# Assessing Energy-related Markets through Multifractal Detrended Cross-correlation Analysis

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Abstract: Regulatory actions aimed the sustainable development force ordinary traders, policymakers, institutional investors to develop new types of risk management strategies, seek better decision-making processes that would allow them more effectively reallocate funds when trading and investing in energy markets such as oil and gas. Due to their supply and demand, they are presented to non-equilibrium, chaotic, long-range dependent, etc. Oil and gas play an important role not only in the financial markets, but they are important in many goods and services, and their extensive usage leads to environmental damage. Thus, the dynamics of the corresponding energy-related indices is a useful indicator of the current environmental development, and their modeling is of paramount importance. We have addressed one of the methods of multifractal analysis which is known as Detrended Cross-Correlation Analysis (DCCA) and its multifractal extension (MF-DCCA) to get reliable and efficient indicators of critical events in the oil and gas markets. For example, we have taken daily data of Henry Hub natural gas spot prices (US$/MMBTU), WTI spot prices (US$/BBL), and Europe Brent spot prices (US$/BBL) from 7 February 1997 to 14 December 2021. Regarding their (multifractal) cross-correlations, we get such indicators as the DCCA coefficient $\rho_{DCCA}$, the cross-correlation Hurst exponent, the maximal, minimal, and mean singularity strength, the width of multifractality, and its asymmetry with the usage of sliding window approach. Our empirical results present that all of the presented indicators respond characteristically during crashes and can be effectively used for modeling current and further perspectives in energy markets and sustainable development indices.

## 1 INTRODUCTION

The largest and most developed countries are aimed at sustainable development. Both natural gas and crude oil prices demonstrate the general pattern of current trends in the world, particularly, in the development of our environment.

There were some discussions about whether natural gas and oil prices appear to be price-related. Compared to natural gas, which tends to be regionally determined, the crude oil market represents the state of the whole world. Therefore, it is discussible which indices of the energy-related market to use for identification of possible trends in the green economy.

Both supply and demand on the energy market form complex, non-stationary, irreversible, non-equilibrium, and multifractal dynamics in these

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markets. These characteristics are reflected in fat-tails (Mandelbrot, 2021) of the probability distribution of these markets and their autocorrelation functions (Aloui and Mabrouk, 2010; Herrer et al., 2017). Mandelbrot presented "fractals" to deal with such irregularities (Mandelbrot, 1967).

Then, there was proposed and revised Rescaled Range Analysis (R/S) by (Hurst, 1951), and it was revised by (Lo, 1991) for studying short- and long-range dependences in a time series. Then, there was proposed Detrended Fluctuation Analysis by (Peng et al., 1994), and (Kantelhardt et al. 2002) extended it to the multifractal version (MF-DFA), which can explore efficiency, short- and long-term memory, etc. over multiple scales. This approach is one of the most reliable in defining multifractal characteristics in non-stationary time series. Except for previous ones, (Podobnik & Stanley, 2008) proposed to study power-law cross-correlations between several series. That method was called Detrended Cross-Correlational Analysis (DCCA). Then, (Zebende 2011) proposed DCCA cross-correlation coefficient for detrended covariance fluctuation functions of time series.

Previously, we devoted our papers to stock, crypto, and sustainable development indices. We studied them using different measures of complexity: Information entropy and its modifications, Recurrence analysis, graph-based measures, irreversibility measures, quantum indicators, and particularly, classical MF-DFA method and random matrix theory to study cooperative behavior among different cryptocurrencies and stock indices (Bielinskyi et al., 2021b; Bielinskyi et al., 2020; Soloviev et al., 2019; Bielinskyi et al., 2021; Bielinskyi et al., 2021). In this paper, we would like to make an analysis of energy-related markets such as WTI and Europe Brent crude oil with Henry Hub natural gas spot markets in terms of the (MF-)DCCA approach. According to this method, we expect to get reliable indicators of crash phenomena in the mentioned market. Such indicators of complexity would be useful for traders, institutional investors, governments, who are looking for better decision-making processes, more effective risk management strategies during trading, and it would be useful for those who care about modeling and forecasting the sustainable development in the world.

## 2 REVISION OF THE PREVIOUS STUDIES

Different studies were devoted to the monitoring and forecasting of the crude oil and natural gas prices, $CO_2$ emissions.

As an example, the study of (Hoayek et al., 2020) aimed to measure the power and efficiency of information reflected in gas prices using different econometric and mathematical models of the information, records, and game theories. In their paper, they studied the dynamics of Henry Hub and National Balancing point gas markets as they are considered to be one of the most developed hubs in the U.S. and Europe. For both markets, the authors chose three indicators: level of competition, price stability, and price uncertainty. Regarding conditional and Shannon entropy, the authors reduced the amount of uncertainty in the given indicators and defined how informative and reliable was their recommendations from given metrics. Their approach emphasized that additional measures need to be applied to the European gas market. For the U.S. gas market, the situation is stable. As authors point out, their study needs additional growth: to include more mathematical/statistical analysis, the greater number of observations, indicators. Also, they mention that problems appear with the probability distribution needed for Shannon entropy and its analogs, which requires additional work on creating methods for the computation of the underlying probability distribution of each indicator. The study made by (Joo et al., 2020) examined the effect of the 2008 global financial crisis on the crude oil market (WTI crude oil spot prices) with the usage of Hurst exponent, Shannon entropy, and the scaling exponent. They investigated how changed efficiency, long-term equilibrium, and collective phenomena before and after the crash. According to their analysis, there was not much difference in volatility of the crude oil market before and after the crash. Period before crash remained efficient according to Hurst exponent $(H = 0.50 \pm 0.01)$, displaying a random walk of WTI prices. After the crash oil prices remained persistent $(H = 0.55 \pm 0.01)$, and then, after 2010, prices started to behave anti-persistently $(H = 0.45 \pm 0.01)$. According to Shannon entropy, the overall market behavior was closer to long-term equilibrium (higher entropy). However, after the crash its entropy started to reduce, indicating the presence of long-term memory effect, dynamics far from equilibrium. Scale-free properties remained after that outbreak, which demonstrates the power-law exponent. This exponent decreased, implying that the probability of observing double returns became higher. (Lautie et al., 2019) investigated price relationships across WTI crude oil futures using the concept of mutual information and information flows. Their study presents rolling window dynamics for mutual information to investigate how it behaves during several structural shocks in this market. Mutual information increased noticeably in 2004 but dropped sharply in 2012-2014. Thus, different parts of the term structure of WTI futures prices became less correlated. Also, the researchers applied the concept of Transfer entropy to study information flows between contracts with different periods. They found that short-dated contracts emit more information, and, after 2012, flows in forward and backward directions were almost the same, but if to look at the whole trading period, they are presented to be more volatile compared to middle-dated contracts. (Hu et al., 2021) said about a common method for evaluating energy use in energy resource exploitation and method for evaluating it which is called energy return on investment. There, they proposed an interpretation of this method in terms of entropy. They considered an energy resource exploitation system to be a kind of dissipative system. Then, they derived a relation between energy return on investment and entropy change. The authors emphasized that future development of energy return on investment and its related indicators must be done in terms of entropy theory.

