MARKOV CHAINS APPLICATION
TO THE FINANCIAL-ECONOMIC TIME SERIES PREDICTION

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In this research the technology of complex Markov chains is applied to predict financial time series. The main distinction of complex or high-order Markov Chains and simple first-order ones is the existing of after-effect or memory. The technology proposes prediction with the hierarchy of time discretization intervals and splicing procedure for the prediction results at the different frequency levels to the single prediction output time series. The hierarchy of time discretizations gives a possibility to use fractal properties of the given time series to make prediction on the different frequencies of the series. The prediction results for world's stock market indices are presented.

Keywords: Prediction, time series, complex Markov chains, discrete time, fractal properties

1. Introduction

Successful modelling and prediction of processes peculiar to complex systems, such as ecological, social, and economical (ESE) ones, remain one of the most relevant problems as applied to the whole complex of natural, human and social sciences ([1–2]). The diversity of possible approaches to modelling such systems and, usually, more than modest success in the dynamics prediction, compel us to look for the reasons of failure, finding them not only in details, but also in the axiomatics, which relates to problem statement, chosen modelling methods, results interpretation, connections with other scientific directions.

With the appearance of quantum mechanics and relativity theory in early twentieth century new philosophical ideas on physical values, measuring procedures and system state have been established, the ones that are completely different from Newtonian notions [3].

For more than 70 years basic concepts of classical and neoclassical economic theories have been discussed by leading scientists, generating new approaches. The general systems theory has acquired recognition in the middle of the 20th century giving way to development of the new, systemic, emergent, and quantum in essence approach to investigation of complex objects, which postulates the limited nature of any kind of modelling and is based upon fixed and closed system of axioms [4].

However, the development of this new philosophical basis of ESE systems modelling is still accompanied with numerous difficulties, and new principles are often merely declared. Current research is devoted to investigation and application of the new modelling and prediction technology, suggested in [5–6], based on concepts of determined chaos, complex Markov chains and hierarchic (in terms of time scale) organization of calculating procedures.

2. Analysis of Prominent Publications Relevant to the Subject

Prediction of financial-economic time series is an extremely urgent task. Modern approaches to the problem can be characterized by the following directions: 1) approximation of a time series using an analytical function and extrapolation of the derived function towards future – so-called trend models [7]; 2) investigation of the possible influence various factors might have on the index, which is being predicted, as well as development of econometric or more complicated models using the Group Method of Data Handling (GMDH) [2, 8]; 3) modelling future prices as the decisions-making results using neuronal networks, genetic algorithms, fuzzy sets [8–9]. Unfortunately, these techniques don't produce stable forecasts, what can be explained by complexity of the investigated systems, constant changes

in their structure. Although we are trying to join these directions in one algorithm, it is the latter option that we prefer, with it consisting in creating a model adequate to the process generating a price time series [10–11]. This very approach gives a chance to approach the complexity of the system, which generates the observed series, develop the model and use its properties as the prognosis.

3. Aims of the Paper, Problem Statement

Let's assume the time series is set by a sequence of discrete levels with constant step of time sampling Δt. We need to generate variants of the time series continuation (prognosis scenarios) according to the relations between the sequences of absolute and relative changes discovered with the help of complex Markov chains.

4. Markov Chains Prediction Technology

Let's suppose there is a sequence of a certain system discrete states. From this sequence we can determine transitions probabilities between the two states. Simple Markov chain is a random process, in which the next state probability depends solely on the previous state and is independent from the rest of them. Complex Markov chain, unlike the simple one, stands for the random process, in which the next state probability depends not only on the current, but also on the sequence of several previous states (history). The amount of states in history is the order of the Markov chain.

Theory of simple Markov chains is widely presented in literature, for example [12]. Developing complex or high order Markov chain's properties is not widely presented in modern scientific publications. It's necessary to mention the papers [13, 14] where properties of complex Markov Chains are developed, but no prediction algorithm is proposed there. The development of prediction method, based on complex Markov chains, is proposed in this paper.

Markov chain of the higher order can be brought to a simple Markov chain by introducing the notion of a "generalized state" and including a series of consequent system's states into it. In this case, tools of simple Markov chains can be applied to the complex ones.

Investigated dynamic series is a result of a certain process. It is assumed that this process is determined, which implies the existence of a causal dependence of further states on history. It is impossible to fix and analyse the infinite history, which puts obstacles in the way of an accurate detection of this influence and making precise predictions.

The problem consists in the maximal use of information, which is contained in the known segment of the time series, and subsequent modelling of the most probable future dynamics scenario.

