Modelling of cryptocurrency market using fractal and entropy analysis in COVID-19

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Abstract. In this article, we present the results of simulation for cryptocurrency market based on fractal and entropy analysis using six cryptocurrencies in the first 20 of the capitalization rating. The application of the selected research methods is based on an analysis of existing methodologies and tools of economic and mathematical modeling of financial markets. It has been shown that individual methods are not relevant because they do not provide an adequate assessment of the given market, so an integrated approach is the most appropriate. Daily values of cryptocurrency pairs from August 2016 to August 2020 selected by the monitoring and modelling database. The application of fractal analysis led to the conclusion that the time series of selected cryptocurrencies were persistent. And the use of the window procedure for calculating the local Hurst coefficient allowed to detail and isolate the persistant and antipersistant gaps. Interdisciplinary methods, namely Tsallis entropy and wavelet entropy, are proposed to complement the results. The results of the research show that Tsallis entropy reveals special (crisis) conditions in the cryptocurrency market, despite the nature of the crises' origin. Wavelet entropy is a warning indicator of crisis phenomena. It provides additional information on a small scale.

Keywords: cryptocurrency market, fractal analysis, wavelet entropy, Tsallis entropy, crisis, COVID-19.

1   Introduction

In the context of digitalization and virtualization of the financial sphere, the problem of forecasting the cryptocurrency market dynamically developing in recent years, is of particular importance. The improvement of tools for modelling and forecasting of the

cryptocurrency market and risk management are necessary in the context of public distrust of cryptocurrency as an innovative financial asset, and the presence of a number of risks inherent in the cryptocurrency transactions (asset security, hedging complexity, high price volatility, lack of guarantees, legal barriers, the limited use as a means of payment).

The COVID-19 pandemic, in which the governments of the world's major economies have been forced to adopt unprecedented security measures, has highlighted this problem. As a result, social, economic, psychological and behavioural crises can occur in the world economy and social spheres [18; 33]. Cash flows have had to be diverted to the treatment of patients, the elimination of the effects of the pandemic, and the development and testing of influenza drugs instead of investing in financial and stock markets. In the research [17] the author draws a parallel between natural disasters, terrorist events, crises, epidemics, pandemics and proves that the effects of the pandemic are particularly pronounced in the financial and banking sectors, in the stock markets and in insurance. Stock market reactions at COVID-19 are the subject of the research [4] in which the researcher identifies the dependency and volatility of stock markets on flu morbidity and mortality. Stock markets have been shown to respond quickly to COVID-19, but the intensity of the reaction has depended more on the number of diseases than on the number of deaths. According to the author, the fluctuating cycle of the stock market's decline and increase is 40-60 days since the onset of the pandemic in the country.

The impact of COVID-19 on stock markets, which is developing, is even more threatening. This leads to instability in the financial, fiscal, and other spheres. Researching the 26 stock markets that are developing, the author in publication [36] has empirically proven that at the onset of the pandemic, with limited economic activity, mobility of the population, cash transfers, trade, commercial service activities, there was a free fall of indices in all the markets studied. Using a Driscoll-Kraay estimate, the researcher concluded that coronavirus had a negative and statistically significant impact on stock markets until mid-April, and then in countries where compensation mechanisms were used, this effect gradually diminished.

Authors of [7] point out that the formation and development of the cryptocurrency market are associated with Bitcoin release and blockchain technology that have some spheres of application; and cryptocurrencies are innovative financial asset that attracts investors.

According to the authors of [22], in assessing stability during a pandemic, there is a higher level of risk to the cryptocurrency market than the stock market. That is, digital currencies exhibit instability, increased irregularity and volatility, indicating a decline in their investment attractiveness during this period.

An important segment of research is the study of the significant impact of the cryptocurrency market in a context of limited communication and commercial activity, where Internet commerce and services are becoming increasingly important. The current development of the economy is characterized by the rapid growth of the pace of informatization of economic processes, the expansion of the scale of e-business, and, in particular, of retail e-trade on a large scale [32]. E-commerce has grown rapidly, spreading comprehensively and offering an increasingly a diverse assortment of goods

and services, e-commerce becomes an instrument for the integration of individuals, enterprises, industries, state institutions, and states into a united community, within which the interaction of partners is effectively and unhindered by means of information and telecommunication technologies [6]. The basis of e-commerce is the new information technologies for commercial operations and the management of production processes. E-commerce, compared to traditional business, has substantial advantages. In particular, the use of new electronic communication channels and new cryptocurrencies significantly reduce costs related to organization and support e-business, and the possibilities of e-commerce allow re-designing business strategy at any moment [30].

Thus, in the context of COVID-19, there is a demand among investors and businessmen for up-to-date information on the state and dynamics of the cryptocurrency market, as well as the possibility of forecasting it. The use of modelling tools for research and forecasting of the cryptocurrency markets using an integrated approach and interdisciplinary methods is therefore a topical scientific and applied challenge.

National and international researches pay great attention to this problem. Different approaches and methods are used for modelling and forecasting the cryptocurrency market and forming the investment portfolio. As this market becomes more and more difficult, the potential investors need tools allowing them to form a highly profitable investment portfolio that may include such an asset as cryptocurrency.

To calculate the cryptocurrency market efficiency, paper [3] offers to use Factor Augmented VAR (TVP-FAVAR)-model considering the impact of a large number of variable factors on a dependent variable and allowing to study the dynamics of more than 45 cryptocurrencies. This approach helps to conclude that the periods of high/low uncertainty in the market correspond to strong/weak link. The authors explain the trend by the increased degree of the market uncertainty associated with the process of cryptocurrency price fluctuations. In this situation, they propose to form a dual investment portfolio, the structure of which can be varied by the dynamic hedge ratio and the dynamic weights of the portfolio. Using the ARFIMA-FIGARCH model with two distributions and the modified logarithmic periodogram method, authors [27] studied the stability of eight biggest cryptocurrency markets and made a conclusion that they were unstable, volatile, had the limited trading horizons and time gaps, complicating the process of attracting investors.

Paper [10] proposes to assess the dynamics of the cryptocurrency profitability and stability by the multivariate stochastic model, which allows to calculate the average currency volatility. Its practical application shows the significant impact of such factors as the volatility of the cryptocurrency market on the one hand, and the rapid growth in demand for it on the other.

