Some applications of cloud technologies in mathematical calculations

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Some applications of cloud technologies in mathematical calculations

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Abstract. This article discusses some use of cloud technology in mathematical calculations using Remote Desktop Ulteo OVD. To use such technologies, it is enough to have access to the Internet through a suitable browser to access an open virtual desktop on a powerful remote computer and then use the resources of the remote computer (server) to solve their problems in processing various information resources – solving mathematical problems, working out texts, translating from one language to another, help on the interpretation of different terms, their origin and more. You can organize access to Ulteo OVD from two servers (Application Server (Windows 2008R2) and Session Manager Server (Linux Ubuntu)), using the Proxmox web-based virtual environment. Gran1, Gran2D, Gran3D software can be installed on the application server. The article also examines in detail some examples of the use of the pedagogical software for educational purposes Gran1. In particular, the calculation of the approximate value of the double integral; graphical two-dimensional problem solving, the so-called linear programming problems; two-dimensional problems, including convex programming – finding the smallest value of a convex downward function (or the highest convexity of a function) on a convex set of inequalities (including linear ones). However, the use in the educational process of any technology, including modern information and communication, as well as the content of training, should be pedagogically balanced, which will allow to avoid any negative effects on the formation of personality of a future member of society, his mental and physical development.

1. Introduction

Extensive use of modern ICT in various spheres of life and activity of people provides opportunities for access to a variety of information from any field of knowledge. There are opportunities to significantly increase people's awareness of various manifestations of the world around them and, as a result, to be better prepared for a successful life and activity in today's conditions.

Educational institutions are not left out either. The introduction of modern ICT in the methodological systems of teaching [11] various disciplines, including mathematics [12], physics [3], chemistry [9], geography [1], history [5], etc., provides opportunities to significantly fundamentalize the content of education, expand and deepen the theoretical knowledge base. In addition, to provide knowledge with practical significance and applicability, to form in students the foundations of professional and general culture, to cultivate in them a sense of concern and responsibility for the safety of the world and people, to be consciously highly cultured, well-educated and polite, about his well-being and peace, the development of his culture and material well-being.

However, the use in the educational process of any technology, including modern ICT, as well as the content of education, should be pedagogically balanced, which will avoid any negative influences on the formation of the future member of society, his mental and physical development. The main thing is

the development of thinking skills, analytical-synthetic, creative [4] and critical [10] thinking of students, the ability to see the essence of various manifestations of the world and the causal links of manifestations of various phenomena and processes, to be able to correctly explain and use them for the environment and for themselves.

Of particular importance is the teaching of natural sciences and mathematics, during which students have to consider and build models of various processes and phenomena, and then explore them, analyzing their various features and characteristics, possibly using different information and communication models to perform calculations or experiments, and based on the results of such analysis, synthesizing the relevant conclusions. This approach to learning allows students to effectively develop logical, critical, creative thinking, scientific worldview, creative approach to solving various problems, their correct vision and ability to explain their nature and essence.

Of particular importance are the cloud technologies for accessing the resources of various powerful computers (servers) via the Internet [6] using not very powerful, including mobile Internet devices – smartphones, tablets, etc. [8] This allows educational institutions to use the resources of remote servers without spending money to purchase their own powerful, and therefore expensive, computers. It is enough to have access to the Internet through the appropriate browser (browser – program for reading) to get to the open virtual desktop on a powerful remote computer (Open Virtual Desktop – OVD [7]) and then use the resources of the remote computer (server) to develop solving their problems in relation to the processing of various information resources – solving mathematical problems, processing texts, translation from one language to another, information on the interpretation of different terms, their origin, and much more.

2. Cloud-based Gran

You can use Ulteo OVD to deploy Remote Desktop. To do this, you need to organize the work of two servers (application server and session manager server), which can be done using a web-oriented virtual environment PROXMOX (figure 1) [2].
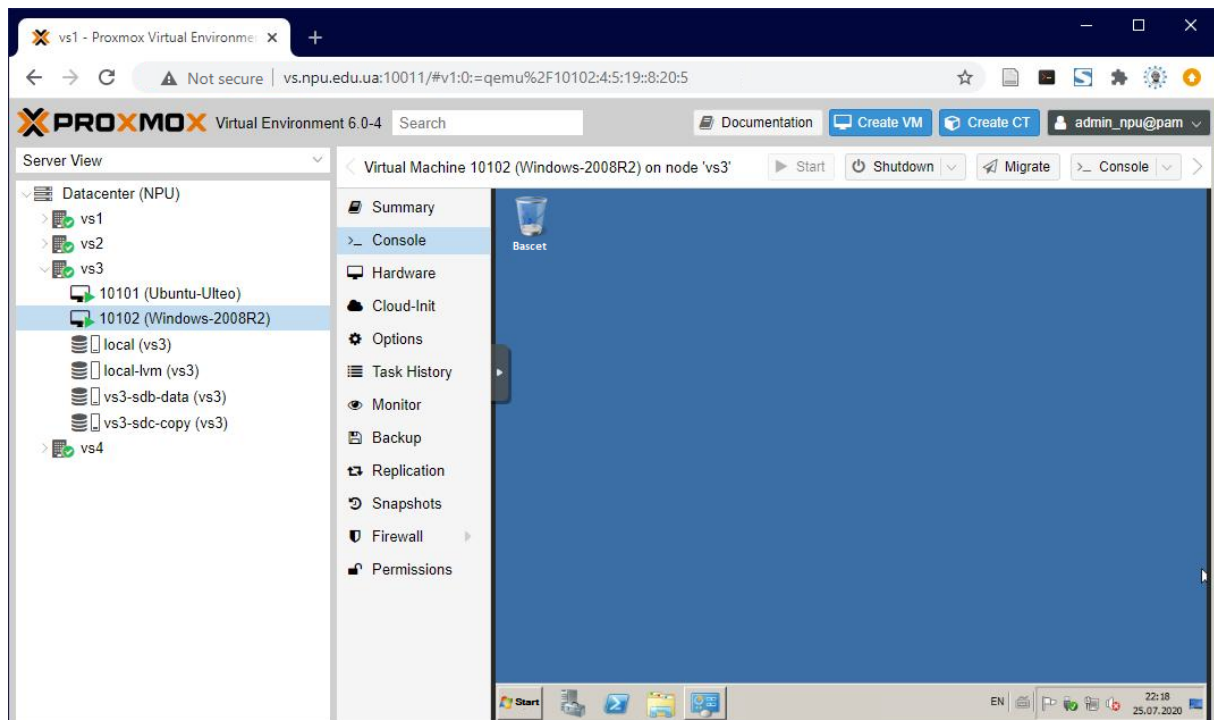

Figure 1. PROXMOX console.

One server, namely the session manager server, is configured using the Linux Ubuntu operating system. The application server is configured using the operating system Windows 2008R2, which can be installed software Gran1, Gran2D, Gran3D.

