Methodology of formation of modeling skills based on a constructive approach (on the example of GeoGebra)

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Abstract

Author's methodology of forming modeling skills involves 4 steps: Step 1 – the teacher step by step constructs the curve by means of cloud based service GeoGebra; Step 2 – the teacher offers a description-definition of the curve and provides a ready-made algorithm by which students model the curve independently in GeoGebra; Step 3 – the teacher offers an algorithm for constructing a curve model, and students need to characterize the properties of the curve or give its definition based on the results, Step 4 – students are offered definitions of curves that they have to model in GeoGebra). An example of realization of the author's methodology is given, the pedagogical experiment on proof of its effectiveness is described.

Keywords

modeling skills, skills development methodology, cloud service, GeoGebra cloud service, constructive approach, modeling, interesting curves, professional training

1. Introduction

Modern science operates with various methods, among which modeling is one of the most popular. Method allows you to move away from the object's ideal representation and use its analogue, which retains the most important characteristics that allow you to talk about the object properties after certain changes or influences on it.

The development of computer technologies has contributed not only to the revival of the modeling method, but also led to the emergence of specialized environments, where it became possible to model various objects (processes) based on a constructive approach. At the same time, the widespread use of smartphones and tablets that have access to the Internet has led to the emergence of cloud services, which also allow you to model objects of different nature [1, 2]. However, methods of using cloud services to develop modeling skills are just beginning

to be developed and implemented, and therefore are not well established and need experimental confirmation.

The importance of developing constructive skills of youth is emphasized in the findings of Laksha [3], Kononenko [4], Ivanina [5], and the formation of modeling skills by individual scientists is associated with the formation of research skills [6].

Analysis of the problem shows that with the advent of specialized mathematics software, its solution has focused not so much on understanding the algorithms of elementary constructions, but on the ability to use computer tools and the ability to visualize the result. The latter, in particular, is emphasized in the finding of Bilousova and Zhytyenyova [7].

Regarding the first (the ability to use tools) it should be noted the emergence of cloud services, including specialized services. Analysis of the latter showed the following.

Different aspects of educational using of cloud technologies and services are examined in the studies of national and foreign researchers. For example, Alkhansa et al. [8] explore advantages of using cloud technologies in research, such as availability of various tools and applications for analyses and collecting data, for managing and organizing references, for communication with peers and experts, the absence of constraints, the access to cloud resources from anywhere any time that has an active Internet connection, etc. They offer the approach to applying of cloud services for enhancing the productivity of university research activities, increasing competitiveness and flexibility of educational institutions. Smith et al. [9] core focus addresses Education as an area of cloud application with a cloud based e-learning system developed to demonstrate the capabilities and effectiveness of cloud technology.

Shyshkina and Popel [10] explore the problems of implementation of cloud technology tools and services in the educational process, describe the current state of development and use of cloud technology services in educational institutions and analyze the content of educational and scientific components of cloud-based educational environment.

Scientific research on the involvement of cloud services for the formation of various skills in specialists in various fields of knowledge is spreading [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23].

Currently, cloud versions of known environments with mathematical modeling capabilities are available, including Maple, MATLAB, WebMathematica, Calculation Laboratory, etc. [24]. We will also add CoCalc [25, 26, 27, 28] and GeoGebra [29, 30, 31, 32, 33, 34, 35] cloud services to this list.

Experience in using GeoGebra cloud service (visualization of mathematical objects; organization of not only analytical but also empirical search for answers in determining individual characteristics of mathematical objects; organization of home computer experiment) allowed to consider GeoGebra as means for formation of skills to model based on constructive approach. However, the analysis of scientific findings confirmed the lack of established methods for its use, and therefore the method of formation of skills to model interesting curves by GeoGebra cloud service based on a constructive approach required testing and experimental confirmation of its effectiveness.

The purpose of the paper is to describe the method of forming the skills to model interesting curves through GeoGebra cloud service based on a constructive approach and to confirm its effectiveness.

2. Material and methods

To test the effectiveness of the methodology of forming modeling skills, a pedagogical experiment, which lasted 3 years (2018–2020) was conducted among pre-service mathematics and computer science teachers.

Makarenko Sumy State Pedagogical University, Borys Grinchenko Kyiv University were the experimental base. The total number of respondents was 86 people (pre-service mathematics and computer science teachers).

The author's methodology was implemented using the freely distributed GeoGebra cloud service (https://www.geogebra.org) in the special courses. The experimental module "Modeling of interesting curves" with a volume of 10 classroom hours (2 hours of lectures and 8 hours of laboratory classes) was introduced.

Two tests of educational achievements (at the beginning and after studying the module) were organized to confirm the effectiveness of the methodology.

Students were offered a typical test problem for modeling a conic by its geometric definition (problems differed from each other in the input conditions, for example, the distance between the foci was different or one of the foci was in the specific point in a given coordinate system, etc.).

The solution of the problem was evaluated by the following indicators (table 1).

Table 1
Indicators of the formation of modeling skills

No	Indicators	Marks
1	Ability to take into account the analytical relationship between the elements	1
2	Ability to take into account the geometric relationship between the elements	1
3	Ability to use the Locus tool	1
4	Ability to use the Trace tool	1
5	Ability to demonstrate the change of the curve shape when changing the input data	1
6	Ability to construct a model visually correct (location, color, size and style of geometric objects)	1
7	Ability to add dynamic text to study numerical characteristics	1
8	Ability to write an algorithm for constructing a model	1
9	Ability to reproduce the steps of the algorithm to construct the model	1
10	Ability to interpret the result of a computer experiment	1

Since the results of the tests were dependent and each time provided for the accumulation of marks, the sign test was used. The number of respondents, whose total score decreased ("−"), did not change ("0") and increased ("+"), was fixed.