In paper [16], researchers use a Lotka-Volterra model with variable intervals to model the number of transactions for Bitcoin, Litecoin and Ripple using two- and three-dimensional models that allow them to get high-precision forecast for Bitcoin and satisfactory level for Litecoin and Ripple.

The investigation of cryptocurrency time series using econometric models of stochastic volatility is shown in papers [9; 29]. Alternative one-dimensional dynamic

linear and multivariate vector autoregressive models are compared in paper [9]. This approach improves the accuracy of the forecast.

The paper of scientists [8] shows that the use of standard GARCH models can make incorrect VAR and ES forecasts, and thus, lead to inefficient risk management and optimization of the investment portfolio. Therefore, the authors propose to use the method of model building (model complex) of VAR and ES recheck based on a confidence model (MCS) to minimize risks and financial losses.

The traditional method of the cryptocurrency market analysis is technical analysis, but in the conditions of high market volatility, it is reasonable to combine the methods of technical analysis with the methods of economic and mathematical modelling. Thus, authors [1] propose a non-parametric model based on technical analysis as an alternative method of assessment and forecasting of the cryptocurrency market. This approach calculates the forecast values of Bitcoin's profitability through a neural network and indicates the speculative nature of the market.

Traditional methods and modelling do not allow to make accurate forecasts and calculations for the development of the cryptocurrency market being a nonlinear complex economic system and to detect the occurrence of crisis phenomena.

Therefore, it is necessary to use the methods being traditionally inherent in other fields of science. Econophysics has rather powerful methodological apparatus for modelling complex socio-economic systems in modern economic science.

Thus, paper [2] points out that it is necessary to apply comprehensive approach for the forecast considering the nonlinear dynamics and the inherent chaos and fractality of the digital currencies. The authors propose the hybrid model for the forecast based on the neural network of long-term memory (LSTM) and empirical wavelet decomposition (EWT), along with the cuckoo search algorithm (CS) for digital currency time series to obtain more accurate forecast values.

Authors [11] use monofractal analysis to investigate the price volatility in the cryptocurrency market and multifractal fluctuation analysis to test the model for stability.

A strong impulse effect in the Bitcoin and Ethereum markets and a reversal effect for Ripple and EOS at high fluctuations were found as a result of the model implementation. The application of this model will help to form effective alternative strategies for the allocation of assets in the investment portfolio.

Article [26] contains a multifractal analysis of the cryptocurrency market's performance level before and after the COVID-19 pandemic. The authors' calculations show that, overall, the cryptocurrency market became more efficient during the pandemic. The research [25] analyzed asymmetrical multifractal oscillations without a trend for Bitcoin and Ethereum, which revealed structural breaks (shifting), long memory, and asymmetric multifractures. Bitcoin has been shown to exhibit low performance on the general upward and downward trends, with the gap between the latter two being small when the temporal scale is reduced and rather, growing as it grows.

Entropy and wavelet analysis are an effective tool for modelling and research of the cryptocurrency market. The authors of article [20] propose to use a multifractional decomposition approach on a time scale for analysis of prices and trading volumes at

different sampling frequencies in high-frequency cryptocurrency markets. The analysis of a large amount of data with the combination of statistical testing showed a heterogeneous multifractal dynamic at certain intervals, so the application of this methodology allowed the authors to obtain a more precise definition of the singularity spectrum. This will allow potential investors to develop an efficient and profitable strategy for high-frequency cryptocurrency trading.

With the help of Renyi entropy, the authors [21] analyze the processes of information exchange and information chaos in the stock markets, cryptocurrency markets, oil and gas markets, precious metals markets before and after a pandemic in order to identify investors' expectations. Empirical research has concluded that investment portfolios containing Bitcoin and precious metals (gold, silver), Bitcoin and Brent have high risk and need to diversify into balanced portfolios, For example, between gas and silver, gold and silver, Brent and silver, Bitcoin and gold.

The application of interdisciplinary and integrated tools for the modelling, forecasting and monitoring of financial markets, in particular stock, currency and cryptocurrency, is also the subject of the work of domestic scientists.

Ukrainian scientists [12] propose the procedure for determining the normalized economic coordinates, economic mass and heterogeneous economic time, based on the basic concepts of general theory of relativity and relativistic quantum mechanics. They are based on the analysis of time series describing socio-economic phenomena and economic interpretation of uncertainty by Heisenberg. The authors confirm that the economic mass of the time series can be an indicator of crisis phenomena.

In the research [14] the authors modelled and monitored cryptocurrency pairs with different capitalization degrees by the recursive and *R/S*-analysis. The usage of the R/S-method led them to conclude that the time series of selected cryptocurrencies were persistent, indicating clear market trends. The highest level of trends' persistence is the currency pair XRP/USD. And with the help of recurrence analysis, crisis periods in the cryptocurrency market were identified.

Therefore, a combination use of different methods of economic and mathematical modelling of the cryptocurrency market allows to increase the accuracy of forecasting, to identify the indicators of crisis phenomena, to analyze the degree of volatility and risk of both individual cryptocurrencies and their pairs.

2 Research methodology

2.1 Fractal analysis

R/S-analysis was first proposed by B. B. Mandelbrot and J. R. Wallis [23] and is based on H. Hurst hydrological studies. According to Hurst, the essence of fractal analysis for time series is to process the structure of a series that reflects a certain process and demonstrates a quantitative degree of chance. In paper [19], this approach was proposed to quantify the nature of self-affine structures.

The method of *R/S*-analysis allows calculating the parameter of self-similarity *H*, which measures the intensity of long-term dependencies in a time series. This metric is based on the analysis of the parameter range and the standard deviation.

Here is an algorithm for *R/S*-analysis. Suppose that the time series $y = \{y_i\}$, $I = 1, 2, \ldots, n$ is given, which must be led to "logarithmic returns". The resulting sequence is divided into initial segments $y = y_1, y_2, \ldots, y_N$.

(i) The average value and standard deviation of S_n are calculated for each segment.

(ii) The cumulative deviation is calculated:

$$x_{t,N} = \sum_{i=1}^{t}(y_i - M_N),\qquad(1)$$

where y_i – increase in a year i, M_N – average, y_i for N periods.

