Complex Systems Theory and Crashes of Cryptocurrency Market

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Abstract. This article demonstrates the possibility of constructing indicators of critical and crash phenomena in the volatile market of cryptocurrency. For this purpose, the methods of the theory of complex systems have been used. The possibility of constructing dynamic measures of complexity as recurrent, entropy, network, quantum behaving in a proper way during actual pre-crash periods has been shown. This fact is used to build predictors of crashes and critical events phenomena on the examples of all the patterns recorded in the time series of the key cryptocurrency Bitcoin, the effectiveness of the proposed indicators-precursors of these falls has been identified. From positions, attained by modern theoretical physics the concept of economic Planck's constant has been proposed. The theory on the economic dynamic time series related to the cryptocurrencies market has been approved. Then, combining the empirical cross-correlation matrix with the random matrix theory, we mainly examine the statistical properties of cross-correlation coefficient, the evolution of the distribution of eigenvalues and corresponding eigenvectors of the global cryptocurrency market using the daily returns of 24 cryptocurrencies price time series all over the world from 2013 to 2018. The result has indicated that the largest eigenvalue reflects a collective effect of the whole market, and is very sensitive to the crash phenomena. It has been shown that both the introduced economic mass and the largest eigenvalue of the matrix of correlations can act like quantum indicator-predictors of falls in the market of cryptocurrencies.

Keywords: Cryptocurrency · Bitcoin · Complex system · Measures of complexity · Crash · Critical events · Recurrence plot · Recurrence quantification analysis · Permutation entropy · Complex networks · Quantum econophysics · Heisenberg uncertainty principle · Random matrix theory · Indicator-precursor

1 Introduction

The instability of global financial systems with regard to normal and natural disturbances of the modern market and the presence of poorly foreseeable financial crashes indicate, first of all, the crisis of the methodology of modeling, forecasting and interpretation of modern socio-economic realities. The doctrine of the unity of the scientific method states that for the study of events in socio-economic systems, the same methods and criteria as those used in the study of natural phenomena are applicable. Significant

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success has been achieved within the framework of interdisciplinary approaches and the theory of self-organization - synergetics. The modern paradigm of synergetics is a complex paradigm associated with the possibility of direct numerical simulation of the processes of complex systems evolution, most of which have a network structure, or one way or another can be reduced to the network. The theory of complex networks studies the characteristics of networks, taking into account not only their topology, but also statistical properties, the distribution of weights of individual nodes and edges, the effects of dissemination of information, robustness, etc. [1–4].

Complex systems are systems consisting of a plurality of interacting agents possessing the ability to generate new qualities at the level of macroscopic collective behavior, the manifestation of which is the spontaneous formation of noticeable temporal, spatial, or functional structures. As simulation processes, the application of quantitative methods involves measurement procedures, where importance is given to complexity measures. I. Prigogine notes that the concepts of simplicity and complexity are relativized in the pluralism of the descriptions of languages, which also determines the plurality of approaches to the quantitative description of the complexity phenomenon [5]. Therefore, we will continue to study Prigogine's manifestations of the system complexity, using the current methods of quantitative analysis to determine the appropriate measures of complexity.

The key idea here is the hypothesis that the complexity of the system before the crashes and the actual periods of crashes must change. This should signal the corresponding degree of complexity if they are able to quantify certain patterns of a complex system. Significant advantage of the introduced measures is their dynamism, that is, the ability to monitor the change in time of the chosen measure and compare it with the corresponding dynamics of the output time series. This allowed us to compare the critical changes in the dynamics of the system, which is described by the time series, with the characteristic changes of concrete measures of complexity. It turned out that quantitative measures of complexity respond to critical changes in the dynamics of a complex system, which allows them to be used in the diagnostic process and prediction of future changes.

Cryptocurrency market is a complex, self-organized system, which in most cases can be considered either as a complex network of market agents, or as an integrated output signal of such a network - a time series, for example, prices of individual cryptocurrency. The research on cryptocurrency price fluctuations being carried out internationally is made more complex by the interplay due to many factors - including market supply and demand, the US dollar exchange rate, stock market state, the influence of crime and the shadow market, and fiat money regulator pressure - that introduces a high level of noise into the cryptocurrency data. Moreover, in the cryptocurrency market, to some extent, the blockchain technology is tested in general. Thus the cryptocurrency prices exhibit such complex volatility characteristics as nonlinearity and uncertainty, which are difficult to forecast and any results obtained are uncertain. Therefore, cryptocurrency price prediction remains a huge challenge.