Let's consider some possibilities of use of resources of the remote server on an example of work with cloud variants of the Gran software educational complex. To get to the virtual desktop (Open Virtual Desktop) on a remote server, you must access the browser services (such as Google Chrome) and in the input line above the desktop enter the address gran.npu.edu.ua (see figure 2), then press the Enter key on the keyboard (or in the list of appropriate symbols on the screen "press" the label with the word "Go" in the case when using a smartphone or other laptop computer where there is no keyboard). As a result, a virtual desktop will open (Ulteo Open Virtual Desktop) (see figure 3) on which in the line "Username" you should select from the proposed list one of the available names, such as "gran", and then in the line "Password" enter password *gran*.
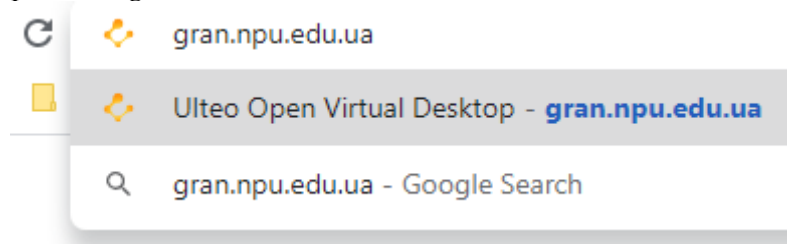

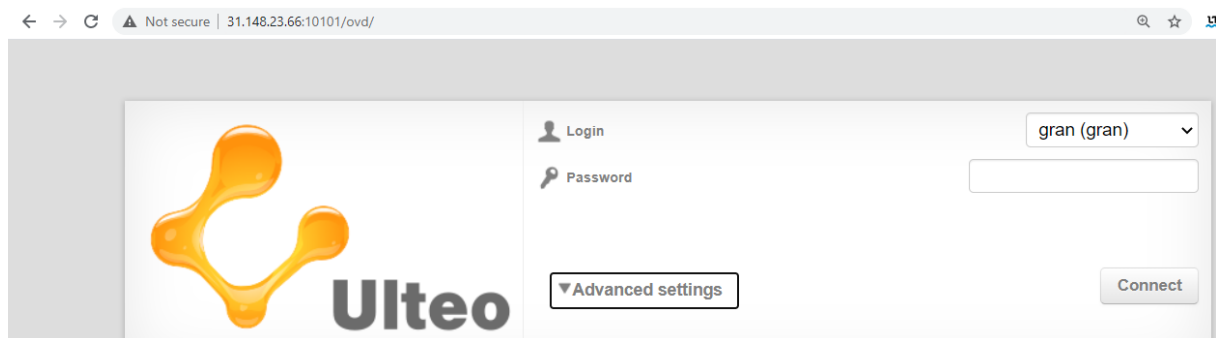
Figure 2. Access to the gran.npu.edu.ua

Figure 3. Ulteo OVD login page.

It should be noted that in this case, the English alphabet (ENG) must be installed on the keyboard. As a result, a window will appear with the message "Ulteo. Loading Open Virtual Desktop" (see figure 4) and soon a window will appear with the designations (through the corresponding images) of five objects: "Basket", "Gran1", "Gran2D", "Gran3D", "Local disk (D)" (see figure 5).
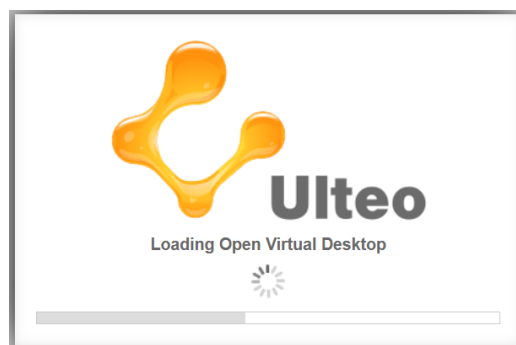

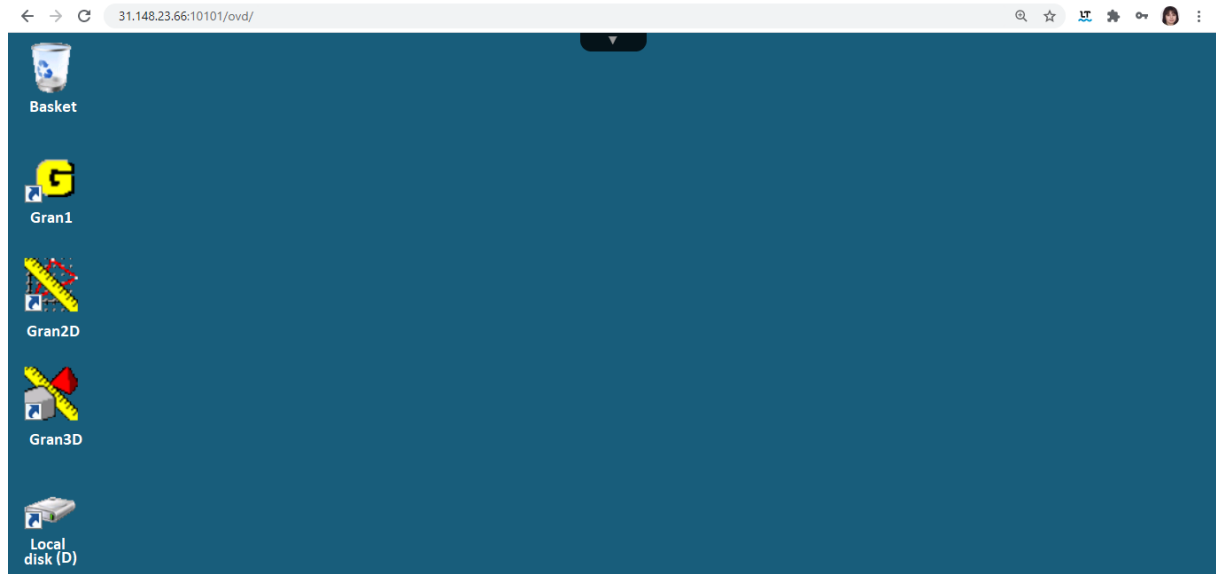**Figure 4.** Ulteo OVD loading screen.

Figure 5. Cloud desktop.

To start working with Gran1, place the cursor on the "G" and double-click "left-click" (or give the appropriate instruction when using a mobile device). As a result, the Gran1 working window will appear on the screen (see figure 6). Then you can start using the services provided in the Gran1 program.
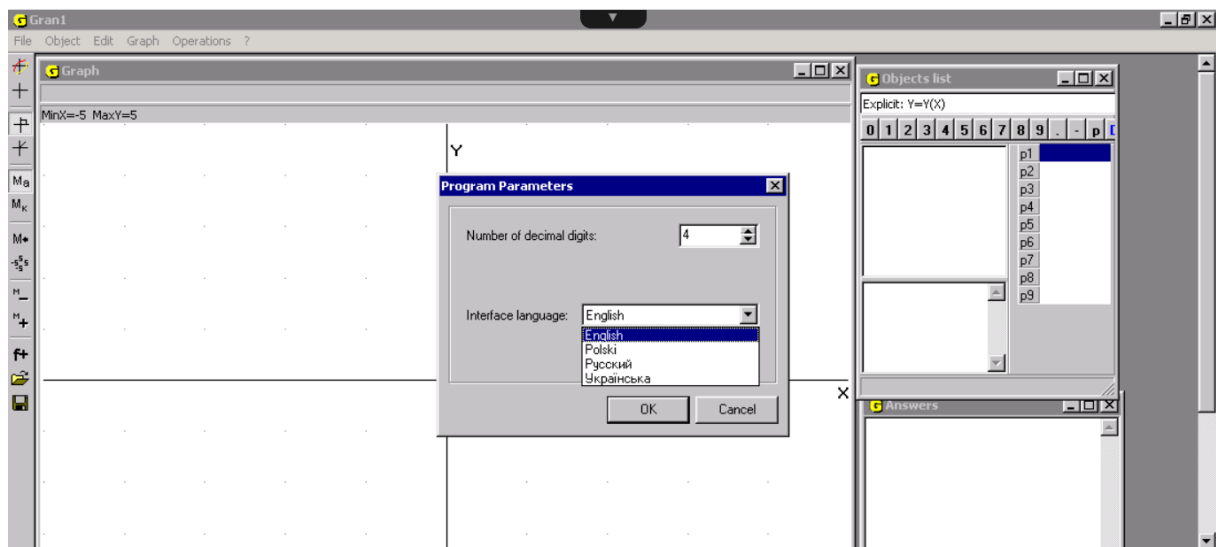

Figure 6. Gran1 starting window.