(iii) Range is determined:

$$R(N) = \max\left(x_{t,N}\right) - \min\left(x_{t,N}\right),\qquad(2)$$

where $R(N)$ – deviation range $x_{t,N}$, $\max(x_{t,N})$ – maximum value for $x_{t,N}$, $\min(x_{t,N})$ – minimum value for $x_{t,N}$.

(iv) From the ratio

$$R/S = (aN)^H,\qquad(3)$$

where R/S – normalized range, N – observation number, a – constant, the Hurst exponent H is derived.

By the value of the Hurst coefficient, we can identify time series and draw conclusions about the minimum forecast of these series behaviour:

(i) $H = 0.5$ – the sequence is white noise; the time series is random. The future values of this series are independent of the previous ones. System status is random walk;

(ii) $0 < H < 0.5$ – anti-persistent or ergodic time series (pink noise), i.e. a series characterized by the so-called "return to average": if there is a decline in the indicator in the studied system over a period of time, an increase should be expected in the next period. The closer H is to 0, the more variable the series is. These changes happen quickly, often, but they are small. Note that such processes are few in reality. System status – flute;

(iii) $0.5 < H < 1$ – persistent or trend resistant time series (black noise, Brownian motion). The time series is characterized by the effect of long-term memory. If the series is growing, it is likely to continue to grow. If it falls today, a decline should be expected tomorrow. The closer the value of H is to 1, the more trend resistance is. The closer the value of N is to 0.5, the noisier the number will be. System status is a trend.

The method *R/S*-analysis has a significant limitation as it shows adequate results only for stationary series. However, most market processes are not static. The Method of Detrended Fluctuation Analysis (DFA), proposed in [28], is devoid of this shortcoming. It involves removing local trends on all segments. The local trend is understood as approximating a polynomial of some fixed order m. Approximation involves linear, quadratic, cubic or higher order polynomials (commonly called DFA1, DFA2, DFA3, etc.). Note that each value of the order of polynomial m corresponds to a separate scaling index $H_{[\text{DFA }(m)]}$.

The advantages of DFA over other methods are that it exhibits long-term correlations of non-stationary time series, as well as ignores obvious random correlations, are a

consequence of non-stationality.

Compared to R/S-analysis, the DFA method gives great possibilities to interpret the scaling index $H_{[\text{DFA}\,(m)]}$ and allows to provide such a classification of series:

(i) $H_{[\text{DFA}(m)]} = 0.5$ – random series;

(ii) if only short-term correlations are present $H_{[\text{DFA}(m)]}$ may differ from 0.5, but tends to follow up to 0.5 when the size of the split segments increases;

(iii) $0.5 < H_{[\text{DFA}(m)]} \leq 1$ – shows persistent long-term correlations;

(iv) $0 < H_{[\text{DFA}(m)]} < 0.5$ – antipersistent series;

(v) $H_{[\text{DFA}(m)]} = 1$ – existing flicker noise $1/f$;

(vi) $H_{[\text{DFA}(m)]} > 1$, correlations exist but cease to reflect power relationships [35].

Application of different orders DFA allows to reveal hidden patterns in the input row.

The time sequences have both persistent and antipersistent gaps.

The local DFA method is based on the use of a mobile window of constant size. The window size shall be sufficient to use the standard DFA method by which the value of the local coefficient is determined. The movable window moves in a sequence with a certain step and the calculations are repeated. The window continues to move until it is fully depleted. Analysis of the local Hurst coefficient allows to identify and interpret significant changes in the dynamics of the system [35].

2.2 Tsallis and wavelet entropy

In economic systems, there is generally a strong interaction between its individual elements. As a result, we have broken the hypothesis of total chaos, and because of this, we have lost the relevance of its characteristics.

The idea of C. Tsallis [38] is to generalize the classical Shannon formula for entropy in the case of complex non-additive statistical systems by defining

$$S_q = \frac{k}{q-1}\left(1 - \sum_{i=1}^{n} p_i^q\right),\qquad(4)$$

where k – is the Boltzmann constant, q – is the real number ($q \in \mathbb{R}$), n – is the total number of configurations, p_i – is the probability corresponding to the microscopic configurations.

Tsallis' entropy is a generalization of the non-additive measure and is calculated by the formula

$$S_q = \frac{1 - \int f(x)^q dx}{q-1}.\qquad(5)$$

For anomalous systems with long memory and (or) long-range interactions, the coefficient q provides insight into the behavior of complex systems: at $q < 1$ in the system there is a predominance of rare phenomena, at $q > 1$ – the advantage of recurring phenomena if $q \to 1$ entropy of Tsallis is reduced to Shannon's entropy. The high values of this coefficient can be considered as a long-term memory parameter because they correspond to long-term relationships between the states of the system [37; 41]. It should be pointed out that for economic series $q < 1$ has not been observed yet.

As already noted, Tsallis' entropy is non-additive: if A and B are two independent states for which it is true that $p\,(A\,B) = p\,(A)\,p\,(B)$

$$S_q(A+B) = S_q(A)+S_q(B) - (1-q)S_q(A)S_q(B). \tag{6}$$

The parameter q is calculated from the cumulative distribution function

$$P_{q,k}(X \geq x) = \left(1 - \frac{(1-q)x}{k}\right)^{\frac{1}{1-q}}, \tag{7}$$

where k – defines limitations. Note that the usage of numerous and mathematical methods of parameter calculation leads to a system of nonlinear equations, the solution of which has a very large bias. Therefore, the research [34] proposes a maximum likelihood method with the previous reduction of the distribution function to the Pareto distribution to estimate the value of the parameter in order to minimize the calculation bias.

An important accent is that Tsallis entropy is calculated for a time series based on the probability distribution of these values, which is substantially non-Gaussian. Such distributions have profitable financial and economic series.

The wavelet transforms provide two-dimensional expansion of the investigated one-dimensional signal, that is, we have the possibility simultaneously to analyze the signal in both physical and frequency spaces. Wavelet transforms [5] are also used to describe the dynamics of complex non-linear processes where different perturbations in the spatial and temporal frequency ranges are possible. It is in the amplitude-time form that the information about the signal is written.