Among these prediction models, one of the most important models is econometric model such as for example autoregressive integrated moving average (ARIMA) that exploit time series stationarity. Because of the presence of local explosive trends, depicted as bubbles, the Bitcoin exchange rate cannot be modelled by any traditional

ARIMA models (see e.g. [6]). Dassios and Li [7] introduce a new diffusion process to describe Bitcoin prices within an economic bubble cycle. In spite of rather a complicated model, forecast bubble results for December, 2017 are disappointing. Tarnopolski [8] has completed the modelling of Bitcoin price using Monte Carlo method based on model of geometric fractional Brownian motion. The Bitcoin price predicted for the beginning of 2018 turned out to be far from reality.

In addition to the classic econometric approaches, artificial intelligence methods (also known as machine and/or deep learning methods) have been used to uncover the inner complexity of cryptocurrency prices. Separate attempts of using both simple artificial neural networks Elmann [9] and method of Bayesian regression [10] are known, as well as more complex methods based on XGboost [11] or on the long short – term memory (LSTM) algorithm for recurrent neural networks [12, 13]. Nowadays combined classical econometric methods as well as methods of machine learning [14, 15] and those which take into consideration the spirit of social networks regarding the state and tendency of cryptocurrency dynamics [16] are becoming more popular.

Thus, lack of reliable models of prediction of time series for the time being will update the construction of at least indicators which warn against possible critical phenomena or trade changes etc. This work is dedicated to the construction of such indicators – precursors based on the theory of complexity.

In this paper, we consider some of the informative measures of complexity and adapt them in order to study the critical and crash phenomena of cryptomarket.

The paper is structured as follows. Section 2 describes previous studies in these fields. Section 3 presents classification of crashes and critical events on the Bitcoin market during the entire period (16.07.2010 – 08.12.2018). Section 4 describes the technique of quantitative recurrent analysis and recurrent measures of complexity as indicators of crashes. The indicator-precursor of crashes based on the calculation of Permutation Entropy is described in Sect. 5. Network measures of complexity and their effectiveness as indicators of cryptomarket crashes are presented in Sect. 6. In Sect. 7, new quantum indicators of critical and crash phenomena are introduced using the Heisenberg uncertainty principle and the Random Matrix Theory. And finally, we discuss our results in Sect. 8.

2 Analysis of Previous Studies

Throughout the existence of Bitcoin, its complexity became much larger. Crashes and critical events that took place on this market as well as the reasons that led to them, did not go unheeded. We determined that there are a lot of articles and papers on that topic which we will demonstrate.

Donier and Bouchaud [17] found that the market microstructure on Bitcoin exchanges can be used to anticipate illiquidity issues in the market, which lead to abrupt crashes. They investigate Bitcoin liquidity based on order book data and, out of this, accurately predict the size of price crashes.

Bariviera [18] demonstrates the dynamics of the intraday price of 12 cryptocurrencies. By using the complexity-entropy causality plane, authors discriminate three different dynamics in the data set. Another paper [19] compares the time-varying weak-form efficiency of Bitcoin prices in terms of US dollars (BTC/USD) and euro (BTC/EUR) at a high-frequency level by using Permutation Entropy. Their research shows that BTC/USD and BTC/EUR markets have been demonstrating more information at the intraday level since the beginning of 2016, and BTC/USD market has been slightly more efficient than BTC/EUR during the same period. And moreover, their research shows that with the higher frequency we have less price efficiency.

Some papers like this one [20] demonstrate how recurrence plots and measures of recurrence quantification analysis can be used to study significant changes in complex dynamical systems due to a change in control parameters, chaos-order as well as chaos-chaos transitions. Santos et al. [21] discuss how to model activity in online collaboration websites, such as Stock Exchange Question and Answering portals because the success of these websites critically depends on the content contributed by its users. In this paper, they represent user activity as time series and perform an initial analysis of these time series to obtain a better understanding of the underlying mechanisms that govern their creation. For this purpose nonlinear modeling via recurrence plots was used, which gives more granular study and deeper understanding of nonlinear dynamics of governing activity of time series and explaining the activity in online collaboration websites.