Some of the studies devoted to oil and gas markets included methods of fractal and multifractal analysis. As an example, (Engelen et al., 2011) studied the spot rate dynamics of Very Large Gas Carriers regarding MF-DFA and rescaled range analysis. Studying logarithmic returns of the daily spot rates, they concluded that freight rates exhibited persistent behavior. Most of the time time-depended Hurst exponent was around 0.7. Comparing multifractal characteristics of initial and shuffled data, they found long-range correlations to prevail rather that fat tails in the probability distribution. The impact of the coronavirus pandemic on the multifractality of gold and oil prices based on upward and downward trends was examined by (Mensi et al., 2020). Such an interesting approach was applied as asymmetric detrended fluctuation analysis to study 15-min interval intraday data. Results presented that as time scale increased, asymmetric multifractality also increased. According to their conclusions, multifractality is especially high for the downtrend of Brent oil and upward trend of gold. That asymmetrical multifractality was strengthened during COVID-19. Interestingly, during the pandemic period, both markets became more inefficient (less complex). Overall, the asymmetric analysis is also a powerful instrument for tracking the investor's sentiments and applying more wise decisions when trading at high-frequency time scales. (Garnier & Solna, 2019) studied WTI and Brent oil price data for the period 1997-2016 with the usage of wavelet-based decomposition, Hurst exponent, and volatilities. The estimated exponent for Brent is 0.46 and for WTI is 0.44, which told about their mean-reverting behavior. The estimated volatilities were 34% for Brent and 32% for WTI. Analysis of Hurst exponent and volatilities using sliding window procedure presents that the nature of both indices is presented to be non-constant. During crashes volatility is the biggest and Hurst exponent increases, indicating that those events are presented to be less efficient (more persistent). Mass Hub, Mid C, Palo Verde, and PJM West are the four major electricity indices of the U.S. that were studied in (Ali et al., 2021) using multifractal analysis. Researchers found the significant presence of multifractality in the electricity market. However, their analysis included a sliding window procedure that presented varying degrees of multifractality. According to their results PJM West had the highest degree of multifractality and Mass Hub had the lowest i.e., it was presented to be the most efficient, while PJM was the least efficient. Moreover, according to the generalized Hurst exponent, at $q = 2$, all indices appeared to be anti-persistent (mean-reverting). The rolling window procedure presents that even not for the whole time series but its sub-series, the dynamics still demonstrate mean-reverting property.

Graph theory plays an important role in different fields of science. Its instruments are of paramount importance when we study collective non-linear phenomena among different indices, especially, for the energy market. (Fang et al., 2018) applied some of the methods for converting time series into a complex network and applied some graph-based indicators such as average shortest path and density with the sliding window procedure. Time series of natural gas, coal, and crude oil were chosen. Between each pair were calculated the correlation coefficients. Also, they defined correlation models based on correlation coefficients and a coarse-graining procedure. They improved the betweenness centrality algorithm to measure the evolution direction of the correlation modes in different clusters of energy prices. Such correlations between clusters were explored for different time lengths of the sliding window. For smaller time windows both positive and negative correlations were observed. When the size of the window increased, positive correlations also became higher. That indicates the interrelationships between the closing prices of the three types of energy to be higher in the long term. Multilayer networks are important for studying complex systems of complex systems. One general graph may consist of several and more subgraphs. (Xu et al., 2020) introduced a multilayer recurrence network for examining energy and carbon markets. Also, after they defined the information linkage coefficient and time-delayed information linkage, they measure interrelationships between carbon and energy markets in different stages of the EU carbon market. Data for the period from 2011 to 2019 were subdivided into four periods and multilayer recurrence networks within each stage were built. The general trend remained U-shaped trend: co-movement of crude oil, coal, natural gas, and carbon prices were decreasing at the first stage, and then it grew progressively during other stages.

Also, there is a study in which (Kassouri et al., 2022) used a method based on wavelet analysis to investigate the interaction between oil shocks and $CO_2$ emissions intensity for the period 1975-2018. Their study presents that wavelet-based for studying the level of co-integration between several markets. Also, they found that supply and demand in the oil market had an inhomogeneous influence on $CO_2$ emissions. The demand-related shocks in the oil market lead to a decrease in $CO_2$ emissions in the U.S. Increase in emissions is followed by uncertainty in the global oil market. One of the main conclusions that we would like to emphasize is that high oil prices for mitigating $CO_2$ do not work for the U.S. case. Thus, policymakers should be aware when attaching the influence of shocks in the oil market to the environment's resilience. (Hussain et al., 2021) employed dynamic copulas and Extreme Value Theory to analyze relationships between oil and stock markets with the highest number of COVID-19 cases. Their study, first of all, confirmed that analyzed data is presented to be non-linear, non-stationary, and heavy-tailed. Moreover, they found that, probably, it was insufficient to represent the influence of COVID-19 on the dependence of two markets. Their findings showed that the degree of dependence between oil and stock markets was shifting. Before the pandemic, their correlation was presented to be higher and became lower during the pandemic. Studying the left and right tails of that dependence, scientists found that for the right tail there was no significant change, while for the left tail there was a significant increase, which told about a higher probability of extreme risks (downward trend) between oil and stock markets. That is, if there was a crisis in the oil market, there would be in the stock market. The study of (Wang et al., 2014) made important research on (multifractal) detrended cross-correlation analysis. In this paper, scientists studied standard and multifractal detrended cross-correlation characteristics for pairs oil-gas, oil-$CO_2$, and gas-$CO_2$. First of all, we would like to note that the cross-correlation scaling exponent i.e., generalized Hurst exponent, demonstrated week persistent behavior for all pairs. Using rolling window dynamics, they presented that in average scaling exponent for almost all pair were close to 0.5, while for oil-$CO_2$ dynamics was more persistent with different window lengths. Cross-correlation coefficient $\rho_{DCCA}$ remained close to zero for scales less than 100 and then started to increase. Thus, for short-term scales correlations were weak, while for long-term scales they were stronger. (Zou and Zhang, 2020) also studied energy and carbon markets using cross-correlation analysis based on multifractal theory. Their relation was presented to be non-linear and multifractal. Also, short-term memory of those markets was significantly stronger compared to long-term memory. Their findings demonstrated that fat-tails of the probability distribution were the main source of multifractality if compare to long-term memory. Under normal circumstances, their dependence was presented to be anti-correlated. (Quantino et al., 2021) devoted their study to Brazilian ethanol and other energy-related commodities such as Brent oil, natural gas prices, $CO_2$ emissions, and sugar for the period 2010-2020. In their study, they also used DCCA with the sliding window algorithm to study correlation characteristics during different periods. For the whole period, they observed weak correlations in short term between Brazilian ethanol and $CO_2$ emissions. For large scales, there are strong correlations for sugar. For oil prices, there are statistically significant correlations up to 128 days, and for natural gas, there are no significant correlations. For rolling window dynamics, there is a need for additional research, but their analysis showed that correlations vary across time.