The essence of the wavelet-analysis method is that an appropriate basis and characteristics of the time series are introduced using the amplitude distribution (series count) in the introduced basis. Any function studied can only be decomposed into a basis if it is required to be orthogonal. This arrangement allows the restoration of the series [15].

Two types of wavelet transform are used in the studies: continuous and discrete.

Continuous wavelet transform (CWT) is directed at quantifying functions at a specific frequency and at a certain point in time. With CWT it is possible to decompose a function into elementary components, which is interesting when studying functions with complex structure. Note that CWT is better suited to studying functions than economic time series [24].

The discrete wavelet transform (DWT) can be used to derive a time series with values in the form of coefficients in the wavelet sequence. These coefficients provide complete information on a number of factors and make it possible to obtain local energy of its constituents at different scales. Note that information can be organized as a hierarchical schema of nested subspaces (multiscale analysis).

Let us give theoretical data of the bases of wavelet entropy and mathematical calculations [31; 40].

Consider time series $X = x_i$, $i = 1, \ldots, M$, the elements of which are represented on the uniform grid with the sampling time t_s (frequency ω_s). Apply the set of scales $1, \ldots, N$ and we will get

$$X(t) = \sum_{j=1}^{N} \sum C_j(k)\Psi_{j,k}(t) = \sum_{j=1}^{N} r_j(t). \tag{8}$$

Here $r_j(t)$ – a value containing information on a series X at frequencies $2^{j-1}\omega_s \le |\omega_s| \le 2^j\omega_s$, Ψ – the analytic function, called the parent wavelet, is the base for the wavelet transform. The spectral coefficients (wavelet coefficients) are defined as $C_j(k) = \langle S, \Psi_{j,k} \rangle$ and allow the energy at each scale $j = 1, ..., N$ to be represented as the energy of the series elements [13].

The sum of the energies of each level gives us the total wavelet energy of the spectrum:

$$E_{tot} = \|X\|^2 = \sum_{j=1}^{N} \sum_k |C_j(k)|^2 = \sum_{j=1}^{N} E_j, \tag{9}$$

where N – maximum expansion level.

The paper [15] shows that the wavelet conversion of the original series is based on wavelet entropy, which is a quantitative measure of the ordered signal and is calculated by the formula:

$$WE = -\sum_{j=1}^{N} p_j ln(p_j), \tag{10}$$

where $p_j = \frac{E_j}{E_{tot}}$ – the relative wavelet energy over which the wavelet energy distribution is determined by the level of the arrangement. Values p_j at different scales form an energy probability distribution:

$$\sum_{j=1}^{n} p_j = 1. \tag{11}$$

Time scale density can be used to determine the characteristics of the time series, its characteristics both in time and in frequency space $\{p_j\}$.

Shannon entropy, which is a measure of the information contained in the distribution, is considered a criterion for analysing and comparing probability distributions. For the determination of normalized total wavelet entropy (*NTWE*) we have [40]:

$$E_{WT} = \frac{-\sum_{j=1}^{N} p_j \cdot ln p_j}{X_{max}}, \tag{12}$$

where $X_{max} = lnN$ the normalizing constant. Depending on the values that will be normalized wavelet entropy, it is possible to characterize the processes occurring in the time series. Normalized total wavelet entropy can be seen as a measure of the degree of chaotic time series, which can provide information about hidden dynamic processes occurring in the system [15].

In order to improve the algorithms for calculating the characteristics of wavelet entropy, the usage of window procedure is meaningful. The wavelet transform is automatically embedded in a movable frequency-time window. For this purpose, the windows of a certain length L are selected, without overlapping, forming i intervals $i = 1, ..., N_T$, де $N_T = \frac{M}{L}$. The corresponding values of the time series of each interval shall be compared to the centre of the window. Therefore, at least one wavelet

coefficient at each scale will be the smallest window length [13].

The moving window is equally good for both low- and high-frequency signal characteristics, which is an advantage of this method. By formula find wavelet energy at scale j for the time window i

$$E_j^{(i)} = \sum_{k=(i-1)\cdot L+1}^{i\cdot L} |C_j(k)|^2, i = 1, \dots, N_T. \tag{13}$$

So, we can find shared power in this time window

$$E_{tot}^{(i)} = \sum_{j=-N}^{-1} E_j^{(i)}. \tag{14}$$

The following formulas can be used to calculate the relative and normalized wavelet entropy reflecting changes over time:

$$p_j^{(i)} = \frac{E_j^{(i)}}{E_{tot}^{(i)}}, E_{WT}^{(i)} = -\sum_{j=-N}^{-1} p_j^{(i)} \cdot \frac{\ln p_j^{(i)}}{X_{max}}. \tag{15}$$

In order to avoid differences in the number of wavelet coefficients at all scales, when the wavelet transform was performed, copies of the wavelet coefficient were added to the time output row on the right and left, so when the wavelet coefficients of C_{ij} are indicated, there are two indexes – scale number and point number respectively.

To calculate wavelet entropy, the Shannon entropy formula was used, which provides a simple method for calculating the probability density of time series energy.

In the paper [31] the indicators of wavelet entropy – scale and point entropy derived from the energy of wavelet coefficients are defined. These types of indicators are based on the Shannon formula approach: scale for entropy, hour for point for scale.

As a result of these calculations, find the wavelet coefficient energy field

$$E_{ij} = C_{ij}^2, \tag{16}$$

which is normalized by the standard average deviation of the output time series

$$\tilde{E}_{ij} = \frac{E_{ij}}{\sigma_t}. \tag{17}$$

The formula for calculating the scale of entropy is:

$$E_{WS} = -\sum_i \left(\left(\sum_j p_{ij} \right) \cdot \frac{\log_2 \sum_j p_{ij}}{\log_2 N} \right), \tag{18}$$

where $p_{ij} = \frac{\tilde{E}_{ij}}{\tilde{E}_{tot}}$ – entropy probability distribution, $E_{tot} = \sum_i \sum_j \tilde{E}_{ij}$, $\log_2 N$ – normalizing multiplier (constant), N – number of time series elements.