Similarly, the Gran2D program, designed to graphically process 2D objects on a plane, and the Gran3D program, designed to graphically analyze 3D objects in 3D space, are loaded.

Let's consider in more detail some examples of application of the pedagogical software of educational purpose Gran1.

3. Application of the cloud-based Gran1 in mathematical calculations

Suppose you need to calculate the approximate value of the double integral.

$$\iint\limits_{G} f(x,y)dxdy \text{, where } G = [a;b)\times [c;d) = \{(x,y)|\ x \in [a;b), y \in [c;d)\}.$$

Divide the segment $[c;d)$ into a number of partial intervals $[y_{i-1}; y_i)$, $i \in \{1,2,\dots,k\}$, $y_0 = c$, $y_k = d$, $y_i - y_{i-1} = h$, where h – some step change of values y_i, and we will give approximately

$$\iint\limits_{G} f(x,y)dxdy$$

through

$$\sum_{i=1}^{k=1} h \int_{a}^{b} (x,y_i)\,dx + h \cdot \frac{1}{2}\left(\int_{a}^{b} f(x,y_0)\,dx + \int_{a}^{b} f(x,y_k)\,dx \right) \qquad (1)$$

or, equivalently

$$\iint\limits_{G} f(x,y)dxdy \approx \sum_{i=1}^{k} h \cdot \frac{1}{2}\left(\int_{a}^{b} f(x,y_{i-1})\,dx + \int_{a}^{b} f(x,y_i)\,dx \right). \qquad (2)$$

For each fixed value of y_i the one-dimensional integral

$$\int_{a}^{b} f(x,y_i)\,dx$$

is easily calculated using the corresponding services provided in the program Gran1, namely the services "Operations", "Integrals", "Integral" (see [12], [13], [14]). Calculating further the sum (1) for a specific value of h, we find the approximate value of the integral

$$\iint\limits_{G} f(x,y)dxdy\,, G = [a;b)\times [c;d).$$

Consider specifically this example. Suppose you need to calculate the approximate value of the double integral

$$\iint\limits_{G} e^{-(x^2+y^2)}dxdy,$$

where $G = [-5;5)\times [-5;5)$. We will calculate the values of one-dimensional integrals

$$\int_{-5}^{5} e^{-(x^2+y^2)}\,dx,$$

by providing variable y_i values $0 + i \cdot h$, $i = \{0,1,2,\dots,k\}$. Given that the function $e^{-(x^2+y^2)}$ is even with respect to the variable y for an arbitrary value of the variable x, the resulting sum

$$\sum_{i=1}^{k} h \cdot \frac{1}{2}\left(\int_{-5}^{5} e^{-(x^2+y_{i-1}^2)}\,dx + \int_{-5}^{5} e^{-(x^2+y_i^2)}\,dx \right)$$

will need to be multiplied by 2 (or change y_i through step h, starting not from $y_0 = 0$, but from $y_0 = -5$).

Note that $f(x, y) = \frac{1}{\pi} e^{-(x^2 + y^2)}$ is the density of the two-dimensional probability distribution, the coordinates of the center of which are $x_1 = 0$, $y_1 = 0$, and the probability scattering variance along the axis OX and along the axis OY is equal to $D_1 = \frac{1}{2}$, $D_2 = \frac{1}{2}$, respectively, standard deviations $\sigma_1 = \frac{1}{\sqrt{2}}$, $\sigma_2 = \frac{1}{\sqrt{2}}$, and $f(x, y) = \frac{1}{\pi} e^{-(x^2 + y^2)}$ can be submitted as $f(x, y) = f_1(x) f_2(y)$, where $f_1(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}$, $f_2(y) = \frac{1}{\sqrt{\pi}} e^{-y^2}$ – the densities of one-dimensional normal probability distributions along the axis OX and the axis OY, respectively. As is known, $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$ (known Euler-Poisson integral).

Let us now turn to the Gran1 program and, specifying the type of dependence between the variables "Explicit", enter the expression $\exp(-(x^\wedge 2 + p1^\wedge 2))$ (see figure 7), where instead of the variable y we specify the variable (dynamic) parameter $p1$. Having set some specific value of the parameter $p1$, the limits of its change "$Min =$", "$Max =$" and the step of changing "$h =$" (in the appropriate windows on the right side of the desktop), refer to the services "Graph", "Plot". Resulting in the window "Graph" will be the graph of function $f(x) = e^{-(x^2 + p1^2)}$ at the set value of parameter $p1$ is constructed (see figures 8–13 for $p1 = 0 \dots 1.25$).
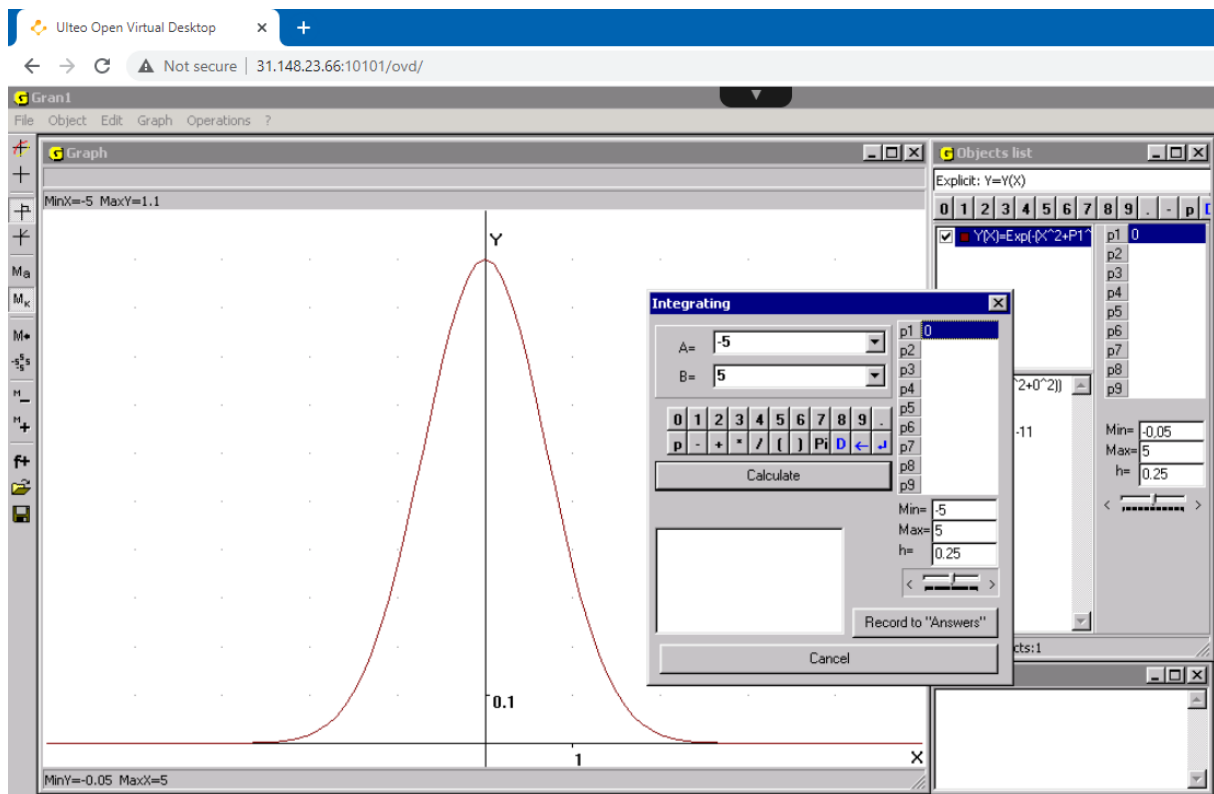

Figure 7. Plot of expression $\exp(-(x^\wedge 2 + p1^\wedge 2))$.