## 3 MATERIALS AND METHODS

Regarding previous studies, we will try to confirm the results of previous researchers, present additional analysis on co-movement between 3 energy-related prices, and construct indicators or indicators-precursors based on the (MF-)DCCA.

The presented work uses daily data of Henry Hub natural gas spot prices (US$/MMBTU), Cushing, OK WTI spot prices FOB (US$/BBL), and Europe Brent spot prices FOB (US$/BBL) (Natural Gas Futures Prices (NYMEX), 1997–2021; Spot Prices for Crude Oil and Petroleum Products, 1986–2021). The sample period of initial data ranged from 7 February 1997 to 14 December 2021. The dynamics of the corresponding data are presented in Figure 1.

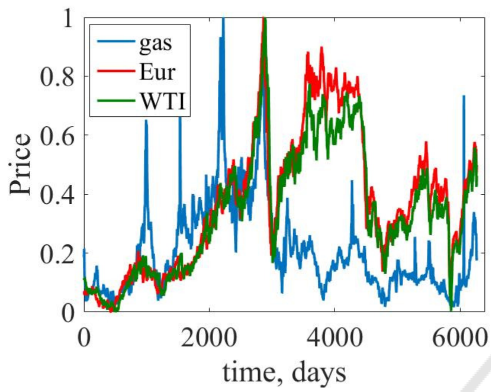



Figure 1: Initial time series of Henry Hub natural gas spot prices (gas), Europe Brent spot prices (Eur), and WTI spot prices (WTI).

According to previous studies, exactly logarithmic (standardized returns) exhibit multifractal characteristics. Therefore, we will calculate further indicators regarding the standardized returns defined by

$$G(t) = [x(t+\Delta t) - x(t)]/x(t)$$

$$\text{and} \tag{1}$$

$$g(t) \equiv [G(t) - \langle G \rangle]/\sigma,$$

where $x(t)$ is a value of our time series; $\Delta t$ is a time shift (in our case $\Delta t = 1$); $\langle G \rangle$ is the average of returns $G$; $\sigma$ is the standard deviation of $G$.

It should be noted that some of the studied values were repeated in our series. Therefore, before calculating returns, we preprocessed our data by smoothing it, using the moving average of 5 days. Figure 2 presents standardized returns of our time series data.

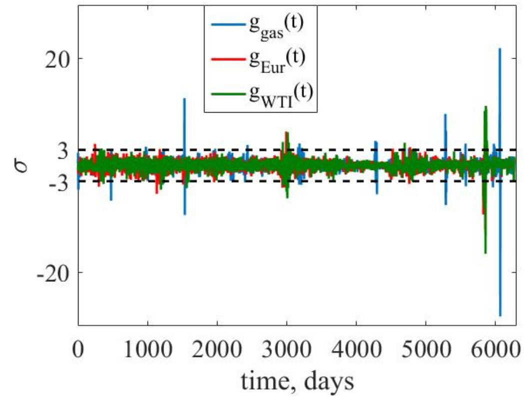



Figure 2: The standardized returns of gas, Eur, and WTI. Events with $\pm 3\sigma$ are marked by dashed lines.

From the figure above it can be seen that most of the time our data is presented to be correlated to each other, but some of the critical events, as an example, of WTI spot market cannot be associated with Euro Brent or Henry Hub prices. Nevertheless, our correlational and multifractal measures should give a more comprehensive and clearer picture.

Also, we can see that most periods in energy markets are defined by events that exceed $\pm 3\sigma$. The WTI returns are characterized by much more extensive crashes. Previous studies pointed out that such events are located in fat-tails of the probability distribution. Figure 3 presents the probability distribution of $g(t)$.

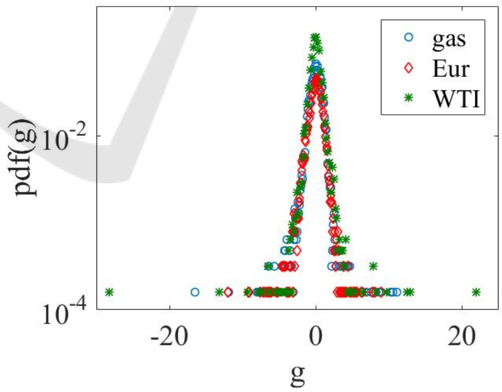



Figure 3: Probability density functions (pdf) of the standardized returns.

Fat-tails, as it was mentioned, are the main source of multifractality and multifractal analysis is of the possible solutions for dealing with such risk phenomena.

Further, we apply multifractal analysis of cross-correlational characteristics for such pairs as WTI-Eur and WTI-Hub. Most of our results are based on the sliding window approach. The idea here is to take a sub-window of a predefined length $w$. For that sub-window, we perform (multifractal-)detrended cross-correlation analysis, get necessary metrics that are appended to the array. Then, the window is shifted by a predefined time step $h$, and the procedure is repeated until the time series is completely exhausted. Our results will be presented for $w \in \{250, 500\}$ and $h = 1$.

# 4 ESTIMATION PROCESS

## 4.1 DCCA Approach

For further calculations we consider two time series $\{x_i \mid i = 1, 2, \dots, N\}$ and $\{y_i \mid i = 1, 2, \dots, N\}$. Then, MF-DCCA considers the following procedure:

- Construct the cumulative time series

$$X(i) = \sum_{k=1}^{i} [x_k - \langle x \rangle]$$

$$\text{and} \tag{2}$$

$$Y(i) = \sum_{k=1}^{i} [y_k - \langle y \rangle]$$

where $\langle x \rangle$ and $\langle y \rangle$ are the mean values of the analyzed time series.

