Computer Modelling and New Technologies, 2011, Vol.15, No.3, 21–26
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HEISENBERG UNCERTAINTY PRINCIPLE AND ECONOMIC ANALOGUES OF BASIC PHYSICAL QUANTITIES

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From positions, attained by modern theoretical physics in understanding of the universe bases, the methodological and philosophical analysis of fundamental physical concepts and their formal and informal connections with the real economic measuring is carried out. Procedures for heterogeneous economic time determination, normalized economic coordinates and economic mass are offered, based on the analysis of time series, the concept of economic Plank's constant has been proposed. The theory has been approved on the real economic dynamic's time series, including stock indices, Forex and spot prices, the achieved results are open for discussion

Keywords: *quantum econophysics, uncertainty principle, economic dynamics time series, economic time*

1. Introduction

The instability of global financial systems depending on ordinary and natural disturbances in modern markets and highly undesirable financial crises are the evidence of methodological crisis in modelling, predicting and interpretation of current socio-economic conditions.

In papers [1–2] we have suggested a new paradigm of complex systems modelling based on the ideas of quantum as well as relativistic mechanics. It has been revealed that the use of quantum-mechanical analogies (such as the uncertainty principle, notion of the operator, and quantum measurement interpretation) can be applied to describing socio-economic processes.

It is worth noting that quantum analogies in economy need to be considered as the subject of new inter-disciplinary direction – quantum econophysics (e.g. [3, 4]), which, despite being relatively young, has already become a part of classical econophysics [5, 6]. However significant differences between physical and socio-economical phenomena, diversity and complexity of mathematical toolset as well as lack of deep understanding of quantum ideology among the scientists, working at the joint of different fields require a special approach and attention while using quantum econophysical analogies.

Our aim is to conduct methodological and philosophical analysis of fundamental physical notions and constants, such as time, space and spatial coordinates, mass, Planck's constant, light velocity from the point of view of modern theoretical physics, and search of adequate and useful analogues in socio-economic phenomena and processes.

2. About Nature and Interrelations of Basic Physical Notions

Time, distance and mass are normally considered to be initial, main or basic physical notions that are not strictly defined. It is thought that they can be matched with certain numerical values. In this case other physical values, e.g. speed, acceleration, pulse, force, energy, electrical charge, current etc. can be conveyed and defined with the help of the three above-listed ones via appropriate physical laws.

Let us emphasize that none of the modern physical theories, including relativistic and quantum physics, can exist without basic notions. Nevertheless, we would like to draw attention to the following aspects.

As Einstein has shown in his relativity theory, presence of heterogeneous masses leads to the distortion of 4-dimensional time-space in which our world exists. As a result Cartesian coordinates of the 4-dimensional Minkowski space (x, y, z, ict), including three ordinary Cartesian coordinates (x, y, z) and the forth formally introduced time-coordinate ict ($i = \sqrt{-1}$ – imaginary unit, c – speed of light in vacuum, t – time), become curvilinear [7].

It is also possible to approach the interpretation of Einstein's theory from other point of view, considering that the observed heterogeneous mass distribution is the consequence of really existing curvilinear coordinates (x, y, z, ict). Then the existence of masses in our world becomes the consequence of geometrical factors (presence of time-space and its curvature) and can be described in geometrical terms.

If we step away from global macro-phenomena that are described by the general relativity theory, and move to micro-world, where laws of quantum physics operate, we come to the same conclusion about the priority of time-space coordinates in the definition of all other physical values, mass included.

To demonstrate it, let us use the known Heisenberg's uncertainty ratio which is the fundamental consequence of non-relativistic quantum mechanics axioms and appears to be (e.g. [2]):

$$\Delta x \cdot \Delta v \geq \frac{\hbar}{2m_0}, \qquad (1)$$

where Δx and Δv are mean square deviations of x coordinate and velocity v corresponding to the particle with (rest) mass m_0, $\hbar$ – Planck's constant. Considering values Δx and Δv to be measurable when their product reaches its minimum, we derive (from (1)):

$$m_0 = \frac{\hbar}{2 \cdot \Delta x \cdot \Delta v}, \qquad (2)$$

i.e. mass of the particle is conveyed via uncertainties of its coordinate and velocity – time derivative of the same coordinate.