In accordance with the purpose of the experiment and the sign test, the null hypothesis was formulated: the author's methodology does not contribute to the formation of modeling skills by means of GeoGebra cloud service. Then the alternative hypothesis was "the author's methodology contributes to the formation of such skills".

The constructed hypotheses define the one-sided sign test for checking the dependent samples.

According to the decision-making rule [36] the null hypothesis of inefficiency / effectiveness of the author's methodology was accepted or rejected.

3. Constructive approach

The constructive approach is characterized by the fact that acquaintance with the properties of concepts begins with constructive activities for their "discovery" and assimilation with a gradual transition to definitions and logical proofs. This, in particular, simplifies the perception of the Geometry course, makes it more accessible, while raising the scientific level through the intensification of educatees' research activities.

The use of the constructive approach contributes to the fact that the activity is manifested in the gradual transition of actions for the construction of objects from the executive level (is characterized by external regulation), then to reproductive (is marked by internal regulation of actions in the construction of known structures), then to the applied level (using the method of construction), and, finally, the creative level (involves the construction of new objects) (figure 1) [37].
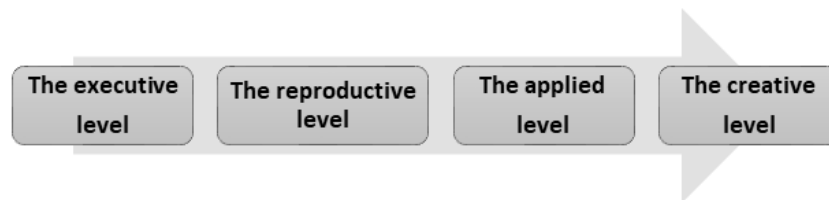
Figure 1: Constructive approach activity levels.

We consider constructive tasks to be one of the means of implementing a constructive approach. Many authors use the term "constructive tasks" in a sense identical to construction problems. However, the peculiarities of the constructive approach require the use for its implementation and other tasks, the solution of which requires the implementation of a particular constructive activity. Therefore, for example, Lisimova [38] considers constructive tasks as tasks for construction, imaging, measurement, geometric design and structural-geometric modeling. Dalinger [39] considers constructive tasks as "problems in the process of solving which reveal the material conditions of geometric figures. Their purpose is to identify the essential features of ideas that are formed through the material conditions of their origin".

Constructive tasks can be a base for establishing new properties of figures (concepts formation), for the consolidation of knowledge, for repetition, control, intensification of research activities. "The peculiarity of constructive tasks is that they can be solved both logically and figuratively or visually effective" [38].

When solving constructive tasks, there is a productive activity that motivates to think independently (methods of constructing have to be developed independently), and not reproductive activity, which is often not an independent mental process, but is a repetition of known steps [40].

Mastering the system of knowledge and skills can take place on two levels: constructive and

analytical. The main feature of the constructive level of mastery of the material is its visual and constructive awareness. This is manifested:

- in the ability to recognize objects that belong and do not belong to the content of this concept, give examples, demonstrate the existence of the studied figures by construction;
- in the knowledge of their most essential properties and the ability to apply known properties in solving problems.

The constructive approach provides "points of support" for Geometry study at a higher abstract level, which is characterized by the ability to formulate definitions of concepts, statements and prove already in formal language, rather than the language of geometric images.

4. Methodology of forming skills to model interesting curves based on a constructive approach in GeoGebra cloud service

Any methodology involves the presence of five elements (figure 2): purpose, content, forms, methods and means. Let us explain them in more detail.
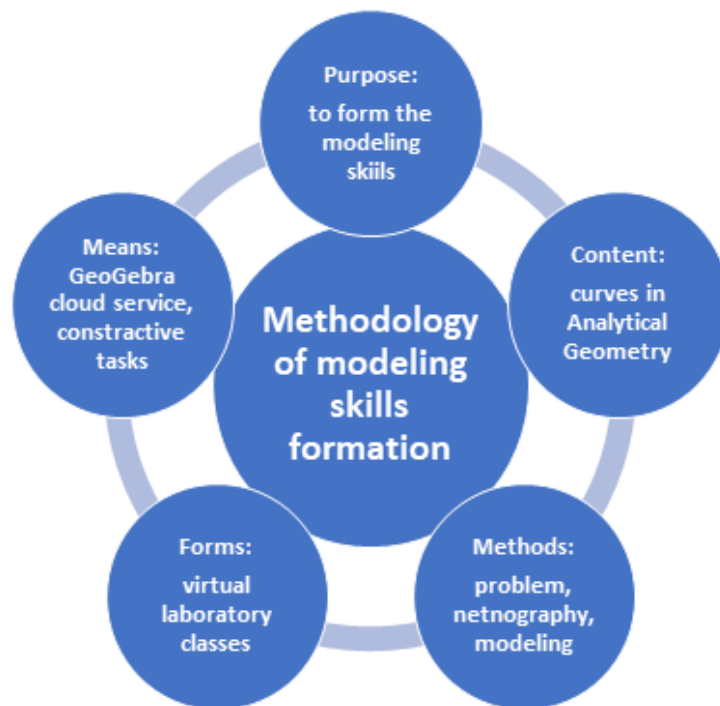
Figure 2: Components of the author's methodology.