- Divide the time series into $N_s \equiv int(N/s)$ non-overlapping segments of equal length $s$. We repeat the procedure from the end of a time series, since $N$ is usually not an integer multiple of $s$, and because of it we may neglect the last part of a time series. Therefore, we will obtain $2N_s$ sub-series.
- Subsequently, we find local trends $\tilde{X}^v(i)$ and $\tilde{Y}^v(i)$ with $m$-order polynomials for each sub-series $v$ ( $v = 1, \dots, 2N_s$ ) and detrend each of those segments. Thus, the detrended covariances of the variances of each box for both time series are given by

$$f^2(v, s) = \frac{1}{s} \sum_{i=1}^{s} \{X[(v-1)s + i] - \tilde{X}^v(i)\} \times \{Y[(v-1)s + i] - \tilde{Y}^v(i)\} \tag{3}$$

for each interval $v, v = 1, \dots, N_s$ and

$$f^2(v, s) = \frac{1}{s} \sum_{i=1}^{s} \{X[N - (v-1)s + i] - \tilde{X}^v(i)\} \times \{Y[N - (v-1)s + i] - \tilde{Y}^v(i)\} \tag{4}$$

for $v = N_s + 1, N_s + 2, \dots, 2N_s$.

- The detrended covariance fluctuation function can be calculated according to

$$F_{DCCA}^2(s) = \frac{1}{2N_s} \sum_{v=1}^{2N_s} f^2(v, s). \tag{5}$$

- By analyzing the log-log plots of $F_{DCCA}(s)$ versus $s$, we can get the scaling behavior of the fluctuation function. Particularly, if time series are power-law cross-correlated, then we get the relation

$$F_{DCCA}(s) \propto s^{h_{xy}}, \tag{6}$$

where $h_{xy}$ is the cross-correlation scaling exponent, which is also known as the Hurst exponent $H$ (Hurst, 1951).

This extension of the Hurst exponent works at the same way:

1) If $h_{xy} > 0.5$ , the cross-correlations between time series are presented to be persistent: an increase (a decrease) in one time series is followed by an increase (a decrease) in other time series.

2) If $h_{xy} < 0.5$ , the cross-correlations between time series are presented to be anti-persistent: an increase in one time series is likely to be followed by a decrease in the other time series.

3) If $h_{xy} \approx 0.5$, both time series follows a random walk, i.e., there are no correlations between them.

4) If $h_{xy} > 1$, both time series are presented to be highly correlated and non-stationary.

Except for the cross-correlational Hurst exponent, the DCCA algorithm proposes to calculate the DCCA cross-correlation coefficient between time series (Zebende, 2011). For each time scale $s$, the DCCA coefficient is defined as

$$\rho_{DCCA}(s) = \frac{F_{DCCA}^2(s)}{F_{DFAx}(s) \times F_{DFAy}(s)}, \tag{7}$$

where $F_{DCCA}^2(s)$ can be found according to equation (5); $F_{DFA}(s)$ is the standard detrended fluctuation function and $-1 \leq \rho_{DCCA}(s) \leq 1$ (Peng et al., 1994). In a similar way to the classical correlation coefficient, $\rho_{DCCA} = 1$ means that time series are positively correlated and co-move synchronically; $\rho_{DCCA} = -1$ denotes that time series move asynchronically (anti-persistently); $\rho_{DCCA} = 0$ presents that there is no correlation between two time series.

In section 5 we will present empirical results related to the rolling window dynamics of $h_{xy}$ and $\rho_{DCCA}$. In the next sub-section, we would like to describe the modified DCCA method which considers multifractal cross-correlation characteristics.

## 4.2 MF-DCCA Approach

Multifractal detrended cross-correlation analysis that was derived from standard DCCA gives multifractal characteristics derived from power-law cross-correlations of time series (Zhou, 2008). This approach modifies standard detrended covariance fluctuation function to $q$th order:

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}[f^2(v,s)]^{q/2}\right\}^{1/q} \quad (8)$$

for $q \neq 0$ and

$$F_q(s) = exp\left(\frac{1}{4N_s}\sum_{v=1}^{2N_s}\ln[f^2(v,s)]\right) \quad (9)$$

for $q = 0$.

As in equation (6), $F_q(s)$ will follow power-law behavior:

$$F_q(s) \propto s^{h_{xy}(q)}, \quad (10)$$

where $h_{xy}(q)$ represents a multifractal generalization of power-law cross-correlation Hurst exponent.

Values of $q$ emphasize the density of small (large) fluctuations. If those values are negative, we make an ascent on scaling properties of small fluctuations. For positive values, scaling properties of the large magnitudes dominate. Generally, if our multifractal characteristics do not depend on $q$ values, the studied time series is presented to be monofractal.

For further calculations, through the multifractal exponent $\tau_{xy}(q) = qh_{xy}(q) - 1$, we can define the singularity strength $\alpha_{xy}(q)$ and the multifractal spectrum $f_{xy}(\alpha)$:

$$\alpha_{xy}(q) = h_{xy}(q) + q\frac{dh_{xy}(q)}{dq} \quad (10)$$

and

$$f_{xy}(\alpha) = q\left[\alpha_{xy}(q) - h_{xy}(q)\right] + 1. \quad (11)$$

Here, $\alpha_{xy}(q)$ can be considered as the local fractal dimension, and $f_{xy}(\alpha)$ can be considered as the "box-counting" dimension of regions with particular singularity strengths.

According to the study of (Ito and Ohnishi, 2020), the greater the level of $q$, the lower the value of $\alpha_{xy}(q)$. If we approach the event with extremely high densities (fluctuations), compared to neighboring boxes (windows), we will have a low value of $f_{xy}(\alpha)$. If critical events would dominate in our system, the singularity spectrum would have a long-left tail that would indicate the dominance of large events. Right-tailed multifractal spectrum would indicate sensitivity to small events. The symmetrical spectrum would show equal distribution of patterns with small and large fluctuations.

Except for those characteristics that were presented before, we would like to calculate the width of the multifractal spectrum which can be defined as

$$\Delta\alpha = \alpha_{max} - \alpha_{min}. \quad (12)$$

The wider it is, the more complex structure, the more uneven distribution we have, and the more violent fluctuations on the surface of our time series. On the contrary, smaller multifractal width indicates that the time series are uniformly distributed. Thus, their structure is much simpler.