The observed process is described as a time series of prices pt with the given sampling time span Δt:

$$p_{\mathrm{ti}} = p\left(\mathrm{t}_0 + i\Delta t\right). \tag{1}$$

Discrete presentation of the time series is in fact a way of existence of this very system. New prices are formed on the basis of contracts or deals, made on the market in certain discrete moments of time, while the price time series is a series of the averaged price levels during the chosen time intervals. While making a decision each trader, who is an active part of the pricing system, works solely with discrete series of the chosen time interval (e.g. minute, 5-minute, hourly, daily, etc.). For $\Delta t \rightarrow 0$ the accuracy of data presentations reaches a certain limit, since for relatively small Δt the price leaps in the moment of deal, while staying unchanged and equal to the last deal during the time between the two deals. Hence, the discreteness of time series has to be understood not only as a limited presentation of activity of the complex financial system, but also as one of the principles of its operation [3, 5–6].

The time series of initial conditions has to be turned into a sequence of discrete states. Let us denote the amount of chosen states as s, each of them being connected to the change in the quantity of the initial signal (returns).

For example, consider the classification with two states, first of which corresponds to positive returns as the price increases, while the second one – to negative as it descends. Generally all possible increments of the initial time series are divided into s groups. Ways of division will be discussed further.

Next we develop predictions for the time series of sampled states. For the given order of the Markov chain and the last generalized state the most probable state is chosen to be the next one. In case if ambiguity occurs while the state of maximum probability is being evaluated, an algorithm is used that allows

reducing the amount of possible prediction scenarios. Therefore we get the series of predicted states that can be turned into a sampled sequence of prognostic values.

Evaluation of increments, prediction, and subsequent restoration are conducted for the given hierarchy of time increments t. To use the given information as effectively as possible, the prediction is conducted for time increments $\Delta t = 1; 2; 4; 8; \ldots$, or a more complex hierarchy of increments and subsequent "splicing" of the results derived from different prediction samplings.

The procedure of prediction and splicing is iterative and conducted starting from smaller increments, adding a prediction with the bigger time increment on every step.

As the sampling time step t increases, the statistics for the investigation of Markov chains decreases, whereas the biggest sampling step, which takes part in the prognostication, limits itself. To supplement the prediction with the low-frequency component the approximation of zero order is being used in the form of a linear trend or a combination of a linear trend and harmonic oscillations [15].

5. Prediction Construction Algorithm

A Let us consider the consequence of operations, required for the prognostic time series construction. To do this we need to set the following parameters:

1) The type of time increments hierarchy (simple – powers of two, complex – product of powers of the first prime numbers).

2) Values of s – the amount of states and r – the order of the Markov chain. These parameters can be individual for every sampling level; finding of optimal parameters is done experimentally.

3) Threshold values δ, and minimal number of transitions N_{min}.

Prediction construction algorithm includes the following steps:

1) Generating hierarchy of time increments – Δt sequence. The maximal of them has to correspond to the length of a prognostic interval N_{max}.

2) For every time increment Δt, as the increments increase, a prediction of states and restoration of the time series along the prognostic states is conducted. Current stage includes following actions:

2.1. Evaluating increments (returns) of the series with Δt sampling. Increment or returns of the time series serves as the basis for states classification [15]. Absolute r_a and relative r_t increments of the time series are considered:

$$r_a = p_t - p_{t-\Delta t};$$ (2)

$$r_t = \frac{p_t - p_{t-\Delta t}}{p_t};$$ (3)

where p_t is the input time series of price dynamics, Δt – sampling interval, which is chosen for subsequent analysis.

2.2. Transforming the time series of increments into the series of state numbers (1..s), i.e. definition of limit values $\{r_{lim;i}\}$, which are used afterwards during transformation of the returns into class numbers.

2.3. Calculating transition probabilities for generalized states.

2.4. Constructing the series of prognostic states using the procedure of defining the most probable next state. The process of prediction implies the following: the last state is chosen (in case of Markov chains of an order $r > 1$ a sequence of r latest states is taken). The probability of transition from current state to all possible states is defined.

From all possible states a state with maximal probability is chosen. It is possible that several states with maximal probability occur, which can be explained by the bimodal probability distribution. The process of decision-making in this case is described in [10–11].

The chosen most probable state is taken as the next prognostic state and the procedure is repeated for the next (last added) state. Thus we receive a time series of prognostic states for the given sampling time Δt.