The purpose of the author's methodology is to form the skill to model interesting curves using cloud services.

Interesting curves of the course of Analytical Geometry are the content on which the author's methodology is realized. Nowadays, their study in the classical course of Analytical Geometry is possible in three ways:

- analytical description and further study of curves – at first analytical (parametric, implicit, explicit) equations, usually of conics, are given and then they are investigated;
- study of curves as locus with a given property – at first the geometric definitions of curves are given, on the basis of which their analytical equations are written (as a rule, these are conics, conchoid of Nicomedes, limaçon of Pascal, strophoid, cissoid of Diocles, lemniscate of Bernoulli, Cassini oval);
- study of curves generated in the kinematic way (as the trajectory of a point), usually cycloidal curves, folium of Descartes, witch of Agnesi, logarithmic spiral.

Due to lack of time, there is only the first way of the study of interesting curves in the course of Analytical Geometry at universities. This limits the possibility to develop the skills to model mathematical objects.

The form of realization of the author's methodology is virtual lecture-practical classes or a combination of visualized lectures and virtual laboratory classes.

Among the teaching methods problem method and netnography method are used.

GeoGebra cloud service, constructive tasks and instructional materials are learning means.

Elementary knowledge in Analytical Geometry is required to use the author's methodology.

In general, the methodology of forming the skills to model interesting curves is as follows: the teacher on a common online platform informs about constructive approaches to the construction of various curves, which were studied in ancient times (the netnography method). He briefly tells about how such constructions were done (compass and ruler, one compass, two compasses, one or two rulers, etc.), or gives an example of practically oriented problems that are solved using interesting curves. After that, the teacher demonstrates one of the described constructions in GeoGebra cloud service, and then asks students to write an algorithm for the above construction (the problem method). Then the teacher asks students according to the algorithm (provided to each student) to reproduce the construction and demonstrate the result, to analyze errors, and if not, to analyze possible limit cases. After that, students are offered the definition of curves generated in mechanical way together with the algorithms of their construction in the cloud service; students must model this type of curve according to the existing algorithm. Then together with students, the task of the following type is carried out: algorithms of curves construction are offered and after their construction, students need to characterize properties of the modeled curves, to give them definitions or kinematic characteristics. After completing this type of task, students are offered only the definition of the curves or their kinematic characteristics, and they must model the curve themselves.

The methodology should be briefly described as follows.

Step 1 – the teacher offers an example of a curve model through the definition (the teacher step by step models the curve), and students must independently compile an algorithm for constructing the model.

Step 2 – the teacher offers a description-definition of the curve and provides a ready-made algorithm according to which students model the curve independently.

Step 3 – the teacher offers an algorithm for constructing a curve model, and students need to characterize the properties of the curve or give its definition based on the results.

Step 4 – students are offered definitions of curves that they have to model them.

We will briefly demonstrate the application of the methodology of forming the skiils to model interesting curves based on a constructive approach in GeoGebra cloud service on the example of the hypocycloids study.

The teacher formulates the problem. The calculation of the action of the kneading machine consists of a description of the trajectory of the point belonging to the working body. The trajectory of the working body does not always fill the container with the substance, as a result, the mixing time increases, which leads to unnecessary energy consumption. Determine the optimal trajectory of the point of the mixer working body [41].

History of the curve discovery (netnography method). Hypocycloid (from Greek words $\upsilon\pi o$ – "under, below" and $\kappa\upsilon\kappa\lambda o\zeta$ – "circle") – is a curve "under the circle". The most famous hypocycloids are the astroid (from Greek "star-shaped") and the deltoid (Steiner curve). The name of the astroid (from "cusp" – exacerbation) was proposed by the Austrian astronomer Carl-Ludwig von Littrow (1811–1877). The shape of the deltoid resembles the capital Greek letter delta, its properties were studied by Leonard Euler in the XVIII century, and then by Jacob Steiner in 1856, after whom it was renamed the Steiner curve.

Definition of the curve. A hypocycloid is a curve generated by the trace of a fixed point on a small circle that rolls within a larger circle.

Next, the teacher demonstrates an example of a ready-made hypocycloid model (figures 3-4), and then offers to describe the algorithm for its construction to students.
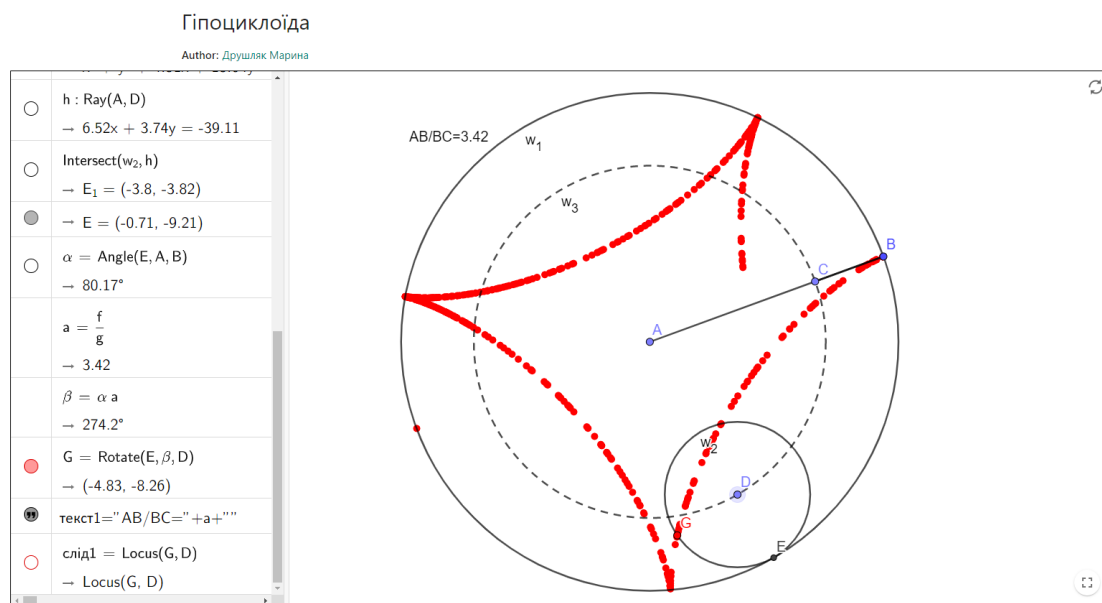
Figure 3: Construction of a hypocycloid using the Trace tool.