The formula for calculating point entropy is:

$$E_{WP} = -\sum_i \left(\left(\sum_j p_{ij} \right) \cdot \frac{\log_2 \sum_j p_{ij}}{\log_2 N} \right). \tag{19}$$

To calculate point entropy, the energy values at each scale would be determined $\tilde{E}_i = \sum_i \tilde{E}_{ij}$ and is the distribution of the probability density of the energies by dividing the energies of the wavelet coefficients by the total energy of the corresponding scale $p_{ij} = \frac{\tilde{E}_{ij}}{\tilde{E}_i}$.

Thus, the technology of using wavelets allows to identify "spikes", sharp changes in the values of quantitative indicators at different periods of time. Wavelet analysis can identify not only obvious anomalies in the investigated system, but also critical values that may be hidden behind relatively small absolute values of the elements.

3 Research findings

Selected six cryptocurrencies in the first 20 capitalization rankings: Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), EOS (EOS), Tether (USDT), USDCoin (USDC). In addition, cryptocurrencies were selected for the following species: those mined (BTC, ETH) – model Proof-of-Work, and those not mined (all others) – model Proof-of-Stake. Cryptocurrencies that those not mined, are also classified as cryptocurrencies (XRP, EOS) and stablecoins (USDT, USDC).

Here are the estimated values of the Hurst indicator, scaling index $H_{[DFA(m)]}$ for cryptocurrency pairs (table 1). The study covers the time span from November 2019 to August 2020. The calculations were made in the MATLAB based data of [39].

Table 1. Results of the Hurst coefficient and scaling index $H_{[DFA(m)]}$ calculations.

Currency pair	Hurst coefficient value (H), which have been calculated by R/S-analysis	Scaling index $H_{[DFA(m)]}$, which have been calculated by DFA
BTC/USD	0.7122	0.54955
XRP/USD	0.7089	0.58829
ETH/USD	0.70511	0.58659
EOS/USD	0.70016	0.58658
USDT/USD	0.58997	0.21414
USDC/USD	0.571	0.15673

It is obvious that the values of the Hurst coefficient (table 1) are exaggerated. Consequently, an integrated approach is needed to evaluate cryptocurrency pairs. For this purpose, the scaling index $H_{[DFA(m)]}$ is calculated, which will allow to clarify the conclusions. The value of the Hurst coefficient and the scaling index range from 0.5 to 1 for the cryptocurrency pairs BTC/USD, XRP/USD, ETH/USD, EOS/USD, indicates the persistence of the time series of the cryptocurrencies, i.e., the markets show clear trends. The XRP/USD currency pair has the highest meaning of the coefficient. In our view, the legal recognition of this cryptocurrency at the state level is an influential factor. It is one of the most popular cryptocurrencies today. It was created to accelerate transactions and currency conversion; it is also one of the cryptocurrencies used to support the technology of the future – Internet of Things.

Regarding the cryptocurrency pairs USDT/USD, USDC/USD with respect to these two indices, it is difficult to draw unambiguous conclusions. As we can see, the values of the indicators show opposite properties. However, as already noted, the Hurst coefficient is overestimated. Therefore, taking into account the obtained values of the scaling index $H_{[DFA(m)]}$, we can state the antipersistence of said rows. This behavior may be caused by the fact that the stablecoins under investigation are pegged to the value of fiat money – a dollar. Since the pandemic, which is in our view a form of crisis, has caused destabilizing developments in the world economy, fluctuations in the currency market, so do dollar-backed stablecoins.

Figure 1 shows the results of local DFA calculation for cryptocurrency pairs from August 2016 to August 2020 (BTC/USD, XRP/USD, ETH/USD, USDT/USD), from October 2018 to August 2020 (USDC/USD) and from July 2017 to August 2020 (EOS/USD). Calculations were performed in MATLAB with the following parameters: window width 100 points, step – 10 points based on data [39].

Thanks to the moving window procedure, we can observe the dynamics of the local Hurst coefficient and draw conclusions about changes in the dynamics of cryptocurrency pairs. The most stable cryptocurrency pair is XRP/USD (figure 2d). The local Hurst coefficient is sensitive to changes in the cryptocurrency market. The characteristic feature of the indicator is a significant rapid decline at the onset of the crisis. In the vicinity of point 25 (figure 1a, b, d), which corresponds to June 2017, the coefficient values are falling rapidly. There was a crisis in the cryptocurrency market – selloff. We also see the reaction to the next crisis of the cryptocurrency market – the "Great Chinese Cold" in the vicinity of Point 35 and the peak of this crisis in December 2017 (the vicinity of Point 70). In March 2020, the World Health Organization announced a pandemic, and at the end of March, partial or complete quarantine was introduced with the support of national Governments. The cryptocurrency market reacted by rapidly dropping local Hurst coefficients in the vicinity of point 120 (figure 1a, b, d). According to the figures. 1e for a pair of Stablecoin USDC/USD observes an extremely unstable dynamic of the local Hurst coefficient, whose value is predominantly less than 0.5, confirming the preliminary conclusion of the antipersistence of the series. For the USDT/USD pair (figure 1.f), the values of the coefficient are between 0.5 and 1, and at most intervals this series exhibits the persistent properties. According to the behavior of the local Hurst coefficient, we can observe the reaction of this cryptocurrency pair to the declaration of a pandemic. Despite the findings, it is worth noting that the cryptocurrency market is speculative. This is confirmed by the dynamics of the local Hurst coefficient for each cryptocurrency pair. As for the stablecoins we should admit that since USDT/USD and USDC/USD are actually provided with the dollar 1:1, there is a perceived artificial correction, and this may be a negative signal to investors, leading to additional risks.

Consider the results of the Tsallis entropy calculations (figure 2) and wavelet entropy (figure 3) for the cryptocurrency pairs studied.