According to the concept [8, 9], space, time, and four fundamental physical interactions (gravitational, electromagnetic, strong and weak) are secondary notions. They share common origins and are generated by the so-called world matrix which has special structure and peculiar symmetrical properties. Its elements are complex numbers which have double transitions in some abstract pre-space.

At the same time, physical properties of space-time in this very point are defined by the nonlocal ("immediate") interaction of this point with its close and distant neighbourhood, and acquire statistical nature. In other words, the observed space coordinates and time has statistical nature.

In our opinion the afore-mentioned conception of nonlocal statistical origin of time and space coordinates can be qualitatively illustrated on the assumptions of quantum-mechanical uncertainty principle using known ratios (e.g. [2]) $\Delta p \cdot \Delta x \sim \hbar$, $\Delta E \cdot \Delta t \sim \hbar$; $\Delta p \cdot \Delta t \sim \frac{\hbar}{c}$.

Interpreting values $\Delta E, \Delta p, \Delta x, \Delta t$ as uncertainties of particle's energy E, its pulse p, coordinate x and time localization t, let us conduct the following reasoning.

While $\Delta x \to 0$ uncertainty of pulse, and thus particle energy, uncertainty formally becomes as big as possible that can be provided only by its significant and nonlocal energetical interaction with the rest of the neighbourhood. On the other side, while $\Delta p \to 0$ the particle gets smeared along the whole space, i.e. becomes delocalised. It might be supposed that the fact of "delocalised" state of the particle takes place in any other, not necessarily marginal Δx and Δp value ratios.

3. Dynamical Peculiarities of Economic Measurements, Economical Analogue of Heisenberg's Uncertainty Ratio

Speaking of economic laws, based on the results of both physical (e.g. quantities of material resources) and economical (e.g. their value) dynamic measurements, the situation will appear to be somewhat different. Adequacy of the formalisms used for mathematical descriptions has to be constantly checked and corrected if necessary. The reason is that measurements always imply a comparison with something, considered to be a model, while there are no constant standards in economics (they change not only quantitatively, but also qualitatively – new standards and models appear). Thus, economic measurements are fundamentally relative, are local in time, space and other socio-economic coordinates, and can be carried out via consequent and/or parallel comparisons "here and now", "here and there", "yesterday and today", "a year ago and now" etc.

Due to these reasons constant monitoring, analysis, and time series prediction (time series imply data derived from the dynamics of stock indices, exchange rates, spot prices and other socio-economic indicators) becomes relevant for evaluation of the state, tendencies, and perspectives of global, regional, and national economies.

Let us proceed to the description of structural elements of our work and building of the model. Suppose there is a set of M time series, each of N samples that correspond to the single distance T, with an equal minimal time step $\Delta t_{\min}$:

$$X_i(t_n), \;\; t_n = \Delta t_{\min} n; \;\; n = 0,1,2,...N-1; \;\; i = 1,2,...M. \qquad (3)$$

To bring all series to the unified and non-dimensional representation, accurate to the additive constant, we normalize them, having taken a natural logarithm of each term of the series:

$x_i(t_n) = \ln X_i(t_n), \;\; t_n = \Delta t_{\min} n; \;\; n = 0,1,2,...N-1; \;\; i = 1,2,...M.$

Let us consider that every new series $x_i(t_n)$ is a one-dimensional trajectory of a certain fictitious or abstract particle numbered i, while its coordinate is registered after every time span $\Delta t_{\min}$, and evaluate mean square deviations of its coordinate and speed in some time window ΔT:

$\Delta T = \Delta N \cdot \Delta t_{\min} = \Delta N, \;\; 1 << \Delta N << N.$

The "immediate" speed of i particle at the moment t_n is defined by the ratio:

$$v_i(t_n) = \frac{x_i(t_{n+1}) - x_i(t_n)}{\Delta t_{\min}} = \frac{1}{\Delta t_{\min}} \ln \frac{X_i(t_{n+1})}{X_i(t_n)}, \tag{4}$$

with variance D_{v_i} and mean square deviation Δv_i.