Let the circle ω_2 roll inside the circle ω_1. Let's construct a curve generated by a point on the circle ω_2.
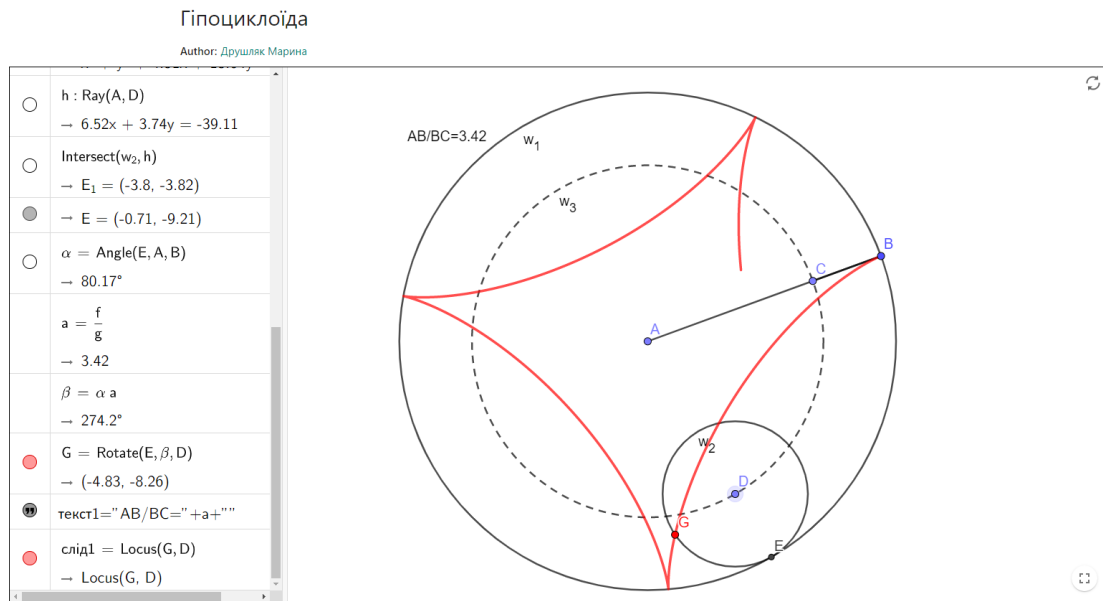
Figure 4: Construction of a hypocycloid using the Locus tool.

Algorithm for hypocycloid model constructing

1. Construct a circle ω_1 with center A passing through point B.
2. Construct the segment AB.
3. Construct an arbitrary point C on the radius AB.
4. Construct the circle ω_3 with center A passing through point C (dotted line).
5. Construct an arbitrary point D on the circle ω_3.
6. Construct the circle ω_2 with center D, the radius of which is equal to the length of the segment CB.
7. Construct the ray AD.
8. Constract the point E – the intersection of the circle ω_2 with the ray AD.
9. Hide the ray AD.
10. Let the center D of the circle ω_2 coincides with the point C in the initial position. If we move the point D around the circle, it will rotate around the point A, and remains tangent to the circle ω_1. Let the circle ω_2 rotate around it when moving the center to create the illusion that the circle ω_2 rolls on the circle ω_1. Only one point of the circle ω_2 should be investigated. Let this point coincides with the point B at the initial moment, and is the point of tangency of the circles ω_1 and ω_2. If the circle ω_2 rolls on the arc BE, then the center D of the circle will return around the point A at the angle $\psi = BAE$. Simple calculations show that all points of the circle ω_2 will rotate around its center at the angle $\psi * \frac{AB}{CB}$.
11. Construct the point G, which is obtained from the point E by rotating around the point D by the angle $\psi * \frac{AB}{CB}$.

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12. Hide the point E.
13.   a) Make the point G leave a trace by rolling the point D on the circle ω_2 (Fig. 3).
         b) Use the Locus tool: the point G is the "pencil point", the point D is the "driver point" (figure 4).

Next, the lecturer at the visualized lecture offers students to explore the analytical equation of the curve, which is described through the parameter.

Let R be the radius of a fixed circle, r be the radius of a rolling circle, $k = \frac{R}{r}$, then the parametric equation of a hypocycloid is

$$x = r*(k-1)*(cos\psi - \frac{cos(k-1)*\psi)}{k-1}), y = r*(k-1)*(sin\psi - \frac{sin(k-1)*\psi)}{k-1}).$$

The modulus of k determines the shape of the hypocycloid: $k = 2$ the hypocycloid is described by Tusi couple (diameter of a fixed circle), $k = 4$ by the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = R^{\frac{2}{3}}$.