Another option is to calculate the proportion of small and large peak values that are addressed to the multifractal spectrum:

$$\Delta f = f(\alpha_{min}) - f(\alpha_{max}), \quad (13)$$

where $f(\alpha_{min})$ and $f(\alpha_{max})$ are the multifractal spectrum's values that correspond to the smallest and the largest singularity values. For $\Delta f < 0$, the larger fluctuation amplitude occurs with a higher possibility and for $\Delta f > 0$, we have the opposite relation (Zhang et al., 2019).

# 5 EMPIRICAL RESULTS AND ANALYSIS

In this section, we would like to present empirical results. which were obtained with the usage of the (MF-)DCCA. Our figures present comparative dynamics of

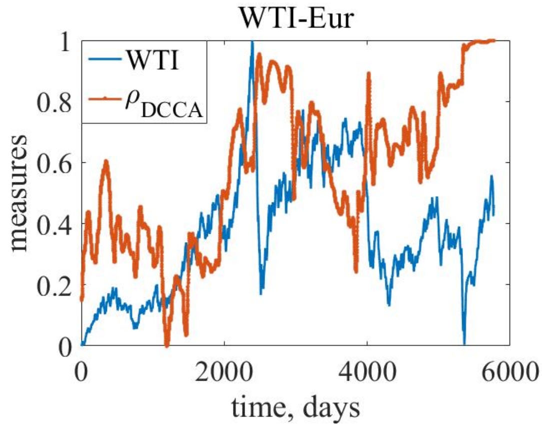
- the cross-correlation coefficient ($\rho_{DCCA}$);
- the generalized cross-correlation Hurst exponent ($h_{xy}$);

- the minimal, maximal, and mean singularity strength ($\alpha_{min}$, $\alpha_{max}$, $\alpha_{mean}$);
- the width of the multifractal spectrum ($\Delta\alpha$);
- the asymmetry of the multifractal spectrum ($\Delta f$).

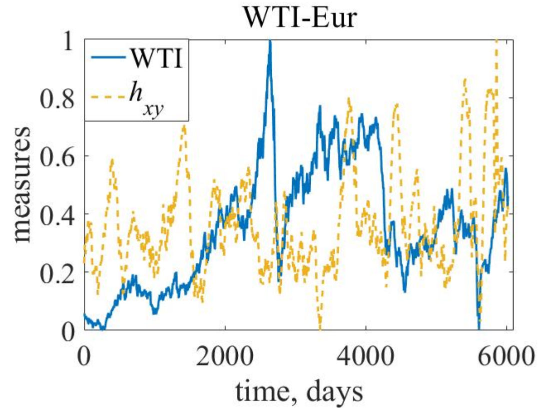
According to our expectations, (MF-)DCCA indicators should behave particularly during crisis events, i.e., increase or decrease during them. The mentioned indicators were calculated for the following parameters:

- sliding windows $w = 250$ days for studying the dynamics of short-term periods for the entire set of the presented here indicators. In this case, we avoid the influence of the dynamics of crises close to each other. At the same time, we get more insufficient statistics;
- sliding window $w = 500$ days for studying long-term behavior of the DCCA coefficient. In this case, the data of previously happened events influence the dynamics of currently studied crashes, but we get more statistics;
- time step $h = 1$ day to get more comprehensive statistics;
- $m = 2$ for fitting local trends in equations (3) and (4);
- time scales $s$ are defined in a range from 10 to 250 and 500 days;
- the values of $q \in [-10; 10]$ with a delay 1 to have a better view on scales with small and large fluctuation density. Nevertheless, the experiments with smaller and larges ranges are possible;

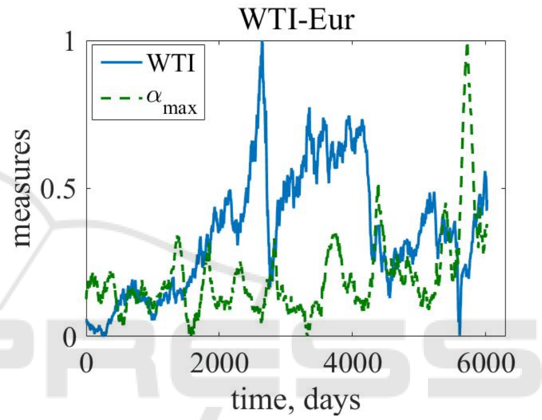
Figure 4 presents the comparative dynamics of the (MF-)DCCA indicators for WTI-Eur pair with $w = 250$ days for all of them and $w = 500$ days for the DCCA coefficient.



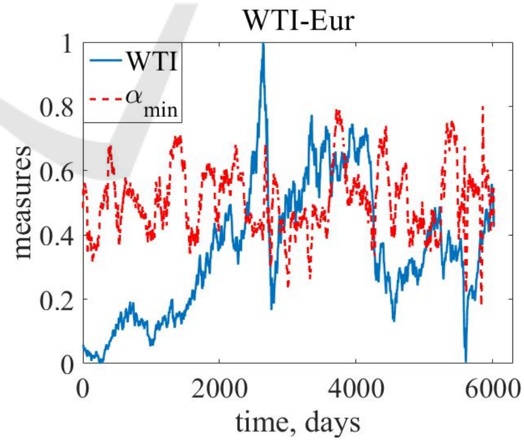

(a)



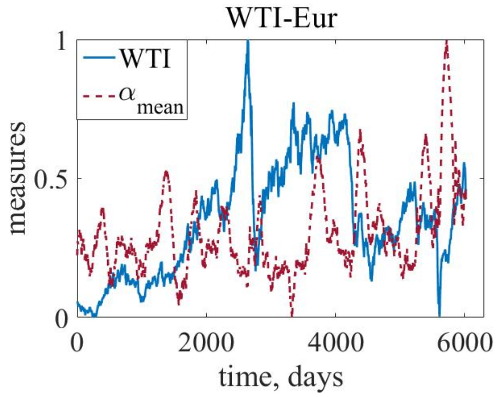

(b)



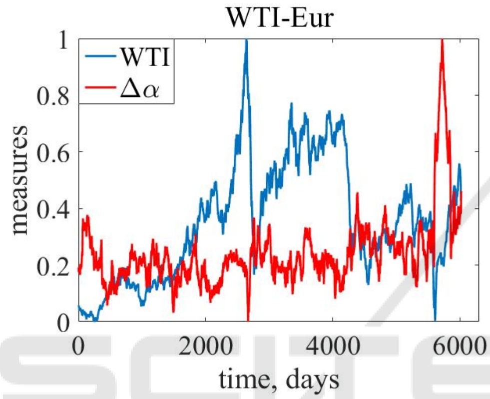

(c)