To evaluate dispersion D_{x_i} coordinates of the i particle are used in an approximated ratio:

$$2D_{x_i} \approx D_{\Delta x_i}, \tag{5}$$

where

$$D_{\Delta x_i} = \; < (x_i(t_{n+1}) - x_i(t_n))^2 >_{n,\Delta N} - (< x_i(t_{n+1}) - x_i(t_n) >_{n,\Delta N})^2 =$$

$$= \; < \ln^2 \frac{X_i(t_{n+1})}{X_i(t_n)} >_{n,\Delta N} - \left(< \ln \frac{X_i(t_{n+1})}{X_i(t_n)} >_{n,\Delta N} \right)^2, \tag{6}$$

which is derived from the supposition that x coordinates neighbouring subject to the time of deviation from the average value $\bar{x}$ are weakly correlated:

$$< (x_i(t_n) - \bar{x})(x_{i+1}(t_n) - \bar{x}) >_{n,\Delta N} \approx 0. \tag{7}$$

Thus we get:

$$\Delta x_i = \sqrt{\frac{D_{\Delta x_i}}{2}} = \frac{1}{\sqrt{2}} \left(< \ln^2 \frac{X_i(t_{n+1})}{X_i(t_n)} >_{n,\Delta N} - \left(< \ln \frac{X_i(t_{n+1})}{X_i(t_n)} >_{n,\Delta N} \right)^2 \right)^{\frac{1}{2}}. \tag{8}$$

It is also worth noting that the value

$$|v_i(t_n)| \cdot \Delta t_{\min} = \left| \ln \frac{X_i(t_{n+1})}{X_i(t_n)} \right|,$$

which, accurate to multiplier $\Delta t_{\min}$ coincides with $|v_i(t_n)|$, is commonly named absolute returns, while dispersion of a random value $\ln(X_i(t_{n+1})/X_i(t_n))$, which differs from D_{v_i} by $(\Delta t_{\min})^2$ – volatility [10].

The chaotic nature of real time series allows to $x_i(t_n)$ as the trajectory of a certain abstract quantum particle (observed at $\Delta t_{\min}$ time spans). Analogous to (1) we can write an uncertainty ratio for this trajectory:

$$\Delta x_i \cdot \Delta v_i \sim \frac{h}{m_i}, \tag{9}$$

or:

$$\frac{1}{\Delta t_{\min}} \left(< \ln^2 \frac{X_i(t_{n+1})}{X_i(t_n)} >_{n,\Delta N} - \left(< \ln \frac{X_i(t_{n+1})}{X_i(t_n)} >_{n,\Delta N} \right)^2 \right) \sim \frac{h}{m_i}, \tag{10}$$

where m_i – economic "mass" of an i series, h – value which comes as an economic Planck's constant.

Having rewritten the ration (10):

$$\Delta t_{\min} \cdot \frac{m_i}{(\Delta t_{\min})^2}\left(< \ln^2 \frac{X_i(t_{n+1})}{X_i(t_n)} >_{n,\Delta N} - \left(< \ln \frac{X_i(t_{n+1})}{X_i(t_n)} >_{n,\Delta N} \right)^2 \right) \sim h \qquad (11)$$

and interpreting the multiplier by $\Delta t_{\min}$ in the left part as the uncertainty of an "economical" energy (accurate to the constant multiplier), we get an economic analogue of the ratio $\Delta E \cdot \Delta t \sim \hbar$.

Since the analogy with physical particle trajectory is merely formal, h value, unlike the physical Planck's constant $\hbar$, can, generally speaking, depend on the historical period of time, for which the series are taken, and the length of the averaging interval (e.g. economical processes are different in the time of crisis and recession), on the series number i etc. Whether this analogy is correct or not depends on particular series' properties.

Generalization the ratios (10, 11) for the case, when economic measurements on the time span T is done in [10]. Thus, h/m_i ratio on the right side of (10) (or (11)) has to be considered a certain unknown function of the series number i , size of the averaging window ΔN , time $\overline{n}$ (centre of the averaging window), and time step of the observation (registration) k .