Tsallis entropy was calculated in the MATLAB using a mobile window procedure and parameters: width of the window – 750 points, step – 5 points.
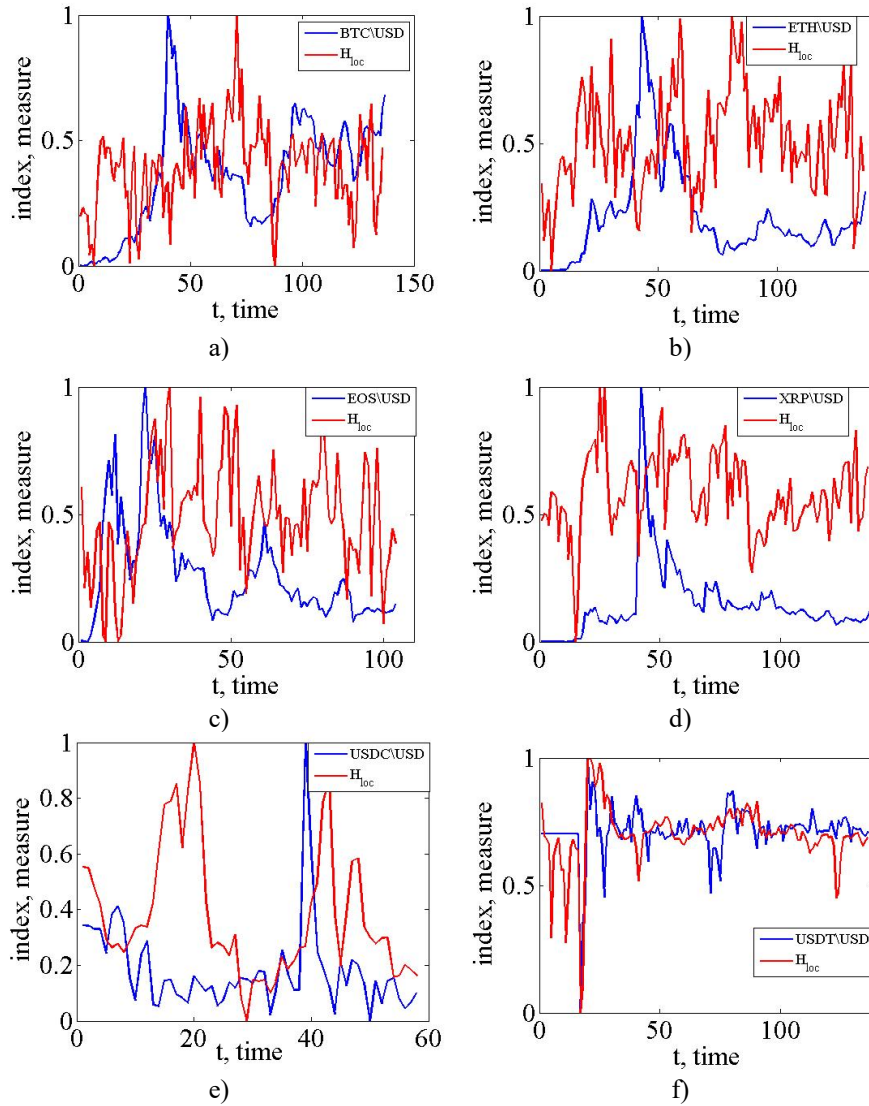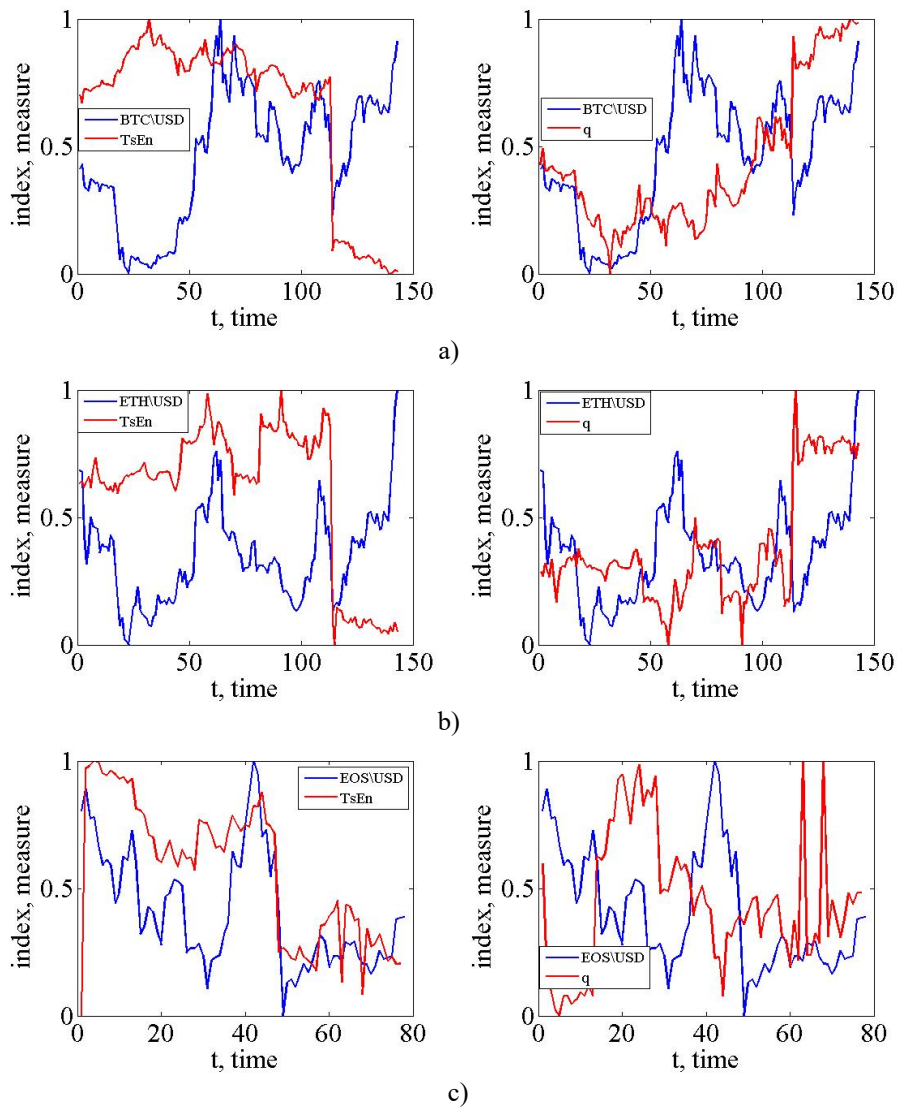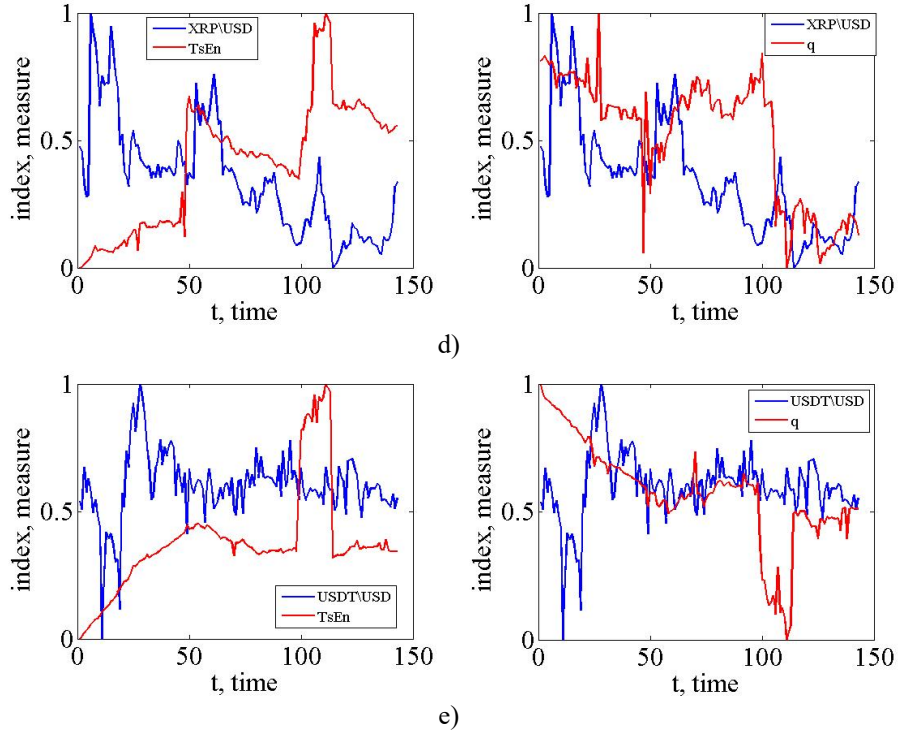
Fig. 1. Relative dynamics of the original series and the local Hurst coefficient for cryptocurrency pairs: a) BTC/USD, b) ETH/USD, c) EOS/USD, d) XRP/USD, e) USDT/USD, f) USDC/USD.