Note that the required numbers in these windows are entered by using the number pad at the top right in the working window, as usual. After entering the required numbers, you should "press" the button "⬅" on the specified panel.

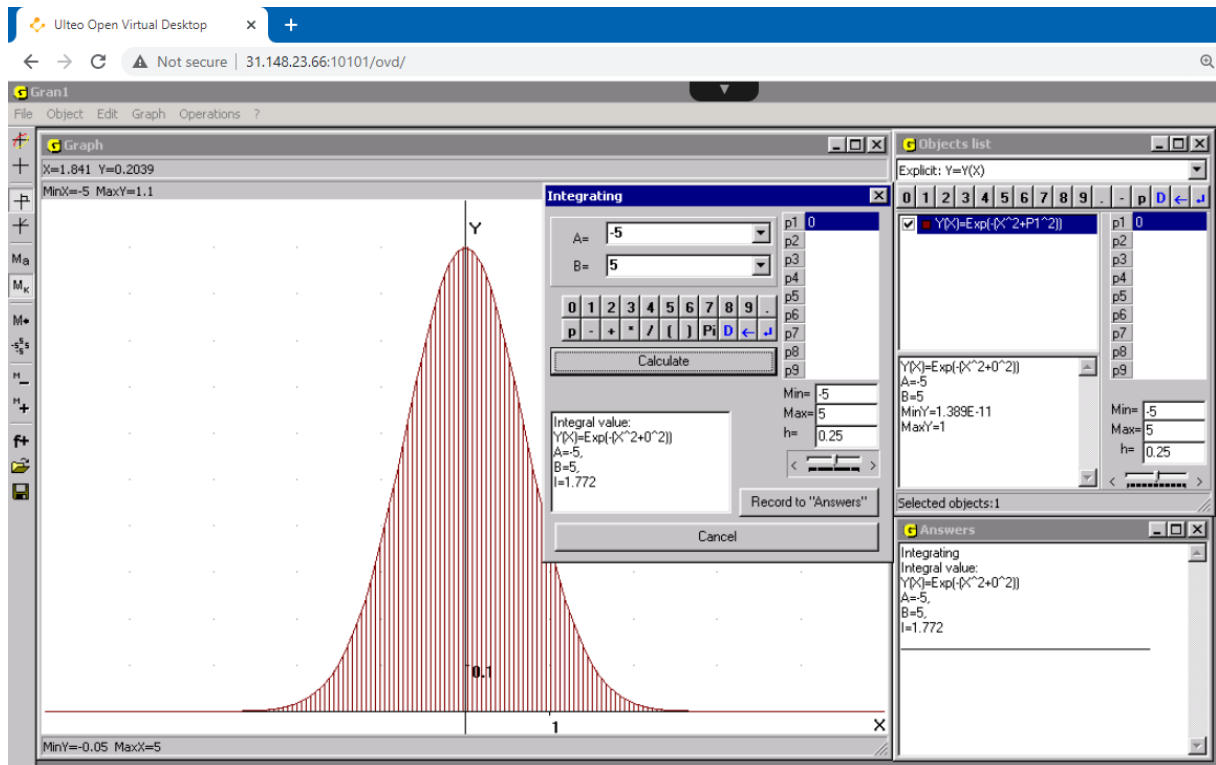Summarizing the obtained 14 values in the table 1, we obtain the set (on condition $h = 0.25$).

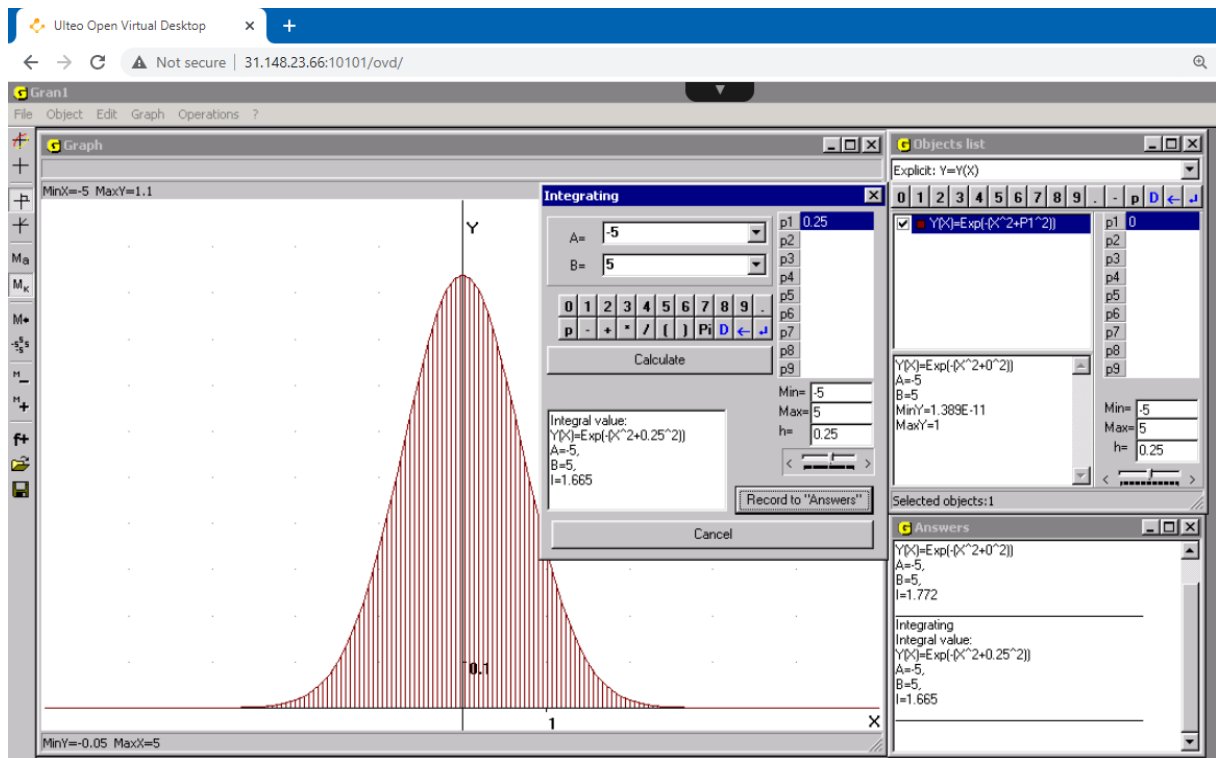**Figure 8.** Graph of function $f(x) = e^{-(x^2 + p1^2)}$ at $p1 = 0$.

