AGM cognitive actions as modal operators of three-valued logic

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Belief revision

- Artificial Intelligence and computer science
- Databases functioning, support and updating
- Update and consolidation of software products
- Information evaluation
- Decision making
- Critical thinking ???

AGM

Carlos Alchourròn, Peter Gärdenfors and David Makinson

- general concepts of belief changes
- cognitive actions defining
- rational agent‘s reasoning

Research objectives

- Consideration of cognitive actions within real logic
- Interpretation of the basic rules of AGM cognitive actions
- Finding out the features of this interpretation
- Get possible answers to some questions about cognitive actions

General Rules for Belief Dynamics

- All beliefs of an agent can be presented as propositions
- A sentence is considered a belief if a subject accepts it as true
- Belief dynamics is possible
- Belief changes should come from natural reasoning
- Belief changes should be rational

Criteria of rationality

Primacy of new information

– the new information is always accepted.

Consistency

– the new epistemic state must be consistent if possible

Minimal loss of previous beliefs

– the attempt to retain as much of the old beliefs as possible

Four components of epistemic theory [Gärdenfors]

- **Epistemic states** (an epistemic state is "in equilibrium" if it is consistent and satisfies the rationality criteria)
- **Epistemic attitudes** (belief statuses: accepted, rejected, indetermined)
- **Epistemic inputs** (external stimuli provoking "belief changes")
- **Criteria of rationality** (are used to determine the behaviour of the belief change)

The epistemological theories are conceptualistic in the sense that they do not presume any account of an 'external world' outside of the individuals' epistemic states. It is true that the epistemic inputs in general have their origin in such a 'reality', but I argue that epistemic states and changes of such states as well as the rationality criteria governing epistemic dynamics can be, and should be, formulated independently of the factual connections between the epistemic inputs and the outer world

Gärdenfors P. *Knowledge in flux: Modeling the dynamics of epistemic states*, p. 9.

Belief set

Belief – a statement that the subject believes to be true – A.

Belief set – a set of statements that the subject believes to be true.

K $\vdash A$ iff A $\in$ K.

Belief status

- Belief A is accepted iff $A \in K$
- Belief A is rejected iff $A \notin K$
- Else the status of A is indetermined

Belief changes = Cognitive actions

Expansion

1. From indetermined to accepted.
2. From indetermined to rejected.

Revision

3. From accepted to rejected.
4. From rejected to accepted.

Contraction

5. From accepted to indetermined.
6. From rejected to indetermined.

AGM expansion and contraction

AGM expansion: **K + A** (**K** an initial belief set, **+** – the sign of expansion and **A** – a belief added to the initial set).

AGM contraction: **K** $\div$ **A** (**K** – an initial belief set, $\div$ – the sign of contraction and **A** – a belief extracted from K).

A sentence **A**, which was not earlier accepted, may be added to the set of sentences, either as a result of new evidence or as a hypothetical assumption in an argument. Or a sentence A that was accepted may be given up as a result of conflicting evidence or a desire to open up for investigation a sentence.

Gärdenfors P. *Knowledge in flux: Modeling the dynamics of epistemic states*, p. 12.

AGM postulates of expansion

Closure	$K + A = Cn(K + A)$	**E 1**
Success	$A \in K + A$	**E 2**
Inclusion	$K \subseteq K + A$	**E 3**
Vacuity	If $A \in K$, then $K + A = K$	**E 4**
Monotonicity	If $K \subseteq H$, then $K + A \subseteq H + A$	**E 5**

AGM postulates of contraction

Closure	$K \div A = Cn(K \div A)$	**C 1**
Success	If $\nvdash A$, then $K \div A \nvdash A$	**C 2**
Inclusion	$K \div A \subseteq K$	**C 3**
Vacuity	If $A \notin K$, then $K \div A = K$	**C 4**
Extensionality	If $A \Leftrightarrow \beta$, then $K \div A = K \div \beta$	**C 5**

Some Interesting Issues in Belief Dynamics

- The problem of the purity of doxastic operations
- The problem of the primacy of doxastic operations
- The problem of connection between doxastic operations

Belief system

- **Epistemic state**: belief set is a set of proposition closed under the logical consequence
- **Valuation function**:
  1. v(A) $=$ t iff $A \in K$
  2. v(A) $=$ n iff $A \notin K$ and $\neg A \notin K$
  3. v(A) $=$ f iff $\neg A \in K$
- **Epistemic inputs**: external validating, internal derivability
- **Commitment function**: if $\vdash A$ then $A \in K$

Expressions and negation

A – statement A belongs to the initial belief set
A & B – statement A and (set of statements, implied) B, belong to the initial belief set
$B \to A$ – statement A belongs to the set of beliefs described by the set of statements B.

- **Non-doxastic expressions**: A, $\neg A$, B, A & B, A $\vee$ B, $\neg A \& B$
- **Doxastic expressions**: +A, $\div A$, $+A \vee B$, $+(A\&B) \to +B$
- **Negation**

$\neg$	p
f	t
n	n
t	f

Implication

$\rightarrow$	t	n	f
t	t	n	f
n	t	t	n
f	t	t	t

$t > n > f$

1. Implication $x \rightarrow y$ is true if $y \geqslant x$.
2. Implication is unknown in other cases if only one of its components is defined.
3. Implication is false if its antecedent is true and its consequent is false.