Study of the curve and establishment of properties (problem method). When the hypocycloid is constructed, we pass to establishment of its properties at the level of practical research.

Note that the curve constructed in this way is open. Let us investigate the conditions under which the constructed hypocycloid will be a closed curve. By changing the radius of the circle ω_3, thereby changing the ratio $k = \frac{R}{r}$ of the radii of fixed and rolling circles, we conclude that the curve will be closed when the ratio $k = \frac{R}{r} \in Z$ (figure 5). In addition, in this case, the hypocycloid consists of k identical arcs, the ends of which lie on a fixed circle (figure 6).
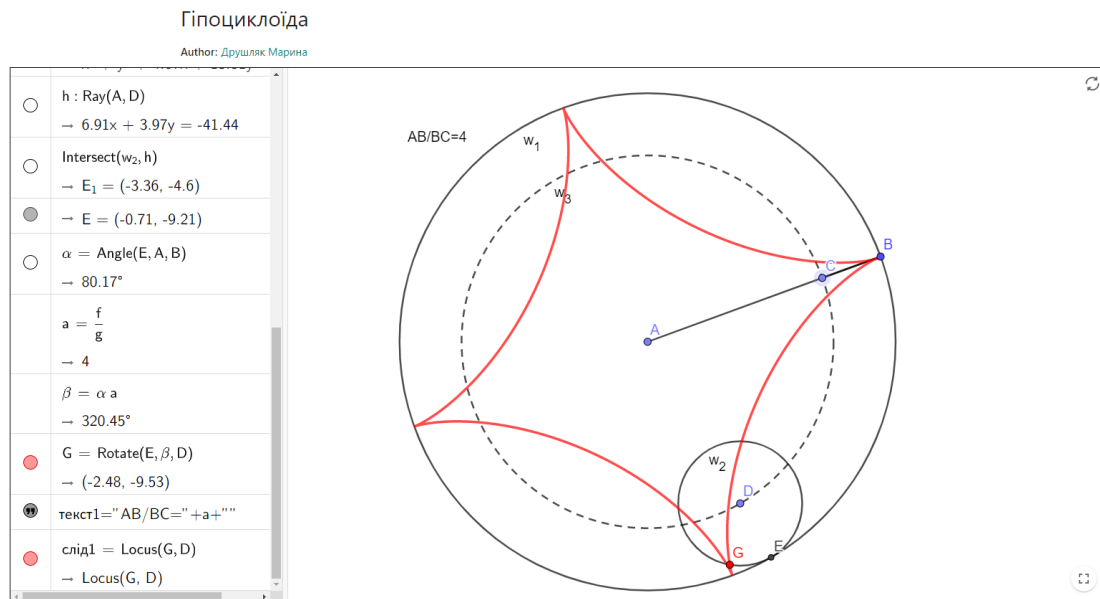
Figure 5: Closed hypocycloid, $k = \frac{R}{r} \in Z$.
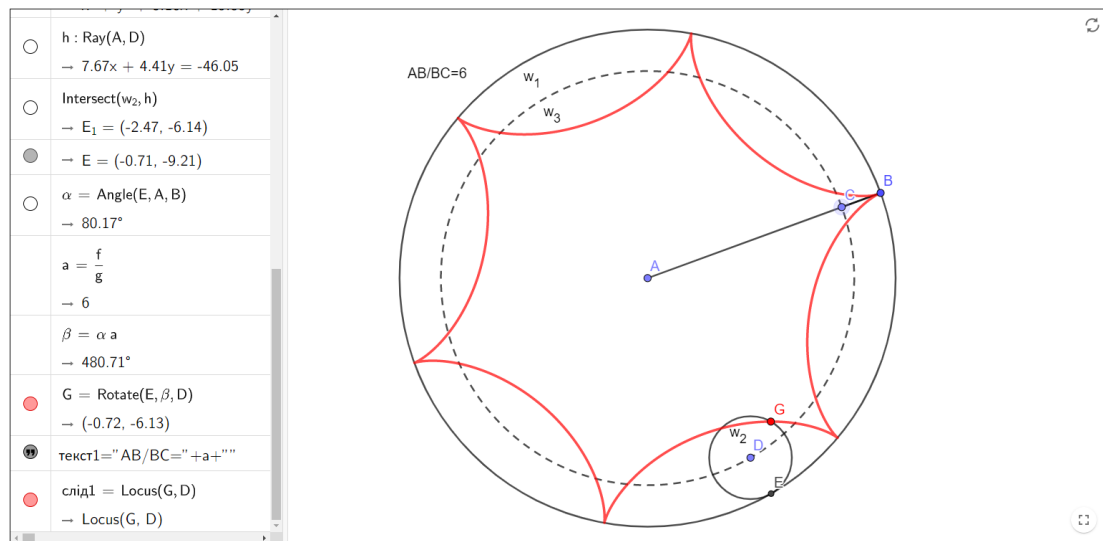
Figure 6: The hypocycloid consists of $\frac{R}{r} = 6$ of identical arcs.

If the radius r of the rolling circle is twice smaller than the radius R of the fixed circle, then the hypocycloid degenerates into a segment (figure 7). This statement is well known as Copernicus theorem.

After the lecture, students are first asked to construct curves models according to the given definition and algorithm, for example, to costruct a cycloid, which is a curve generated by a point of a circle rolling on a straight line, reproducing all the steps of the algorithm. Not only instructions but also possible consultations are provided.