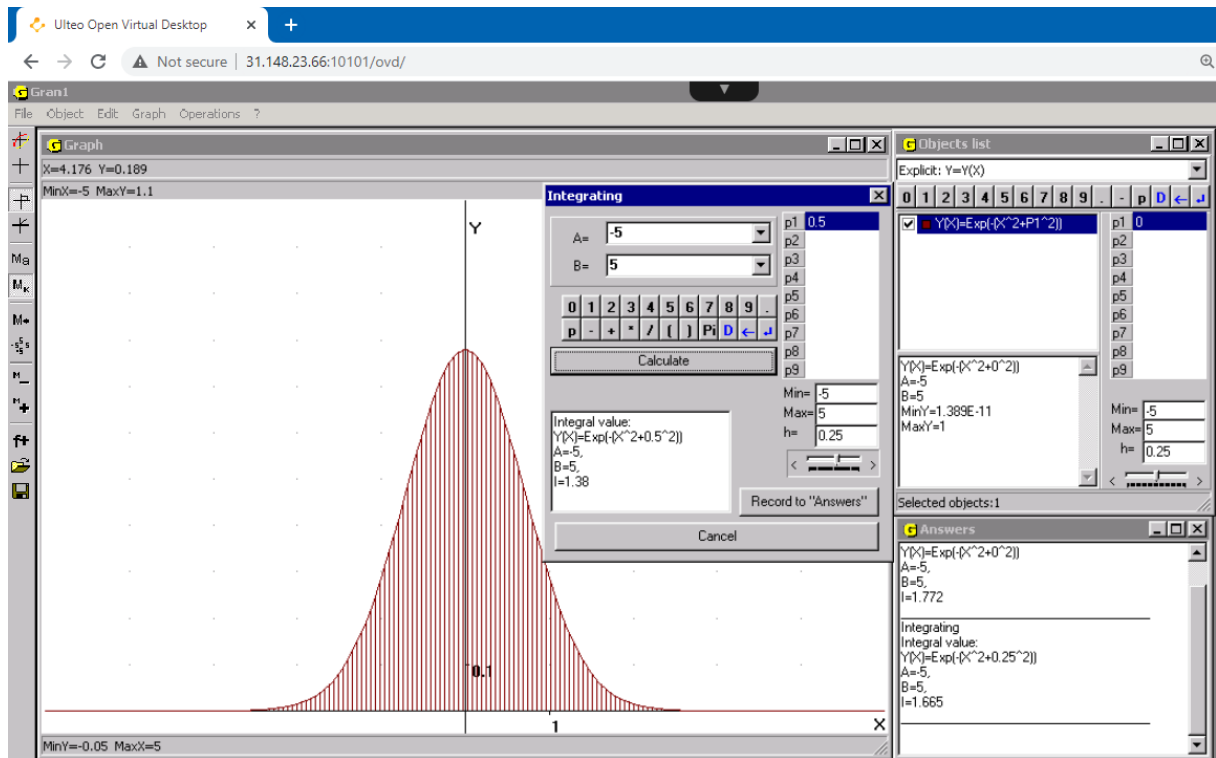**Figure 9.** Graph of function $f(x) = e^{-(x^2 + p1^2)}$ at $p1 = 0.25$.

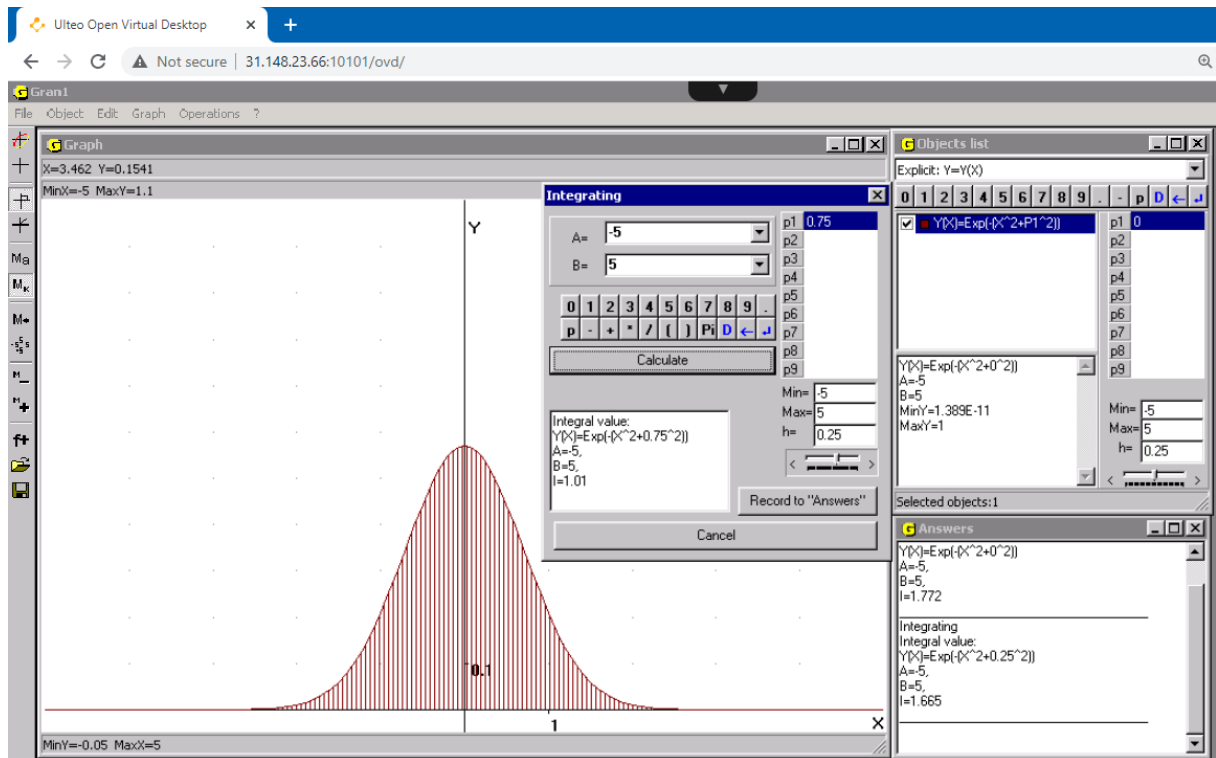**Figure 10.** Graph of function $f(x) = e^{-(x^2 + p1^2)}$ at $p1 = 0.5$.