Conjunction and Disjunction

- **Conjunction**: $p\&q \equiv \neg(\neg p \vee \neg q)$
  v($p\&q$) = min(v(p), v(q))

- **Disjunction**: $p \vee q \equiv (p \to q) \to q$
  v($p \vee q$) = max(v(p), v(q));

Expansion

$+$	p
t	t
f	n
f	f

0. A cannot be indetermined.

1. From indetermined to accepted.

$+$A if A $=$ true

2. From indetermined to rejected.

$+\neg A$ if A $=$ false

Expansion

The first type is when the epistemic attitude 'A is indetermined' is changed into either 'A is accepted' or 'not-A is accepted' (that is, 'A is rejected'). I call this kind of change an expansion, because it consists in adding a new belief (and its consequences) to the belief set without retracting any of the old beliefs.
(Gärdenfors P. *KiF*, p. 47)

Operator + is fully expressed in propositional logic

$+p \equiv \neg(p \rightarrow \neg p)$

+p	$\neg$	$(p$	$\rightarrow$	$\neg p)$
t	**t**	t	f	f
f	**f**	n	t	n
f	**f**	f	t	t

Contraction

- **A is indetermined**: $\neg + A \ \& \ \neg + \neg A$

The second kind of change occurs when one of the attitudes 'A is accepted' or 'A is rejected' is changed into 'A is indetermined'. This kind of change is called a contraction, because it consists in giving up the belief in A (or the belief in not-A). This kind of change can be made by an agent in order to open up for investigation some proposition that contradicts what the agent previously believed. (Gärdenfors P. *KiF*, p. 47)

Contraction

$\div A = \neg{+}A \mathbin{\&} \neg{+}\neg A$

$\div$	p
f	t
t	n
f	f

0. A cannot be determined.
1. From accepted to indetermined.
2. From rejected to indetermined.

the agent is not (no longer) sure about A at all

$\div A \equiv \div\neg A$

All possible states

$$A \vee \div A \vee \neg A$$

A	$\vee$	$\div A$	$\vee$	$\neg A$
(t)		f		f
n		(t)		n
f		f		(t)

Iteration and Composition

$$+ + A \equiv +A \qquad \div\div A \equiv \bot$$
$$+ \div A \equiv \div A \qquad \div + A \equiv \bot$$

Introduction rules

Expansion Introduction

If $\vdash A$ then $\vdash +A$

Contraction Introduction

If $\vdash +A$ and $\vdash +\neg A$ then $\vdash \div A$

Identifying the status of the sentence

$+A$ – A is exactly true (only true): $A \in K$, $K \to A$, $K\&A$;

$+\neg A$ – A is exactly false (only false): $\neg A \in K$, $K \to \neg A$, $K\&\neg A$;

$\div A = \div\neg A$ – A is not defined: $A \notin K$ and $\neg A \notin K$

$\neg+A$ – A is not true (false or indefinite): $A \notin K$.

$\neg+A$	$=$	$\neg A$	$\vee$	$\div A$
f		f	**f**	f
t		n	**t**	t
t		t	**t**	f

Implication and doxastic commitments

$+A \to A$

Expanding a belief set by A, the agent must accept A as his belief

$A\&B \to A$
If an agent maintain a belief $A\&B$, then she also must maintain a belief A.

$+A \to \neg{\div}A$
If we have expanded the set of beliefs by A, then we commit not to remove A.

AGM postulates of expansion

Closure	$K + A$ – belief set $+(A \to B) \to (+A \to +B)$	**E 1**
Success	$A \in (K + A)$ $+A \to A$	**E 2**
Inclusion	$K \subseteq (K + A)$ $B\& + A \to B$	**E 3**
Vacuity	If $A \in K$ then $(K + A) = K$ $A \to (+A \to A)$ or $(B\&A) \to (+A \to B)$	**E 4**
Monotonicity	If $K \subseteq H$ then $(K + A) \subseteq (H + A)$ $(B \to C) \to ((B\& + A) \to (C\& + A))$	**E 5**

AGM postulates of contraction

Closure	$K \div A$ – belief set $\div(A \to B) \to (\div B \to A)$	**C 1**
Success	If $\nvdash A$ then $K \div A \nvdash A$ $\div A \to \neg + A$	**C 2**
Inclusion	$K \div A \subseteq K$ $\neg(\div A \to B) \ \to \ \neg + B$	**C 3**
Vacuity	If $A \notin K$ then $K \div A = K$ $(B \& \neg + A) \to (\div A \to B)$	**C 4**
Extensionality	If $A \Leftrightarrow B$ then $K \div A = K \div B$ $(A \leftrightarrow B) \leftrightarrow (\div A \leftrightarrow \div B)$	**C 5**

Trouble with contraction

Conjunctive inclusion

$$\text{If } K \div (A \wedge B) \nvdash A \text{ then } K \div (A \wedge B) \subseteq K \div A$$
$$\neg(\div(A \,\&B) \;\rightarrow A) \;\rightarrow (\div A \;\rightarrow\; \div(A \wedge B))$$

Conjunctive factoring

$$\div(A \,\&B) \;\rightarrow\; \div A \;\vee\; \div B$$

Recovery

$$K \subseteq (K \div A) + A \qquad \textbf{C 6}$$
$$(B\& \div A) \rightarrow (+A \rightarrow B)$$

Faces of minimality

Minimality 1 $\div(A \to B)\& \div B \to \neg(+A \to +B)$

Minimality 2 $B\&(\div A \to \div B) \to (\div A \to (B \to A))$

Minimality 3 $B\&\neg+(A \to B) \ \to \ (\div A \to B)$

Conclusion

- $\div A$ and $+A$ comes from Gerdenfors' requirements for belief changes
- $\div A$ and $+A$ satisfies the necessary postulates
- $\div A$ partially satisfies additional postulates
- $\div A$ does not reflect the properties of the AGM contraction very well and has some strange properties
- $\div A$ vs $/$ and $\neg + A$

- 1. **On the purity of doxastic operations**: every contraction must be preceded by an expansion.
- 2. **On the primacy of doxastic operations**: the expansion operator is primary.
  3. **On the problem of connection**: a contraction operator can be expressed by the expansion and negation.