The next lesson tasks such as: "Establish the relationship between the number of arcs of the epicycloid, generated by a point of a circle rolling on the outside of another circle, and the ratio of the radii of these circles", are offered.

Then, at the next two lessons, tasks such as: "A point and a line, the distance between which is equal to a, are given. A line is drawn through an arbitrary point X of the line and the point. The points at a distance b from point X are marked. Find the locus of such points".

The described methodology differs from the traditional one, as the latter usually involves only step 2 and step 4, which does not allow students to understand the algorithms involved in the constructing the model, to develop the ability to compare step-by-step construction ideas and the algorithm steps. There is also no practice of qualitative analysis of the algorithm (which is provided by step 3 and which is supported by the "step-by-step playback service" in GeoGebra). This is what we consider fundamentally important for effective formation of skills to model and what GeoGebra service makes possible.

Let us note also that in terms of distance learning, it is GeoGebra service that allows the demonstration of constructions and the ability to work with models at anytime from anywhere.
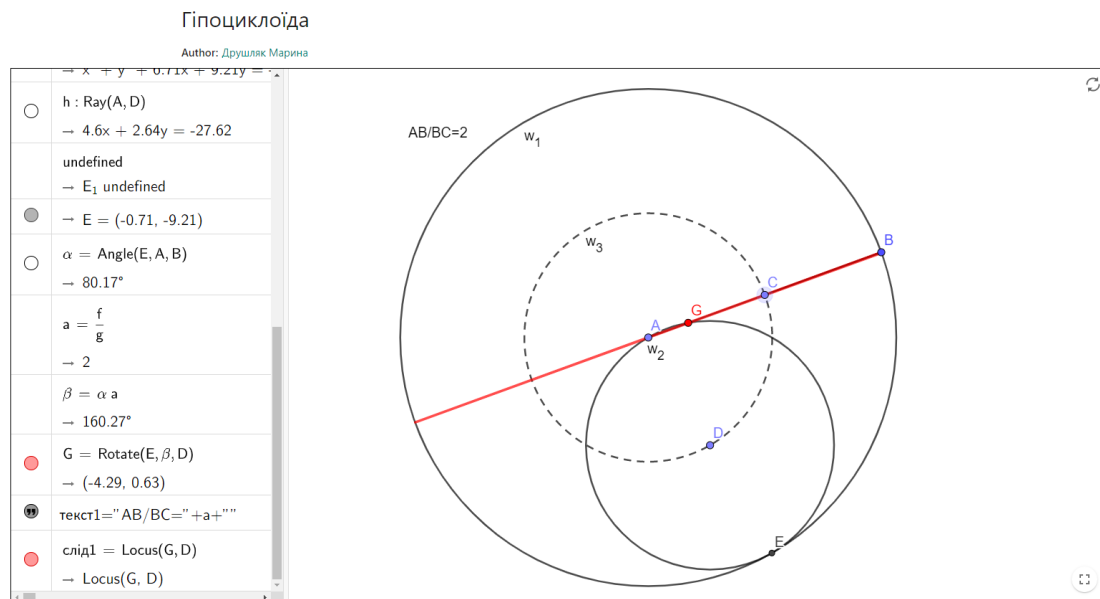
Figure 7: Degenerate hypocycloid, $\frac{R}{r} = 2$.

5. Statistical analysis of experiment results

Verification of the effectiveness of the author's methodology was carried out based on two tests. Students had to solve the problem 1 (for example, to construct a curve, for each point of which the sum of distances to two given points is constant) before studying the topic and the problem 2 (for example, to construct a curve, each point of which is equidistant from the point lying in the third quarter, and a line that forms an acute angle with the abscissa) after studying the topic.

The evaluation of solutions was carried out through the assessment of skills: to take into account the analytical and geometric relationship between the elements; to use the Locus and Trace tools; to visualize the model successfully; to add dynamic text to study numerical characteristics; to write an algorithm for constructing a model; to reproduce the steps of the algorithm to construct the model; to interpret the result of a computer experiment (in more detail in table 1).

The results of the controls are presented in table 2.

These marks were used to determine the number of respondents whose total score decreased ("−"), did not change ("0") and increased ("+") (table 3).

According to the decision-making rule [36] we have: the value of $T_{exp} = 46$ (this is the number of "+" signs in the sample), $n = 67$ (this is the number of respondents who have changed the results), the acceptance interval of the hypothesis H_0 is $[26, 41]$ at the significant level of 0.05.

Since T_{exp} is not included in the acceptance interval of the hypothesis H_0, we reject the hypothesis H_0 and accept the alternative one with the conclusion that the author's methodology contributes to the formation of modeling skills. Since the value of T_{exp} is beyond to the segment on the right, it is necessary to conclude there is the positive dynamics of the number of such