Figure 11. Graph of function $f(x) = e^{-(x^2 + p1^2)}$ at $p1 = 0.75$.
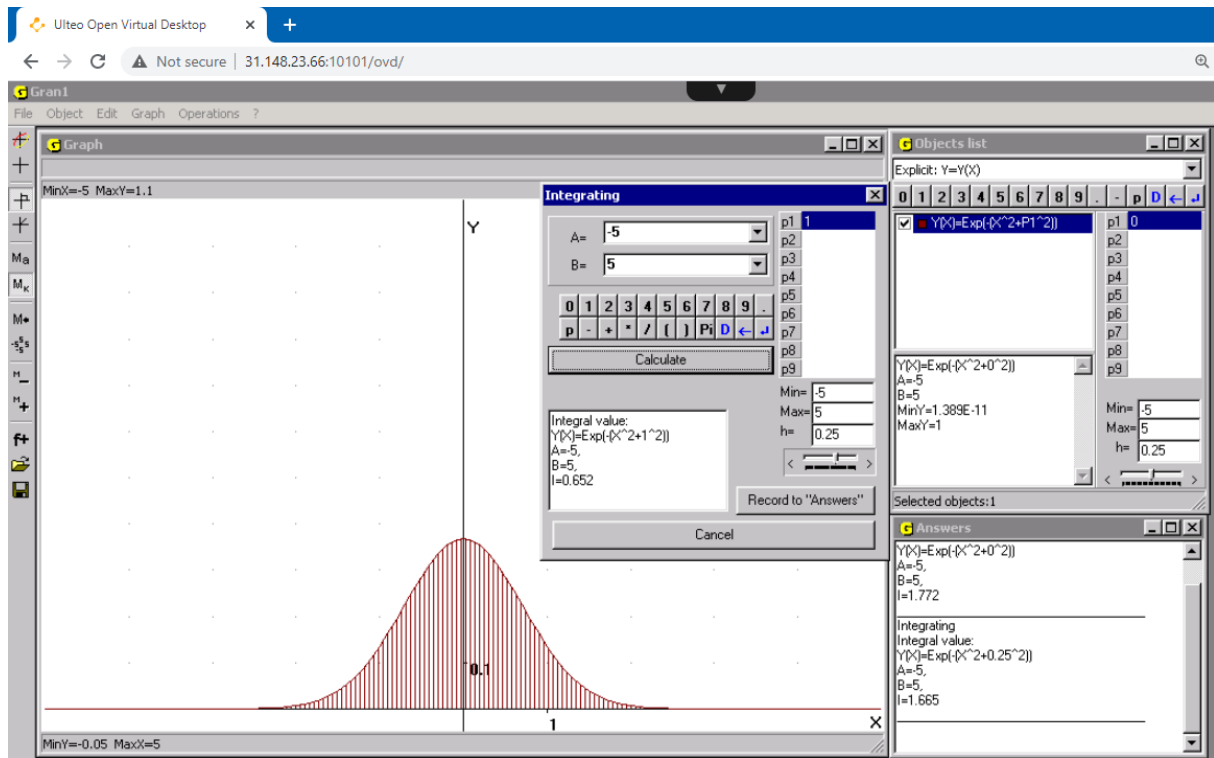
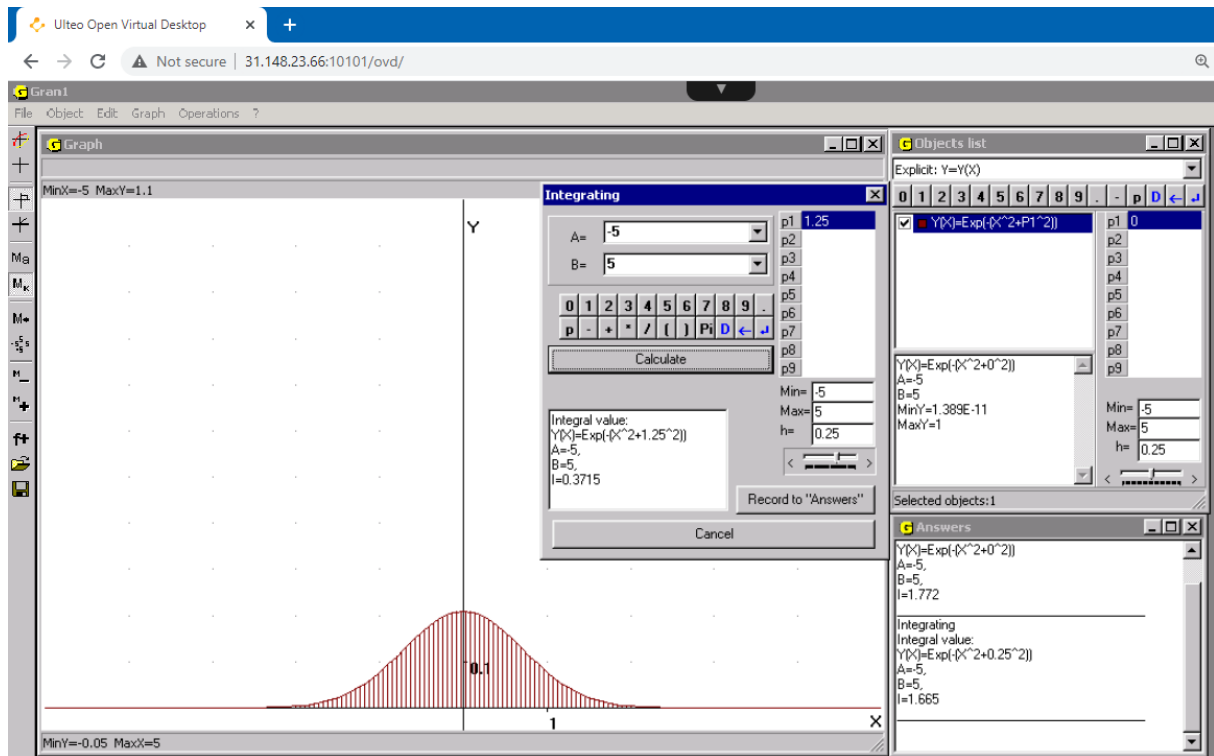**Figure 12.** Graph of function $f(x) = e^{-(x^2 + p1^2)}$ at $p1 = 1$.

Figure 13. Graph of function $f(x) = e^{-(x^2 + p1^2)}$ at $p1 = 1.25$.

Table 1. $I(p1)$ values.

$p1$	0	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25
$I(p1)$	1.772	1.665	1.380	1.010	0.652	0.3715	0.1868	0.0829	0.03246	0.01122	0.003422	0.000920	0.000218	0.000046

Calculating now the sum of the found values of the integral

$$I(p1) = \int_{-5}^{5} e^{-(x^2+p1)}dx$$

for different (specified in the table) parameter values $p1$, we will receive

$$\sum_{p1=0}^{3.25} I(p1) = 7.168.$$

Given formula (1) (or (2)), we find

$$\sum_{p1=0}^{3.25} I(p1) - \frac{I(0)}{2} = 7.168 - \frac{1.772}{2} = 6.282.$$

Multiplying the obtained value 6.282 by the step size $h = 0.25$, we obtain $6.282 \cdot 0.25 = 1.571$. Multiplying the obtained value 1.571 by 2 (since the parameter $p1$ was changed not from -5 to 5, but from 0 to 5, and taking into account that if $p1 > 3.25$ practically $I(p1) = 0$), as a result we will receive $1.571 \times 2 = 3.142$, that is

$$\iint_{G} e^{-(x^2+y^2)}dxdy = 3.142 \approx \pi.$$

Almost the same results are obtained in the case $h = 0.1$ or $h = 0.05$. Because,

$$\iint_{R^2} e^{-(x^2+y^2)}dxdy \approx \iint_{G} e^{-(x^2+y^2)}dxdy = \pi,$$

for

$$\iint_{R^2} e^{-(x^2+y^2)}dxdy = \int_{-\infty}^{\infty} e^{-x^2}dx \int_{-\infty}^{\infty} e^{-y^2}dy \approx \int_{-5}^{5} e^{-x^2}dx \int_{-5}^{5} e^{-y^2}dy = \sqrt{\pi}\sqrt{\pi} = \pi,$$

or

$$\frac{1}{\pi}\int_{R^2} e^{-(x^2+y^2)}dxdy = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-x^2}dx \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-y^2}dy \approx\approx \frac{1}{\sqrt{\pi}}\int_{-5}^{5} e^{-x^2}dx \frac{1}{\sqrt{\pi}}\int_{-5}^{5} e^{-y^2}dy \approx 1$$

due to the fact that with a normal probability distribution with density $f(x) = \frac{1}{\sqrt{\pi}}e^{-x^2}$ standard quadratic deviation $\sigma = \frac{1}{\sqrt{2}}$, and the probability of getting outside $(x_c - 3\sigma, x_c + 3\sigma)$ is almost zero, the probability of falling outside the set $G = [-5;5] \times [-5;5]$ is almost zero, and

$$P(G) = \iint_{G} \frac{1}{\pi}e^{-(x^2+y^2)}dxdy \approx 1.$$

Another variant of the approximate calculation of the double integral

$$\iint\limits_{G} f(x,y)dxdy,$$

where $f(x,y)$ function of the form $f(x^2 + y^2)$, G – plural form $G = \{(x,y)|x^2 + y^2 \leq R^2\}$, can be such. Put $y = 0$ and using the services of Gran1, build a graph of the function $y = f(x,0)$. Next, starting from the point $(0, f(0,0))$ and shifting towards the increase of the abscissa x, we embed in the constructed graph of the polyline function $f(x,0)$, placing the vertices of the polyline on the graph as close as possible to each other and placing such vertices along the graph of the function within the abscissa x in the set G. Next, we turn to the services of the Gran1 program "Operations", "Polygonal lines processing", "Solid of rotation volume and surface area, axis OY". As a result, we obtain the approximate value of the integral

$$\iint\limits_{G} f(x,y)dxdy.$$

Suppose, for example, you need to calculate the double integral

$$\iint\limits_{G} \frac{1}{\pi} e^{-(x^2+y^2)}dxdy,$$

where $G = \{(x,y)|x^2 + y^2 \leq 5\}$.