To get at least an approximate, yet obvious, formula of this function and track the nature of dependencies, we postulate the following model presentation of the right side (10):

$$\frac{h}{m_i} \cong \frac{\tau(\overline{n}, \Delta N_\tau) \cdot H_i(k, \overline{n}, \Delta N_H)}{\Delta t_{\min} \cdot m_i}, \qquad (12)$$

where

$$\frac{1}{m_i} = \ < \varphi_i(n,1) >_{(0 \le n \le N-2)}, \qquad (13)$$

m_i is a non-dimensional economic mass of an i – numbered series,

$$\tau(\overline{n}) = \frac{< \varphi_i(n,1,\Delta N_\tau) >_{(\overline{n}-\Delta N_\tau/2 \ < n < \ \overline{n}+\Delta N_\tau/2),\ (1 \le i \le M)}}{< \left(< \varphi_i(n,1,\Delta N_\tau) >_{(\overline{n}-\Delta N_\tau/2 \ < n < \ \overline{n}+\Delta N_\tau/2),\ (1 \le i \le M)} \right) >_{\overline{n}}} \qquad (14)$$

– local physical time compression ($\tau(\overline{n}) < 1$) or magnification ($\tau(\overline{n}) > 1$) ratio, which allows to introduce the notion of heterogeneous economic time (for a homogenous $\tau(\overline{n}) = 1$), $H_i(k, \overline{n})$ – non-dimensional coefficient of the order of unit, which indicates differences in the dependence of variance $D_{\Delta x_i}$ (see (13) taking into account the case of $k \ge 1$) on the law $D_{\Delta x_i} \sim k$ for the given i and $\overline{n}$.

$$\varphi_i(n, k, \widetilde{N}) = \frac{1}{k}\left(\ln^2 \frac{X_i(t_{n+k})}{X_i(t_n)} - \left(< \ln \frac{X_i(t_{n+k})}{X_i(t_n)} >_{n,\widetilde{N}} \right)^2 \right) \qquad (15)$$

and the multiplier $1/\Delta t_{\min}$ on the right side (12) can be considered as an invariant component of an economic Planck's constant h :

$$\overline{h} = 1/\Delta t_{\min} \ . \qquad (16)$$

As you can see, $\overline{h}$ has a natural dimension "time" to the negative first power.

4. Experimental Results and Their Discussion

To test the suggested ratios and definitions we have chosen 9 economic series with $\Delta t_{\min}$ in one day for the period from April 27, 1993 to March 31, 2011. The chosen series correspond to the following groups that differ in their origin:

1) stock market indices: USA (S&P500), Great Britain (FTSE 100) and Brazil (BVSP);
2) currency dollar cross-rates (chf, jpy, gbp);
3) commodity market (gold, silver, and oil prices).

Figure 1 shows averaged coefficients of time $\tau(t)$ compression-expansion (formula (14)) for three groups of incoming series: currency (Forex), stock, and commodity markets.

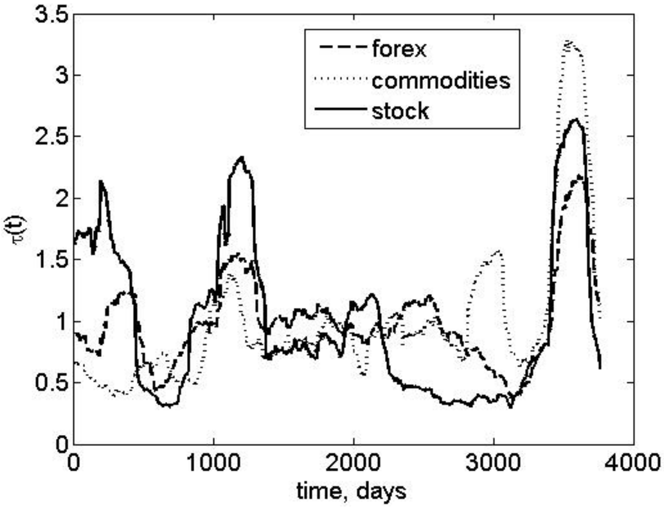

Figure 1. Coefficients of time compression-expansion, market "temperature". The explanation is in the text

The theory show that $\tau(t)$ exists in proportion to the averaged square speed (according to the chosen time span and series), i.e. average "energy" of the economical "particle" (as it is in our analogy), and can be thus interpreted as the series "temperature". Crises are distinguished with the intensification of economic processes (the "temperature" is rising), while during the crisis-free period their deceleration can be observed (the "temperature" is falling), what can be interpreted as the heterogeneous flow of economic time. $\tau(t)$ dependencies shown on Figure 1 illustrate all afore-mentioned. Note that local time acceleration-deceleration can be rather significant.

In Table 1 we give the values of a non-dimensional economic mass of the m_i series, calculated using (13) for all 9 incoming series, as well as average masses of each group [10].