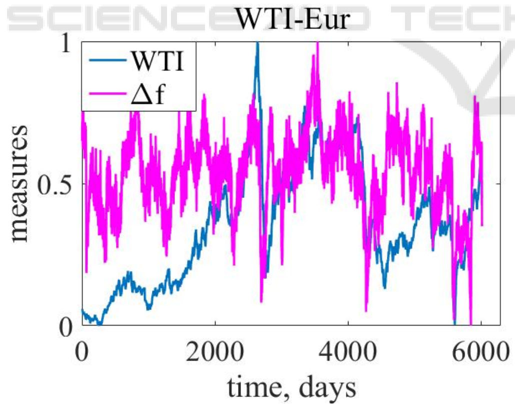

(d)




(e)




(f)




(g)

Figure 4: The comparative dynamics of pair WTI crude oil spot prices and Europe Brent crude oil prices (WTI-Eur) with the DCCA coefficient (a), the cross-correlated generalized Hurst exponent (b), the maximal singularity strength $\alpha_{max}$ (c), the minimal singularity strength $\alpha_{min}$ (d), the mean singularity strength $\alpha_{mean}$ (e), the width of the singularity spectrum $\Delta\alpha$ (f), and its asymmetry $\Delta f$ (g).

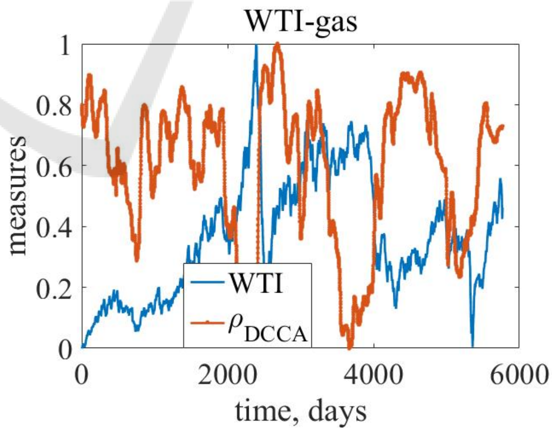
Generally, according to Figure 4, we can that our indicators respond in a particular way to our crashes.

The cross-correlational coefficient, in Figure 4 (a) demonstrated co-movement of our time series for a long-term period. From Figure 4 (b) we can see that $h_{xy}$ increases before the crash, i.e., they demonstrate persistent behavior, and decreases after it, that is, both time series become more mean-reverting during the crash. Before the critical event, both commodities seem attractive for trading, but the crash that may be caused by certain geopolitical events forces users to transfer their funds from those energy commodities to another product.

Figure 4 (c-f) demonstrates that singularity exponents and the width of $f(\alpha)$ become higher. It means that during critical phenomena different time scales in the studied time series respond inhomogeneously: their cross-correlated dynamics start to exhibit different patterns and more fluctuated (rough) behavior.

Figure 4 (g) demonstrates a decrease during critical events. That is a signal that the ends of $f(\alpha)$ become more uneven. If $\Delta f$ decrease, it means that the multifractal spectrum has a longer left-tail. More left-tailed $f(\alpha)$ demonstrates multifractal predominance of the fluctuations with large magnitudes. In the opposite case, if $\Delta f$ increases, our spectrum can be distributed more symmetrically or closer to the right side. In other words, fluctuations can be distributed homogeneously or small fluctuations will have greater density.

Next, in Figure 5, let us present (MF-)DCCA measures for WTI-gas pair.




(a)