Using the services of Gran1, turn to the services "Object", "Create", enter the expression of a clearly defined dependence and then, turning to the services "Graph", "Plot", build a graph of a given relationship between abscissas and ordinates of points on the graph (see figure 14). Next, set the type of relationship between the variables "Polygonal line", refer to the services "Object", "Create", and in the window that appears, specify the method of specifying the vertices of the broken "Data from screen". After that, we will indicate on the constructed graph the point – the vertex of the polygonal line, for which it is necessary to place the mouse cursor in the corresponding point on the graph and press the left mouse button. As a result, such a point will be marked, and it will be given the appropriate number. After entering the last point, click "OK" in the upper right corner of the graphics window (see figure 15) and then refer to the services of Gran1 "Graph", "Plot".

To obtain the desired approximate value of the double integral

$$\iint\limits_{G} \frac{1}{\pi} e^{-(x^2+y^2)}dxdy \ , G = \{(x,y)|x^2 + y^2 \leq 5\},$$

which in the geometric interpretation is the volume under the surface $Z = f(x,y) \geq 0$ over the area G, we turn to the services of the Gran1 program "Operations", "Polygonal lines processing", "Solid of rotation volume and surface area, axis OY". As a result (see figure 16), we obtain an approximate value of the integral

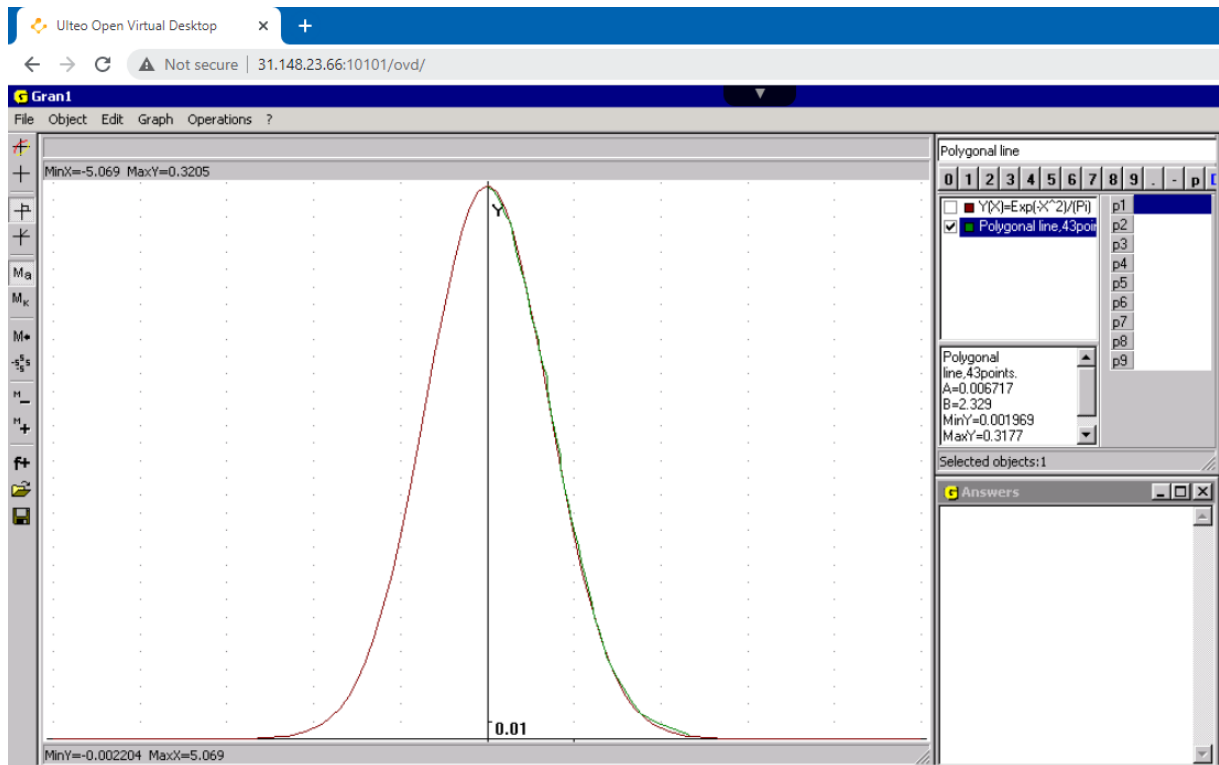$$\iint\limits_{G} \frac{1}{\pi} e^{-(x^2+y^2)}dxdy - V = 0.9849.$$

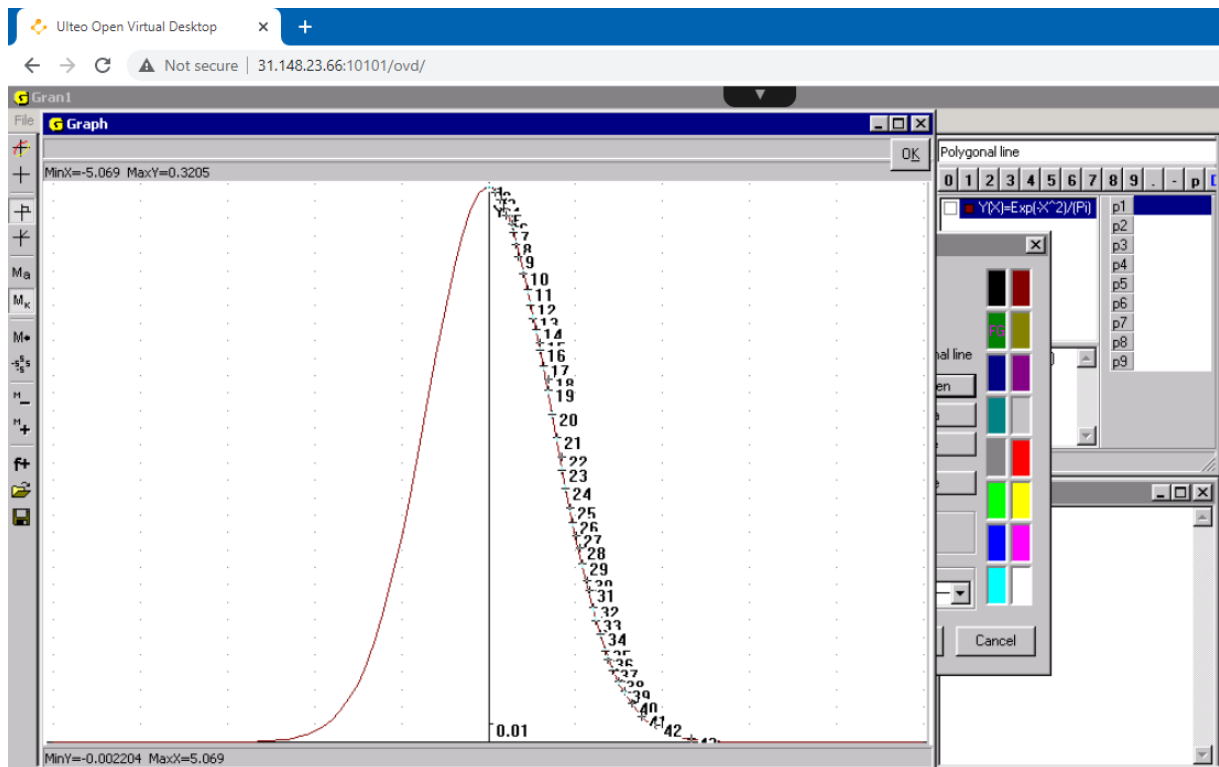**Figure 14.** "Polygonal line" plot.