Table 2
The results of students' tests

No	Test 1	Test 2	No	Test 1	Test 2	No	Test 1	Test 2
1	3	4	30	3	5	59	4	5
2	4	5	31	4	4	60	3	4
3	5	4	32	4	6	61	3	4
4	4	3	33	4	6	62	5	4
5	5	3	34	4	7	63	5	5
6	5	5	35	5	4	64	5	4
7	5	6	36	5	3	65	6	4
8	6	6	37	5	3	66	5	6
9	4	5	38	5	5	67	4	7
10	3	4	39	6	6	68	3	7
11	3	3	40	5	7	69	2	6
12	3	3	41	4	5	70	7	5
13	4	2	42	4	4	71	5	8
14	4	6	43	6	4	72	4	8
15	5	6	44	7	5	73	3	9
16	4	7	45	6	6	74	3	8
17	5	6	46	5	7	75	4	7
18	4	5	47	7	7	76	5	6
19	7	4	48	8	7	77	5	6
20	6	5	49	8	6	78	5	6
21	6	3	50	3	6	79	6	7
22	5	5	51	4	7	80	5	6
23	7	6	52	4	7	81	5	8
24	7	6	53	4	7	82	6	7
25	7	7	54	1	7	83	5	6
26	4	4	55	2	7	84	6	6
27	5	3	56	4	5	85	5	5
28	5	5	57	4	4	86	3	6
29	5	5	58	3	4	-	-	-

Table 3
Dynamics of marks according to the results of students' tests

Dynamics	Negative "–"	Without changes "0"	Positive "+"	Changes $n=$"–"+"+"
Members	21	19	46	25

students who have developed the skills to model interesting curves using GeoGebra cloud service.

6. Conclusions

1. Author's methodology of forming modeling skills involves 4 steps: Step 1 – the teacher offers an example of a curve model through the definition (the teacher step by step constructs the curve), and students must write the constructing algorithm independently; Step 2 – the teacher offers a description-definition of the curve and provides a ready-made algorithm by which students model the curve independently; Step 3 – the teacher offers an algorithm for constructing a curve model, and students need to characterize the properties of the curve or give its definition based on the results, Step 4 – students are offered to define curves that they have to model). The methodology effectiveness was confirmed by the sign test at the significant level of 0.05.

2. GeoGebra cloud service is effective for being perceived as a means for the formation of students' (pre-service mathematics and computer science teachers) modeling skills. At the same time, we consider it is also possible to use this methodology in the conditions of traditional training with GeoGebra installed on PC.

References

[1] O. Markova, S. Semerikov, M. Popel, CoCalc as a learning tool for neural network simulation in the special course "Foundations of mathematic informatics", CEUR Workshop Proceedings 2104 (2018) 388–403.

[2] S. Semerikov, I. Teplytskyi, Y. Yechkalo, O. Markova, V. Soloviev, A. Kiv, Computer simulation of neural networks using spreadsheets: Dr. Anderson, welcome back, CEUR Workshop Proceedings 2393 (2019) 833–848.

[3] E. Laksha, The role of students' constructive skills in preparation for professional activity, in: 2008 Conference Creativity and research in mathematics education, 2008, pp. 41–43.

[4] N. Kononenko, Features of the development of the constructive component of mental activity in the field of geometry in schoolchildren, Scientific notes of the Trans-Baikal State University. Series: Physics, mathematics, engineering, technology (2010) 128–131.

[5] T. Ivanina, The role of constructive activity in the development of the child, Bulletin of the Krasnoyarsk State Pedagogical University (2010) 19–23.

[6] V. Pikalova, L. Novakovska, Formation of research skills of students in studying the topic second level curves, in: All-Ukrainian Conference Designing a learning environment as a methodological problem, 2007, pp. 107–109.

[7] L. I. Bilousova, N. V. Zhytyenyova, Functional approach to the use of technology of visualization for intensification of learning process, Information Technologies and Learning Tools 57 (2017) 38–49. URL: https://journal.iitta.gov.ua/index.php/itlt/article/view/1525. doi:10.33407/itlt.v57i1.1525.

[8] A. Alkhansa, A. Shakeabubakor, E. Sundararajan, A. R. Hamdan, Cloud computing services and applications to improve productivity of university researchers, International Journal of Information and Electronics Engineering 5 (2015) 153–157. doi:10.7763/IJIEE.2015.V5.521.

[9] A. Smith, J. Bhogal, M. Sharma, Cloud computing: Adoption considerations for business

and education, in: 2014 International Conference on Future Internet of Things and Cloud, 2014, pp. 302–307. doi:`10.1109/FiCloud.2014.54`.

[10] M. P. Shyshkina, M. V. Popel, Cloud based learning environment of educational institutions: The current state and research prospects, Information Technologies and Learning Tools 37 (2013) 66–80. URL: https://journal.iitta.gov.ua/index.php/itlt/article/view/903. doi:`10.33407/itlt.v37i5.903`.

[11] P. Merzlykin, M. Popel, S. Shokaliuk, Services of SageMathCloud environment and their didactic potential in learning of informatics and mathematical disciplines, CEUR Workshop Proceedings 2168 (2017) 13–19.

[12] M. Popel, S. Shokalyuk, M. Shyshkina, The learning technique of the SageMathCloud use for students collaboration support, CEUR Workshop Proceedings 1844 (2017) 327–339.

[13] O. Bondarenko, O. Pakhomova, V. Zaselskiy, The use of cloud technologies when studying geography by higher school students, CEUR Workshop Proceedings 2433 (2019) 377–390.

[14] I. Kholoshyn, O. Bondarenko, O. Hanchuk, E. Shmeltser, Cloud ArcGIS Online as an innovative tool for developing geoinformation competence with future geography teachers, CEUR Workshop Proceedings 2433 (2019) 403–412.

[15] A. Kiv, V. Soloviev, S. Semerikov, CTE 2018 – How cloud technologies continues to transform education, CEUR Workshop Proceedings 2433 (2019) 1–19. URL: http://ceur-ws.org/Vol-2433/paper00.pdf.