Table 1. Economic series masses

Incoming series		Economic mass	Average economic mass of the group
Commodity market	gold	$2{,}816 \cdot 10^4$	$4{,}983 \cdot 10^3$
	silver	$4{,}843 \cdot 10^3$	
	oil	$2{,}777 \cdot 10^3$	
Currency market	jpy	$2{,}148 \cdot 10^4$	$2{,}499 \cdot 10^4$
	gbp	$3{,}523 \cdot 10^4$	
	chf	$2{,}180 \cdot 10^4$	
Stock market	S&P 500	$6{,}251 \cdot 10^3$	$4{,}748 \cdot 10^3$
	FTSE 100	$6{,}487 \cdot 10^3$	
	BVSP	$1{,}507 \cdot 10^3$	

As you can see from the Table 1, the stock market is distinguished with the lowest mass value, while the currency one shows the maximum number. Oil price series has the lowest mass on the commodity market, gold – the highest one. As for the currency market, British pound (gbp) has the highest value and Japanese yen rates (jpy) demonstrate the minimum mass of the group, although the dispersion is lower than that of the commodity market. The smallest spread is peculiar to the currency market. Dynamic and developing Brazilian market (BVSP) has the lowest mass, while the maximum value, just like in the previous case, corresponds to Great Britain (FTSE 100). It is explained by the well-known fact: Britain has been always known for its relatively "closed" economy as compared with the rest of the European and non-European countries.

The last group of experimental data corresponds to the dependence of Planck's economic constant (calculated for different series) on time $\Delta t = k\Delta t_{\min}$ (time between the neighbouring registered observations), which is characterized by $H_i(k,\bar{n})$ coefficient [10].

On Figure 2 integral dependencies $H_i(k)$ for stock market are depicted. As you can see there are no obvious regularities, which can be explained by various crises and recessions of the world and national economies that took place during the investigated period.

To decide whether it is possible for local regularities of Planck's economic constant dependence on Δt to appear, we have chosen relatively small averaging fragments, $\Delta N = 500$, which approximately equals two years. Corresponding results for some of these fragments on stock markets are given on Figure 3. Evidently figure show clear tendencies of $H_i(k,\overline{n})$ recession and rise for given type of the market (unlike integral dependencies $H_i(k)$).

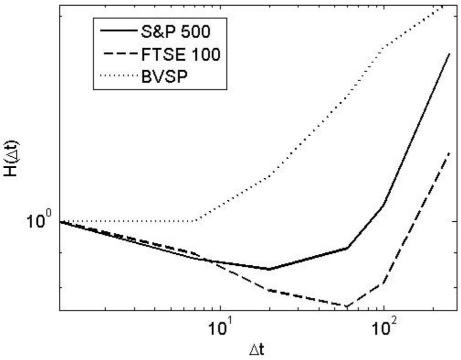

Figure 2. Integral coefficient $H_i(k)$ dependences for stock market

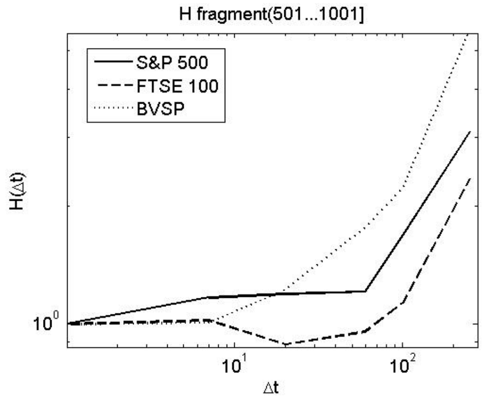

Figure 3. Local coefficient $H_i(k,\overline{n})$ dependences for stock market (averaging time span from 12.06.1995 to 15.07.1997, 500 daily values)

5. Conclusions

We have conducted methodological and philosophical analysis of physical notions and their formal and informal connections with real economic measurements. Basic ideas of the general relativity theory and relativistic quantum mechanics concerning space-time properties and physical dimensions peculiarities are used as well. We have suggested procedures of detecting normalized economic coordinates, economic mass and heterogeneous economic time. The afore-mentioned procedures are based on socio-economic time series analysis and economical interpretation of Heisenberg's uncertainty principle. The notion of economic Planck's constant has also been introduced. The theory has been tested on real economic time series, including stock indices, currency rates, and commodity prices.

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Received on the 11th of September, 2011