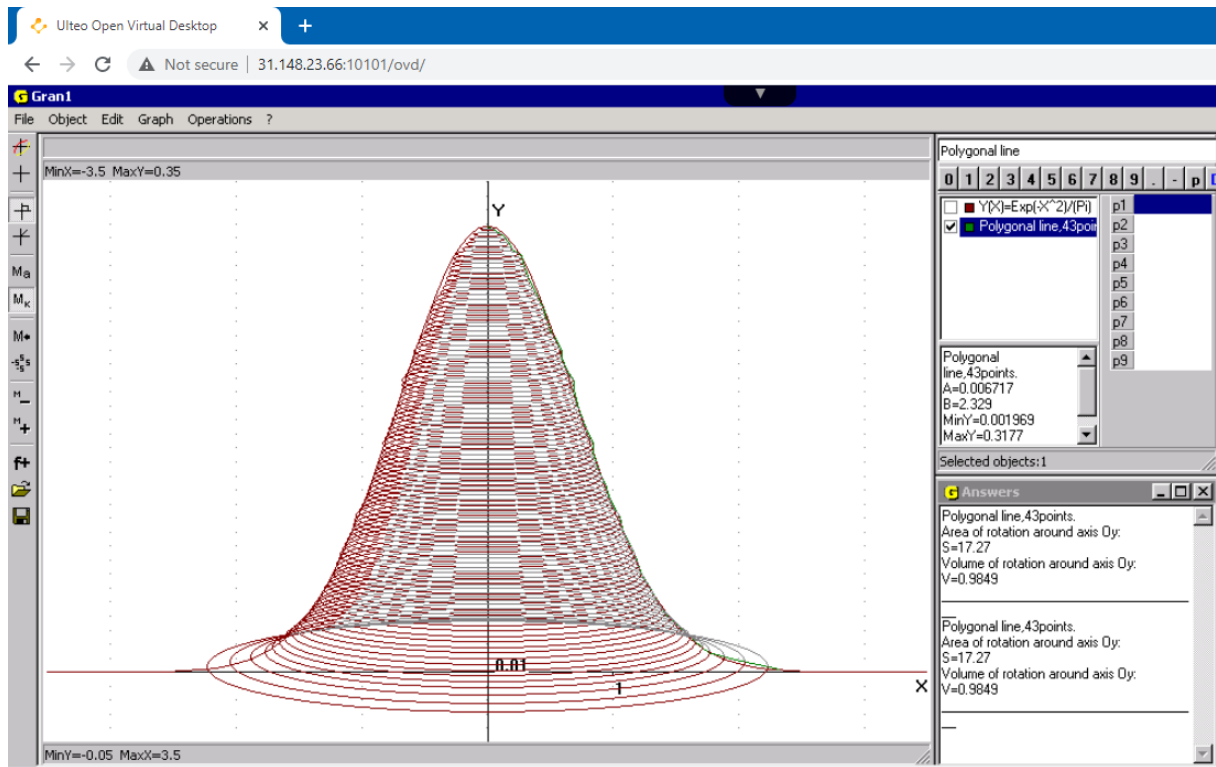**Figure 15.** Plot with numerical markers.

Figure 16. Approximate value of the integral $\iint_G \frac{1}{\pi} e^{-(x^2+y^2)} dx dy - V = 0.9849$.
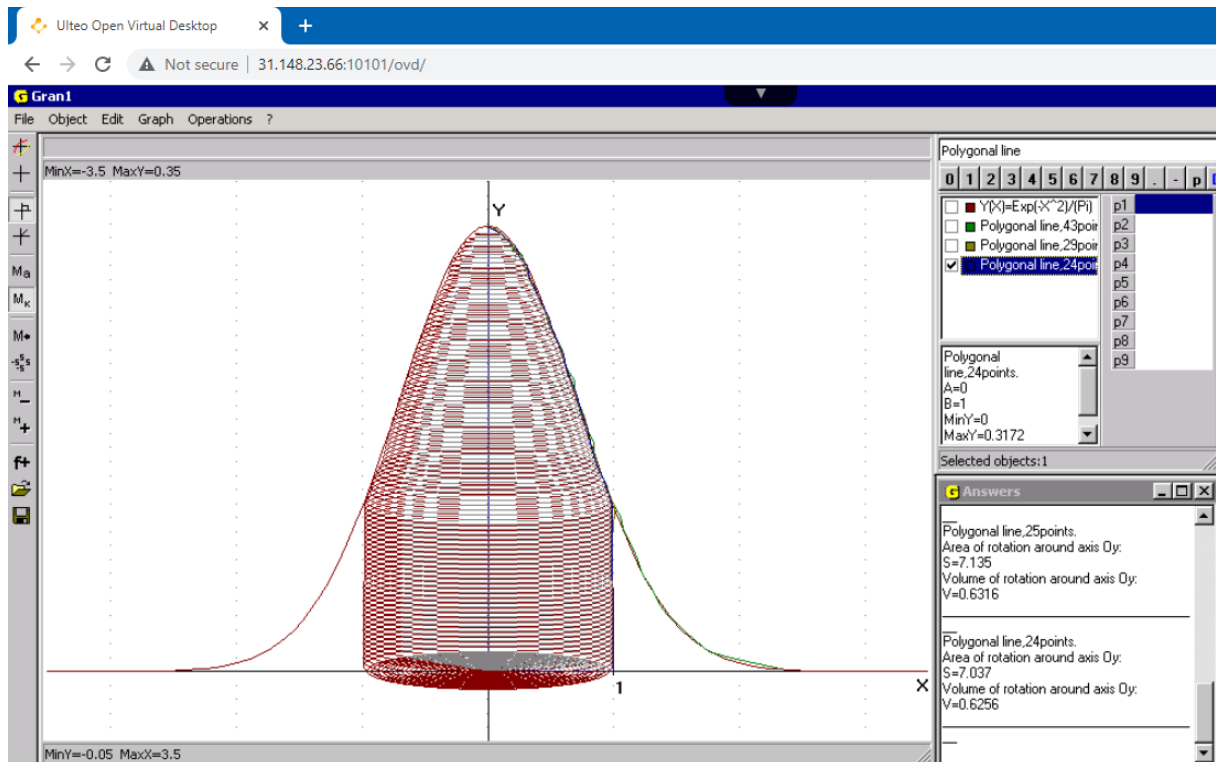

Figure 17. Example of calculating the probability of hitting a circle of radius 1 with the center.

In case, where necessary to calculate the volume of the body under the surface $Z = f(x, y) \geq 0$ over a circle of radius $r < R$, should be achieved, so that the abscissa of the rightmost vertex of the polygonal line is equal to r and further indicate another vertex on axis OX with coordinates $(r, 0)$. Then find the volume of the body of rotation of the thus formed polygonal line around the axis OY. In figure 17 shows an example of calculating the probability of hitting a circle of radius 1 with the center. Provided that on the plane XOY the probabilities are distributed with density $f(x, y) = \frac{1}{\pi} e^{-(x^2+y^2)}$. This probability is approximately equal to 0.6256.

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