Tsallis entropy (TsEn) and q are indicators of critical phenomena in complex systems. The characteristics of these indicators are the rapid decline (TsEn) and the rapid growth (q) of values at the time of the crisis.

The following conclusions can be drawn from this analysis. For cryptocurrency pairs BTC/USD, ETH/USD in the vicinity of point 120 (March 2020), the characteristic

behavior of the Tsallis entropy index and the coefficient q is observed. These values have not returned to the pre-pandemic values. Although the value of these cryptocurrencies has reached the pre-crisis period, the entropy rate for them is affected by the crisis period. For the cryptocurrency pair XRP/USD, the rapid change in entropy occurred earlier, and in this case, it worked as a warning indicator of a crisis event.


a)

b)

c)

Fig. 2. Relative dynamics of Tsallis entropy (left) and *q* coefficient (right) and cryptocurrency pairs: a) BTC/USD, b) ETH/USD, c) EOS/USD, d) XRP/USD, e) USDT/USD.
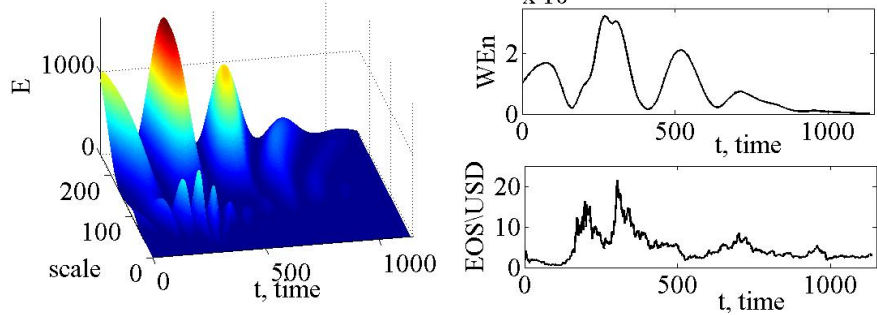
Tsallis' entropy behavior and the coefficient *q* for the cryptocurrency pair USDT / USD has also changed rapidly in the vicinity of point 110, but the return of these values to the pre-crisis values in a very short period of time confirms the artificial corrections of the stablecoins, provided by the dollar.
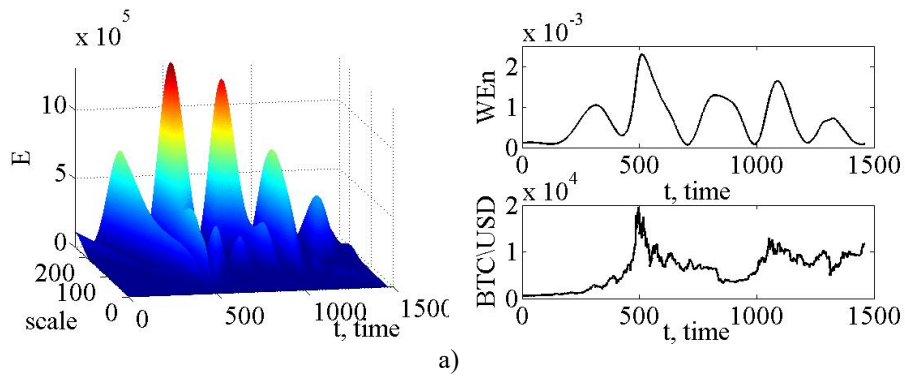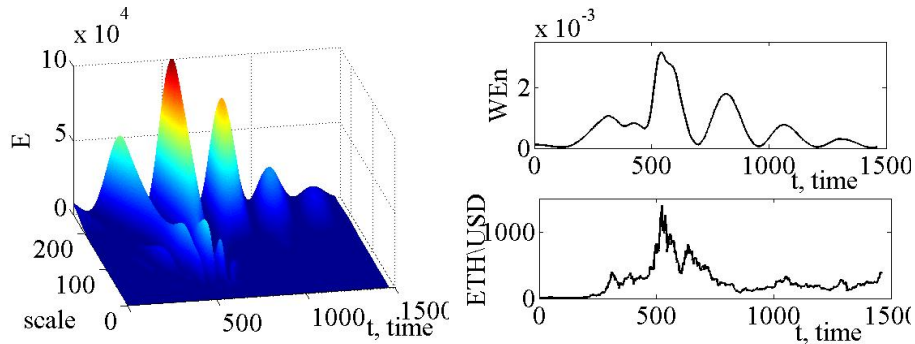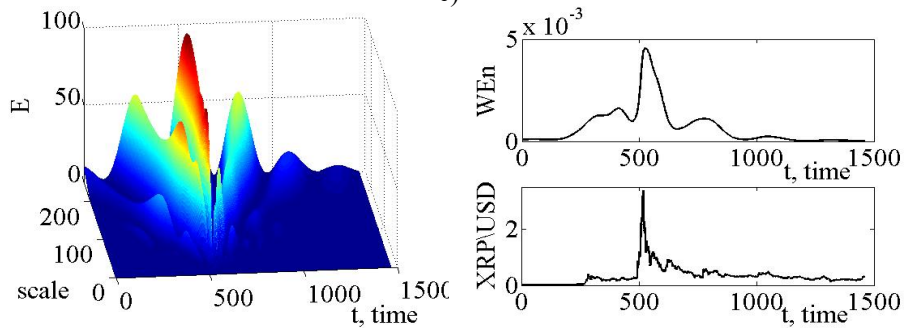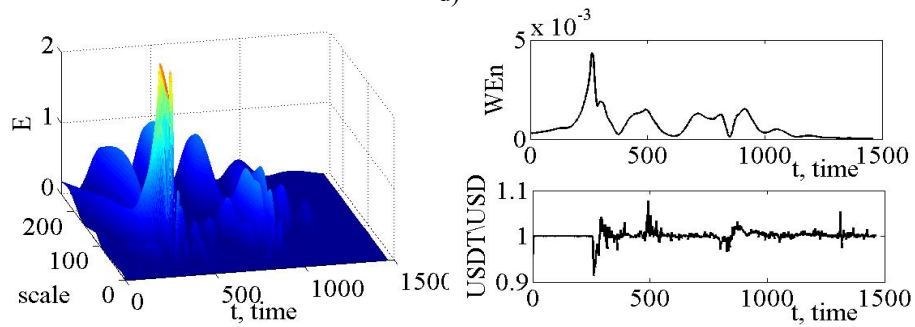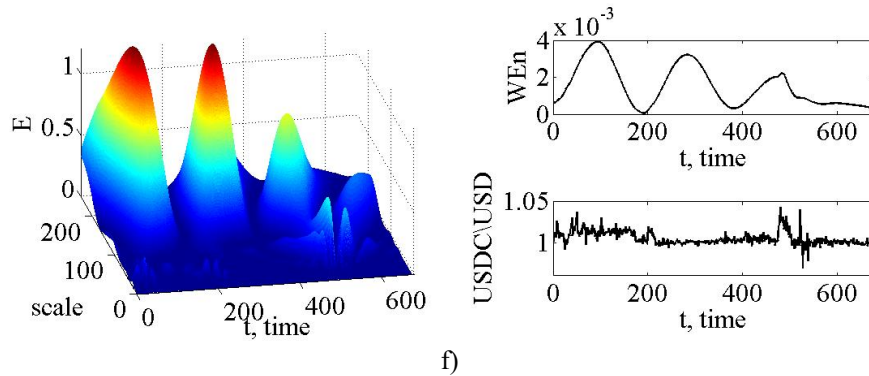
b)

c)

d)

e)

Fig. 3. Wavelet coefficient energy (left) and point wavelet entropy dynamics (right) for cryptocurrency pairs a) BTC/USD, b) ETH/USD, c) EOS/USD, d) XRP/USD, e) USDT/USD, f) USDC/USD.

Wavelet entropy is a reliable indicator of crisis phenomena in social and economic systems. As with the previous indicator, wavelet entropy has characteristics of behaviour before and after the crisis. Empirically proved the response of "law of three waves" in the pre-crisis period: on the energy surface (as on the point wave entropy graph) characteristic increasing oscillations are formed, with the maximum of each subsequent wave being greater than the previous one. The signal of a possible crisis is the exceedance of the wavelet entropy of the third wave maximum of the previous wave. In the post-crisis period fluctuations are markedly attenuated. Wavelet entropy (WEn) was calculated with the following parameters: width of the window – 750 points, step – 5 points, scale – 300 points.