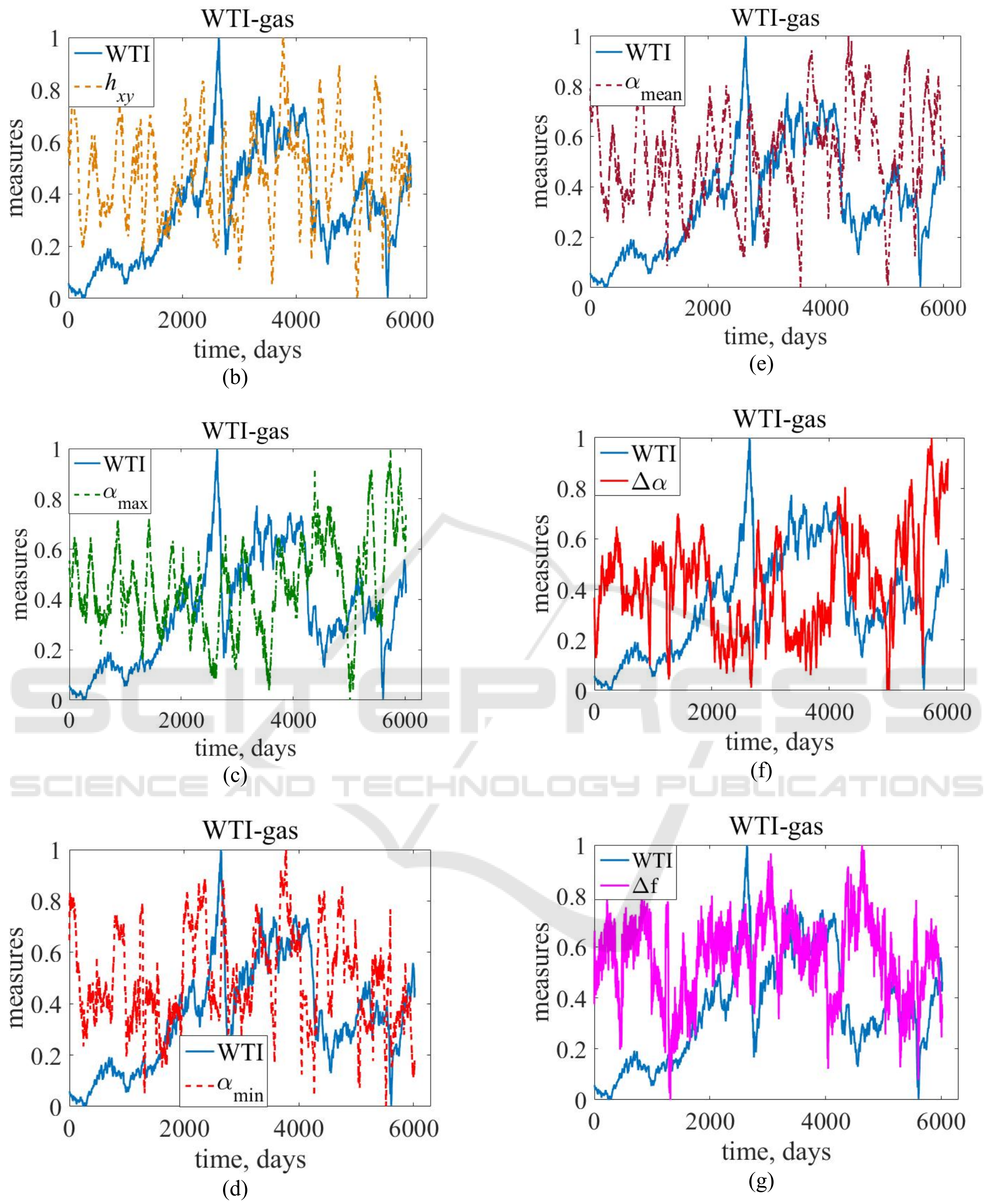



Figure 5: The comparative dynamics of pair WTI crude oil spot prices and Henry Hub natural gas spot prices (WTI-gas) with the DCCA coefficient (a), the cross-correlated generalized Hurst exponent (b), the maximal singularity strength $\alpha_{max}$ (c), the minimal singularity strength $\alpha_{min}$ (d), the mean singularity strength $\alpha_{mean}$ (e), the width of the singularity spectrum $\Delta\alpha$ (f), and its asymmetry $\Delta f$ (g).

According to the results in Figure 5, we can see the same patterns in our indicators. The DCCA coefficient grows during abnormal phenomena for short- and long-term periods. The cross-correlational Hurst exponent demonstrates anti-persistent behavior of time series during crisis. Their multifractality becomes stronger and wider. Finally, $f(\alpha)$ demonstrates left-tailed asymmetry during critical phenomena for both time series.

## 6 CONCLUSIONS

Energy-related markets incorporate necessary information about sustainable development not only in the particular state but in the whole world in general. Policymakers and ordinary traders should have full knowledge about all the supply and demand shocks, which lead to irreversible, non-equilibrium, chaotic, and, studied in this paper, multifractal properties.

In this paper, we have analyzed previous studies related to the topic of the analysis of complex phenomena in energy-related time series, and considering it, we have applied the (MF-)DCCA method to present own analysis of these markets and their varying efficiency.

In this study, we have analyzed (multifractal) cross-recurrent characteristics of such systems as daily data of Henry Hub natural gas spot prices, WTI spot prices, and Europe Brent spot prices. We have compared WTI with Euro Brent and WTI with Henry Hub natural gas.

Using the sliding window approach, we have calculated such measures as the cross-correlation coefficient for long-term scale, the Hurst exponent, the minimal, maximal, and mean singularity exponents, the width of the multifractal spectrum, and its asymmetry. All of the presented indicators give reliable information on the shocks in the energy markets. As expected, the correlation coefficients demonstrate collective behavior between studied time series during crisis events. The Hurst exponent $h_{xy}$ as the classical one increases before the crash, demonstrating trending behavior and decreases during it. Multifractal indicators presented that time series demonstrate extensive multifractality during crisis states.

These results may be useful for regulators, governments, institutional investors who invest or trade in energy-related markets. This will help them to develop portfolios for better decision-making processes during worldwide trends aimed at improving sustainable development. In the future, on the basis of such indicators of the cross-correlation and multifractal properties, it will be possible to create highly reliable risk management systems that will allow to identify and forecast crashes more precisely.

## REFERENCES

Zebende, G. (2011). DCCA cross-correlation coefficient: Quantifying level of cross-correlation. *Physica A: Statistical Mechanics and Its Applications, 390*(4), 614–618. https://doi.org/10.1016/j.physa.2010.10.022

Peng, C. K., Buldyrev, S. V., Havlin, S., Simons, M., Stanley, H. E., & Goldberger, A. L. (1994). Mosaic organization of DNA nucleotides. *Physical Review E, 49*(2), 1685–1689. https://doi.org/10.1103/physreve.49.1685

Zhou, W. X. (2008). Multifractal detrended cross-correlation analysis for two nonstationary signals. *Physical Review E, 77*(6). https://doi.org/10.1103/physreve.77.066211

Ito, M. I., & Ohnishi, T. (2020). Evaluation of the Heterogeneous Spatial Distribution of Population and Stores/Facilities by Multifractal Analysis. *Frontiers in Physics, 8*. https://doi.org/10.3389/fphy.2020.00291

Zhang, X., Liu, H., Zhao, Y., & Zhang, X. (2019). Multifractal detrended fluctuation analysis on air traffic flow time series: A single airport case. *Physica A: Statistical Mechanics and Its Applications, 531*, 121790. https://doi.org/10.1016/j.physa.2019.121790

Mandelbrot, B. B. (2021). *The Fractal Geometry of Nature*. Echo Point Books & Media, LLC.

Aloui, C., & Mabrouk, S. (2010). Value-at-risk estimations of energy commodities via long-memory, asymmetry and fat-tailed GARCH models. *Energy Policy, 38*(5), 2326–2339. https://doi.org/10.1016/j.enpol.2009.12.020

Herrera, R., Rodriguez, A., & Pino, G. (2017). Modeling and forecasting extreme commodity prices: A Markov-Switching based extreme value model. *Energy Economics, 63*, 129–143. https://doi.org/10.1016/j.eneco.2017.01.012

Mandelbrot, B. (1967). The Variation of Some Other Speculative Prices. *The Journal of Business, 40*(4), 393. https://doi.org/10.1086/295006

Hurst, H. E. (1951). Long-Term Storage Capacity of Reservoirs. *Transactions of the American Society of Civil Engineers, 116*(1), 770–799. https://doi.org/10.1061/taceat.0006518

Lo, A. W. (1991). Long-Term Memory in Stock Market Prices. *Econometrica, 59*(5), 1279. https://doi.org/10.2307/2938368

Kantelhardt, J. W., Zschiegner, S. A., Koscielny-Bunde, E., Havlin, S., Bunde, A., & Stanley, H. (2002). Multifractal detrended fluctuation analysis of nonstationary time series. *Physica A: Statistical Mechanics and Its Applications, 316*(1–4), 87–114. https://doi.org/10.1016/s0378-4371(02)01383-3

Podobnik, B., & Stanley, H. E. (2008). Detrended Cross-Correlation Analysis: A New Method for Analyzing Two Nonstationary Time Series. *Physical Review Letters, 100*(8). https://doi.org/10.1103/physrevlett.100.084102

Bielinskyi, A. O., Khvostina, I., Mamanazarov, A., Matviychuk, A., Semerikov, S., Serdyuk, O., Solovieva, V., & Soloviev, V. N. (2021b). Predictors of oil shocks. Econophysical approach in environmental science. *IOP Conference Series: Earth and Environmental Science*, *628*(1), 012019. https://doi.org/10.1088/1755-1315/628/1/012019

Bielinskyi, A., Semerikov, S., Serdiuk, O., Solovieva, V., Soloviev, V., & Pichl, L. (2020). Econophysics of sustainability indices. In A. E. Kiv (Ed.), Proceedings of the Selected Papers of the Special Edition of International Conference on Monitoring, Modeling & Management of Emergent Economy (M3E2-MLPEED 2020) (pp. 372–392). CEUR-WS.org.