[16] O. Markova, S. Semerikov, A. Striuk, H. Shalatska, P. Nechypurenko, V. Tron, Implementation of cloud service models in training of future information technology specialists, CEUR Workshop Proceedings 2433 (2019) 499–515.

[17] P. Nechypurenko, T. Selivanova, M. Chernova, Using the cloud-oriented virtual chemical laboratory VLab in teaching the solution of experimental problems in chemistry of 9th grade students, CEUR Workshop Proceedings 2393 (2019) 968–983.

[18] M. Volikova, T. Armash, Y. Yechkalo, V. Zaselskiy, Practical use of cloud services for organization of future specialists professional training, CEUR Workshop Proceedings 2433 (2019) 486–498.

[19] I. Kholoshyn, O. Bondarenko, O. Hanchuk, I. Varfolomyeyeva, Cloud technologies as a tool of creating Earth Remote Sensing educational resources, CEUR Workshop Proceedings 2643 (2020) 474–486.

[20] A. Kiv, M. Shyshkina, S. Semerikov, A. Striuk, M. Striuk, H. Shalatska, CTE 2019 - When cloud technologies ruled the education, CEUR Workshop Proceedings 2643 (2020) 1–59. URL: http://ceur-ws.org/Vol-2643/paper00.pdf.

[21] O. Korotun, T. Vakaliuk, V. Soloviev, Model of using cloud-based environment in training databases of future IT specialists, CEUR Workshop Proceedings 2643 (2020) 281–292.

[22] V. Y. Velychko, E. H. Fedorenko, N. V. Kaidan, V. N. Soloviev, O. V. Bondarenko, The support of the process of training pre-service mathematics teachers by means of cloud services, CEUR Workshop Proceedings (2020, in press).

[23] T. A. Vakaliuk, O. V. Korotun, S. O. Semerikov, The selection of cloud services for ER-diagrams construction in IT specialists databases teaching, CEUR Workshop Proceedings (2020, in press).

[24] M. Shyshkina, Formation and development of a cloud-oriented educational and scientific environment of a higher educational institution, UkrINTEI, Kyiv, 2015.

[25] M. Popel, CoCalc cloud service as a means of forming professional competencies of a mathematics teacher, Vydavnychyj centr Kryvorizkogo nacionalnogo universytetu, Kryvyi Rih, 2018.

[26] I. Lovianova, D. Bobyliev, A. Uchitel, Cloud calculations within the optional course Optimization Problems for 10th-11th graders, CEUR Workshop Proceedings 2433 (2019) 459–471.

[27] K. Vlasenko, O. Chumak, D. Bobyliev, I. Lovianova, I. Sitak, Development of an online-course syllabus "Operations research oriented to cloud computing in the CoCalc system", CEUR Workshop Proceedings 2740 (2020) 278–291.

[28] D. Y. Bobyliev, E. V. Vihrova, Problems and prospects of distance learning in teaching fundamental subjects to future mathematics teachers, Journal of Physics: Conference Series 1840 (2021) 012002. URL: https://doi.org/10.1088/1742-6596/1840/1/012002. doi:10.1088/1742-6596/1840/1/012002.

[29] Geogebra, 2020. URL: https://www.geogebra.org.

[30] O. V. Semenikhina, M. G. Drushliak, Y. V. Khvorostina, Use of GeoGebra cloud service in future math teachers' teaching, Information Technologies and Learning Tools 73 (2019) 48–66. URL: https://journal.iitta.gov.ua/index.php/itlt/article/view/2500. doi:10.33407/itlt.v73i5.2500.

[31] E. Semenikhina, M. Drushlyak, Y. Bondarenko, S. Kondratiuk, N. Dehtiarova, Cloud-based service GeoGebra and its use in the educational process: the BYOD-approach, TEM Journal 8 (2019) 65–72. doi:10.18421/TEM81-08.

[32] Y. Botuzova, GeoGebra dynamic models at the mathematics lessons as a STEM-approach, Physical and Mathematical Education (2018) 31–35.

[33] T. Kramarenko, O. Pylypenko, V. Zaselskiy, Prospects of using the augmented reality application in STEM-based Mathematics teaching, CEUR Workshop Proceedings 2547 (2020) 130–144.

[34] T. Kramarenko, O. Pylypenko, I. Muzyka, Application of GeoGebra in Stereometry teaching, CEUR Workshop Proceedings 2643 (2020) 705–718.

[35] O. Hrybiuk, System of dynamic mathematics of GeoGebra as a means of supporting general and special abilities of students in the process of research learning: practical work experience, Physical and Mathematical Education (2020) 37–51.

[36] M. Grabar, K. Krasnjanskaja, Application of mathematical statistics in pedagogical researches. Nonparametric methods, Pedagogika, Moscow, 1977.

[37] L. Tuholko, The development of constructive activity of students in the X-XI grades in teaching geometry, Ph.D. thesis, Belorus State University, Minsk, 2018.

[38] O. Lisimova, The system of design tasks as a method for studying planimetry in grade 7, Ph.D. thesis, Saint Petersburg, 1997.

[39] V. Dalinger, Methodological features of teaching students planimetry in a stereometric environment, Modern high technologies (2012) 59–61.

[40] A. Chuhnov, Constructive problems in olympiads in mathematics and computer science, Computer tools in education (2018) 56–62. doi:10.32603/2071-2340-2018-6-56-62.

[41] A. Dashkevich, Geometric modeling of schemes of action of planetary kneading machines, Ph.D. thesis, Kyiv, 2008.