ICT tools at this stage:

- between the potential use of multimedia tools and the lack of psychological and pedagogical foundations for their use;
- between the need of students and teachers for personal ICT tools and the lack of offers from manufacturers;
- between the need of personal computer manufacturers for an adapted version of UNIX and the lack of hardware for its operation.

These contradictions marked the beginning of the third stage, in which they were resolved.

2.3. 1981-1989: The dawn of PC

The spread of personal computers is linked to the third stage, which spanned 1981 to 1989. The lower stage border (1981) corresponds to the debut of IBM PC [48] and MS DOS [36], which had a profound impact on the development of the current market for software tools for mathematics education.

One of the most popular strategies for raising higher education students' learning achievement levels is computer-assisted learning, so user, computer, program, and data personalization was the stage's dominant idea.

During these years, a significant amount of work was devoted to the psychological and pedagogical substantiation of learning with the use of personal computers and the development of computer-based training courses in mathematics. The most popular programming languages at that period, according to an analysis of publications, were BASIC, Pascal, FORTRAN, and Algol. As a result, at the Massachusetts Institute of Technology, advanced mathematics students wrote computer programs to carry out research on a variety of mathematical problems. The development of computer-based teaching methods for applied mathematics has been made possible by developments in numerical methods, computing, and creative applied mathematics research [68].

Engelbrecht and Harding [14] reveals a number of advantages of using computers in teaching mathematics: development of practical skills; variety of presentation of educational material; use in the process of modeling and programming.

John J. Wavrik, professor emeritus of Department of Mathematical Sciences at the University of California, San Diego [28] and author of the Computer Algebra course for students majoring in symbolic computing (mid-1980s), was one of the first to argue that the use of computers is necessary in the teaching mathematics, as their use contributes to improving the level of students' academic achievements [73].

His article "Computers and the Multiplicity of Polynomial Roots" is "not describe the assorted twists and turns of fate that lead a worker in a pure mathematics area like algebraic geometry to become involved with computers. It is quite likely, however, that as personal computers become more common an algebraist who acquires one will at some stage make a stab at using it for research work. ... It is natural to hope that computers can be used to reduce the difficulty of computation. As it turns out, the process is not as simple as it might first appear. ... We would like to have machine assistance, for example, in computing invariants for specific instances of the objects of study" [73, p. 34]. Wavrik [73] notes that "the problem of efficiently computing the greatest common divisor of two polynomials with integer coefficients has received a great deal of attention. ... Computer systems designed for algebraic computation generally allow "infinite precision" integer arithmetic and these algorithms can be implemented on systems of this type. Machines which perform fixed precision arithmetic often allow an accuracy of only between 6 and 16 decimal digits before roundoff occurs" [73, p. 46]. "This article includes a computer program to serve as an example of implementation on a real machine. The program is written in BASIC – the most common language found on microcomputers" [73, p. 50]. The author points out that "the purpose of this article is to give readers some of the flavor of algebraic computation. A program is included, not to provide readers with a piece of ready-made software, but rather to give them ideas for developing their own program" [73, p. 55].

Pea [47] describe the AlgebraLand [8], a software program in which students are freed from hand calculations associated with executing different algebraic operations and allowed to focus on high level problem-solving strategies they select for the computer to perform. AlgebraLand is said to enable students "to explore the problem space faster" as they learn equation solving skills [47, p. 92].

If we look at the history of ICT use at Hope College as an example, we can trace its evolution during the third stage [62]:

- 1981 – Translation of FORTRAN programs into BASIC for the TRS-80 Radio Shack Computer in order to take advantage of the better graphics
- 1982 – First International Conference on Teaching Statistics (ICOTS I), a poster of Tanis [60] "The Use of Microcomputers for Understanding Concepts in Probability and Statistics"
- 1983 – Annual Meeting of the Mathematical Association of America, Tanis [61] made a presentation "Using Microcomputers to Illustrate Concepts in Probability and Statistics" using TRS-80 graphics
- 1985 – Translation and adaptation of statistics programs to the IBM PC
- 1986 – Math 212, a laboratory for Introductory Statistics using IBM PCs and BASIC, was introduced

At this stage, a number of computer mathematics systems were developed, namely: Cayley (1982 [9]), FORM (1984 [70]), Fermat (1985 [32]), PARI/GP (1986 [46]), MathCAD (1986 [15]), GAP (1986 [44]), Buchmora (1987 [65]), CoCoA (1987 [10]), Mathomatic (1987 [35]), Mathematica (1988 [76]), Derive (1988 [53]).

In the 1989 report of the U.S. National Academy of Sciences on the future of mathematics education [42] (as it was seen at the end of the third stage) indicates that "As calculators have surpassed human capacity for arithmetic calculations, so now are symbolic computer packages overtaking human ability to carry out the calculations of calculus. Until recently, computers could only operate numerically (with rounded numbers) and graphically (with visual approximations). But now they can operate *symbolically* just as people do, solving equations in terms of x and y just as we teach students in school mathematics. Symbolic computer systems compel fundamental rethinking of what we teach and how we teach it" [42, p. 52]. "Calculators and computers compel reexamination of priorities for mathematics education" [42, p. 61]: the report considered electronic spreadsheets, numerical analysis packages, symbolic computer systems, and sophisticated computer graphics to be the leading tools of teaching mathematics, and interactive textbooks, remote classrooms, and integrated learning environments to be promising.

"Texts, software, computer networks, and databases will blend in coming years into a new hybrid educational and information resource" [42, p. 67] – this forecast marked the end of the third and the beginning of the fourth stage of the development of the theory and methodology of computer-assisted mathematics instruction for engineering students in the United States.

Thus, the characteristic features of the third stage of development of the theory and methods of using ICT in teaching mathematics to engineering students in the United States include:

1) wide use of mathematical libraries, computer mathematics systems and problem-oriented languages;
2) wide introduction of personal and personalized ICT tools in teaching mathematics;
3) use of general-purpose ICTs (text editors, spreadsheets, databases, etc.) to support the teaching of mathematics.

A number of contradictions can be identified in the development of ICT tools at this stage:

- between the potential of using global computer networks and the lack of personal access to them;
- between the necessity of transferring hypertext and hypermedia systems for educational purposes to the network environment and the lack of development of corresponding tools;
- between the need of students and teachers in transferring educational materials to the network and the lack of development of psychological and pedagogical foundations for them.

These contradictions marked the beginning of the fourth stage, in which they were resolved.

3. Conclusion and future work

The research findings enable us to draw the conclusion that the teaching methodology is gradually evolving, taking into account the scientific and technological advancements of the time, at each stage of the development of computer-assisted mathematics instruction for engineering students in the United States. New ICTs induce changes in the theory and practices of teaching mathematics and the introduction of new objectives, tools, forms, and strategies for structuring mathematics education and enhancing its content. These improvements have a good effect on mathematics teaching, greatly enhancing student abilities.

Future work include the further investigation and in-depth examination of the fourth, fifth, and sixth stages of the history and development of computer-assisted mathematics instruction for engineering students in the United States.

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