Figure 3 shows that for one cryptocurrency there are no "three waves" observed, either at the beginning of the pandemic or at other periods. The results of the wavelet entropy calculation allow to refine the preliminary conclusions of the cryptocurrency market according to other indicators. In our opinion, the cryptocurrency market has been shocked by the COVID-19 outbreak and unprecedented measures. However, the ability to explore the time series relative to scale provides additional information about hidden processes in this market. In fact, disturbances for all cryptocurrency pairs are present on a small scale. This confirms once again the high activity within the market and it is likely to be speculative.

4   Conclusions

We can conclude based on the results of the cryptocurrency market research that cryptocurrencies have long-term potential and prospects in today's globalized economy being subject to crisis.

The analysis of the cryptocurrency market conducted in the article on the basis of fractal analysis leads to the conclusion that the cryptocurrency pair XRP/USD has the highest level of trends' persistence. Stable antipersistence detected for cryptocurrency pair USDC / USD. The results of calculating the local Hurst coefficient at the time of

the official declaration of a pandemic by the World Health Organization, for virtually all the cryptocurrency pairs under investigation, justify anti-persistence. However, in the future, the cryptocurrency market is witnessing a return of pre-pandemic trends.

Interdisciplinary methods of financial market research, namely Tsallis entropy and wavelet entropy, are proposed to compare and refine the results of the calculations. The results of the calculations show that the cryptocurrency market has indeed reacted to a crisis such as COVID-19. It should be noted, however, that no significant impact has been discovered and the cryptocurrency market is now considered to be more susceptible to speculation.

The proposed methodology for monitoring and modelling the cryptocurrency market is of practical importance as it will allow potential investors to form a profitable portfolio with a high level of reliability and stability over time.

In addition, the results of the research can be useful to all scientists who are working on the effects of COVID-19 on this market.

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