Soloviev, V. N., & Belinskiy, A. O. (2019) Complex Systems Theory and Crashes of Cryptocurrency Market. In: Ermolayev V., Suárez-Figueroa M., Yakovyna V., Mayr H., Nikitchenko M., Spivakovsky A. (eds) Information and Communication Technologies in Education, Research, and Industrial Applications. ICTERI 2018. Communications in Computer and Information Science, vol 1007. Springer, Cham. https://doi.org/10.1007/978-3-030-13929-2_14

Hoayek, A., Hamie, H., & Auer, H. (2020). Modeling the Price Stability and Predictability of Post Liberalized Gas Markets Using the Theory of Information. *Energies, 13*(11), 3012. https://doi.org/10.3390/en13113012

Joo, K., Suh, J. H., Lee, D., & Ahn, K. (2020). Impact of the global financial crisis on the crude oil market. *Energy Strategy Reviews, 30*, 100516. https://doi.org/10.1016/j.esr.2020.100516

Lautier, D. H., Raynaud, F., & Robe, M. A. (2019). Shock Propagation Across the Futures Term Structure: Evidence from Crude Oil Prices. *The Energy Journal, 40*(3). https://doi.org/10.5547/01956574.40.3.dlau

Hu, Y., Chen, Y., Tang, S., Feng, L., & Huang, C. (2021). An Explanation of Energy Return on Investment From an Entropy Perspective. *Frontiers in Energy Research, 9*. https://doi.org/10.3389/fenrg.2021.633528

Engelen, S., Norouzzadeh, P., Dullaert, W., & Rahmani, B. (2011). Multifractal features of spot rates in the Liquid Petroleum Gas shipping market. *Energy Economics, 33*(1), 88–98. https://doi.org/10.1016/j.eneco.2010.05.009

Garnier, J., & Solna, K. (2019). Emergence of turbulent epochs in oil prices. *Chaos, Solitons & Fractals, 122*, 281–292. https://doi.org/10.1016/j.chaos.2019.03.016

Ali, H., Aslam, F., & Ferreira, P. (2021). Modeling Dynamic Multifractal Efficiency of US Electricity Market. *Energies, 14*(19), 6145. https://doi.org/10.3390/en14196145

Fang, W., Gao, X., Huang, S., Jiang, M., & Liu, S. (2018). Reconstructing time series into a complex network to assess the evolution dynamics of the correlations among energy prices. *Open Physics, 16*(1), 346–354. https://doi.org/10.1515/phys-2018-0047

Xu, H., Wang, M., & Yang, W. (2020). Information Linkage between Carbon and Energy Markets: Multiplex Recurrence Network Approach. *Complexity, 2020*, 1–12. https://doi.org/10.1155/2020/5841609

Kassouri, Y., Bilgili, F., & Kuşkaya, S. (2022). A wavelet-based model of world oil shocks interaction with CO2 emissions in the US. *Environmental Science & Policy, 127*, 280–292. https://doi.org/10.1016/j.envsci.2021.10.020

Hussain, S. I., Nur-Firyal, R., & Ruza, N. (2021). Linkage transitions between oil and the stock markets of countries with the highest COVID-19 cases. *Journal of Commodity Markets*, 100236. https://doi.org/10.1016/j.jcomm.2021.100236

Wang, G. J., Xie, C., Chen, S., & Han, F. (2014). Cross-Correlations between Energy and Emissions Markets: New Evidence from Fractal and Multifractal Analysis. *Mathematical Problems in Engineering, 2014*, 1–13. https://doi.org/10.1155/2014/197069

Zou, S., & Zhang, T. (2020). Cross-correlation analysis between energy and carbon markets in China based on multifractal theory. *International Journal of Low-Carbon Technologies, 15*(3), 389–397. https://doi.org/10.1093/ijlct/ctaa010

Quintino, D. D., Burnquist, H. L., & Ferreira, P. J. S. (2021). Carbon Emissions and Brazilian Ethanol Prices: Are They Correlated? An Econophysics Study. *Sustainability, 13*(22), 12862. https://doi.org/10.3390/su132212862

*Natural Gas Futures Prices (NYMEX)*. (1997–2021). [Dataset]. U.S. Energy Information Administration. https://www.eia.gov/dnav/ng/ng_pri_fut_s1_d.htm

*Spot Prices for Crude Oil and Petroleum Products*. (1986–2021). [Dataset]. U.S. Energy Information Administration. https://www.eia.gov/dnav/pet/pet_pri_spt_s1_d.htm

Bielinskyi, A. O., Serdyuk, O. A., Semerikov, S. O., & Soloviev, V. N. (2021, December). Econophysics of cryptocurrency crashes: a systematic review. In A. E. Kiv, V. N. Soloviev, & S. O. Semerikov (Eds.), Selected and Revised Papers of 9th International Conference on Monitoring, Modeling & Management of Emergent Economy (M3E2-MLPEED 2021) (pp. 31–133).

Bielinskyi, A. O., Hushko, S. V., Matviychuk, A. V., Serdyuk, O. A., Semerikov, S. O., & Soloviev, V. N. (2021, December). Irreversibility of financial time series: a case of crisis. In A. E. Kiv, V. N. Soloviev, & S. O. Semerikov (Eds.), Selected and Revised Papers of 9th International Conference on Monitoring, Modeling & Management of Emergent Economy (M3E2-MLPEED 2021) (pp. 134–150).

Mensi, W., Sensoy, A., Vo, X. V., & Kang, S. H. (2020). Impact of COVID-19 outbreak on asymmetric multifractality of gold and oil prices. Resources Policy, 69, 101829.