2.5. Restoring the value series from the state series with Δt sampling.

2.6. Splicing the prediction of Δt sampling with the time series derived from splicing of the previous layers (with the lesser step Δt). In case if the current time series is the first one, the unchanged time series will come as a result of splicing.

3) To splice the last spliced time series with the continuation of the linear trend, created along all previously known points.

The time series, spliced with the linear trend, is the result of prediction. Detailed algorithm description is published in [10–11]. Our software for time series forecasting by the proposed methods is available from our website: http://kafek.at.ua/MarkovChains1_2_20100505.rar.

6. Results of Stock Indices Prediction

In this section we offer the results of stock indices prediction. The stock's indices databases are available from http://finance.yahoo.com. Prediction's time series with different input learning set's length are shown on Figure 1. The time of prediction series beginning on the next figure is the point 1000 and corresponds to December 1, 2011. The mean and the standard deviation for the all prediction series is shown on Figure 2.
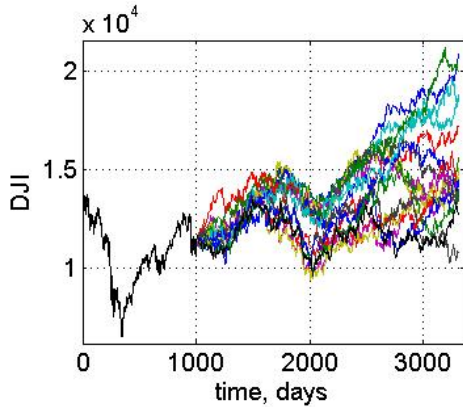

Figure 1. Prediction series, calculated with different learning set's length
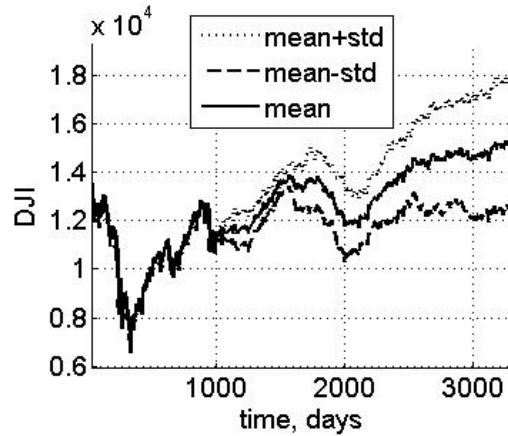
Figure 2. Mean value and standard deviations for prediction time series

The normalization procedure is proposed in order to compare indices and its prediction series with different absolute values. The normalized values calculated with the following formula:

$$y_n(t) = \frac{y(t) - \min\{y(t)\}}{\max\{y(t)\} - \min\{y(t)\}}.$$

(4)

Normalized prediction time series are shown on Figure 3. Americas, Asia and Europe are weighted average of country's stock indices predictions, weighted with GDP values for the corresponding countries. Americas includes Brazil, Mexico, Canada, Argentina, and USA. Europe consists of Great Britain, Germany France, Netherlands, Portugal, Italy, Ireland, Greece and Spain. Asian stock indices: China, Korea, Japan, India, New Zealand. For details of the averaging procedure see [11].
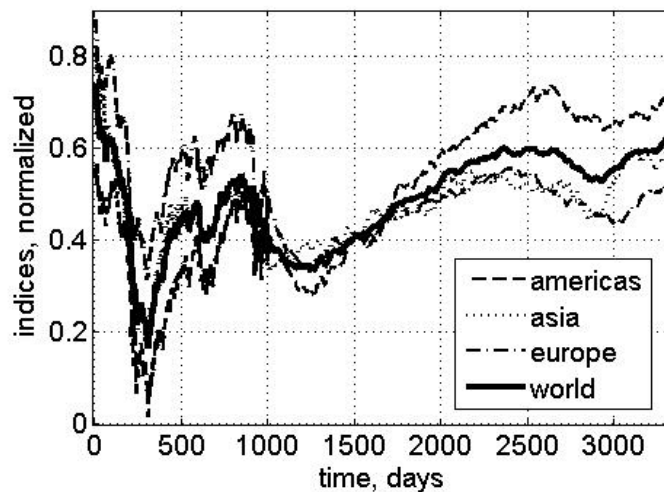

Figure 3. Mean values of normalized World's powerful economies indices prediction series

7. Conclusions and Further Work

The current paper suggests an algorithm of time series prediction based on complex Markov chains. Hierarchy of time increments principle allows using the information, which is contained in the time series during the prognosis construction, to its fullest. Experimental work on stock market indices time series prediction shows the efficiency of the algorithm and confirms the relevance of further research of the offered method.

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