Taking to the account studies on network analysis we can notice different papers on this topic [22–24]. Di Francesco Maesa et al. [22] have performed on the users' graph inferred from the Bitcoin blockchain, dumped in December 2015, so after the occurrence of the exponential explosion in the number of transactions. Researchers first present the analysis assessing classical graph properties like densification, distance analysis, degree distribution, clustering coefficient, and several centrality measures. Then, they analyze properties strictly tied to the nature of Bitcoin, like rich-get-richer property, which measures the concentration of richness in the network. Bovet et al. [23] analyzed the evolution of the network of Bitcoin transactions among users and built network-based indicators of Bitcoin bubbles.

In this article [24], authors consider the history of Bitcoin and transactions in it. Using this dataset, they reconstruct the transaction network among users and analyze changes in the structure of the subgraph induced by the most active users. Their approach is based on the unsupervised identification of important features of the time variation of the network. Applying the widely used method of principal component analysis to the matrix constructed from snapshots of the network at different times, they show how changes in the network accompany significant changes in the price of Bitcoin.

Separately, it is necessary to highlight the work of Sornette [25, 26], who built a precursor of crashes based on the generation of so-called log-periodic oscillations by the pre-crashing market. However, the actual collapse point is still badly predicted.

Thus, construction of indicators – precursors of critical and crash phenomena in the cryptocurrency market remains relevant.

3 Data

Bitcoin, despite its uncertain future, continues to attract investors, crypto-enthusiasts, and researchers. Being historically proven, popular and widely used cryptocurrency for the whole existence of cryptocurrencies in general, Bitcoin began to produce a lot of news and speculation, which began to determine its future life. Similar discussions began to lead to different kinds of crashes, critical events, and bubbles, which professional investors and inexperienced users began to fear. Thus, we advanced into action and set the tasks:

(1) Classification of such bubbles, critical events and crashes.
(2) Construction of such indicators that will predict crashes, critical events in order to give investors and ordinary users the opportunity to work in this market.

At the moment, there are various research works on what crises and crashes are and how to classify such interruptions in the market of cryptocurrencies. Taking into account the experience of previous researchers [26–30], we have created our classification of such leaps and falls, relying on Bitcoin time series during the entire period (16.07.2010 – 08.12.2018) of verifiable fixed daily values of the Bitcoin price (BTC) (https://finance.yahoo.com/cryptocurrencies).

For our classification, crashes are short, time-localized drops, with strong losing of price per each day, which are formed as a result of the bubble. Critical events are those falls that could go on for a long period of time, and at the same time, they were not caused by a bubble. The bubble is an increasing in the price of the cryptocurrencies that could be caused by certain speculative moments. Therefore, according to our classification of the event with number (1, 3–6, 9–11, 14, 15) are the crashes that are preceded by the bubbles, all the rest - critical events. More detailed information about crises, crashes and their classification in accordance with these definitions is given in the Table 1.

Accordingly, during this period in the Bitcoin market, many crashes and critical events shook it. Thus, considering them, we emphasize 15 periods on Bitcoin time series, whose falling we predict by our indicators, relying on normalized returns and volatility, where normalized returns are calculated as

$$g(t) = \ln X(t + \Delta t) - \ln X(t) \cong [X(t + \Delta t) - X(t)]/X(t), \tag{1}$$

and volatility as

$$V_T(t) = \frac{1}{n}\sum_{t'=t}^{t+n-1} |g(t')|$$

Besides, considering that $g(t)$ should be more than the $\pm 3\sigma$, where sigma is a mean square deviation.

Calculations were carried out within the framework of the algorithm of a moving window. For this purpose, the part of the time series (window), for which there were calculated measures of complexity, was selected, then the window was displaced along the time series in a one-day increment and the procedure repeated until all the studied series had exhausted. Further, comparing the dynamics of the actual time series and the corresponding measures of complexity, we can judge the characteristic changes in the dynamics of the behavior of complexity with changes in the cryptocurrency. If this or that measure of complexity behaves in a definite way for all periods of crashes, for example, decreases or increases during the pre-crashes period, then it can serve as an indicator or precursor of such a crashes phenomenon.

Table 1. BTC Historical Corrections. List of Bitcoin major corrections $\geq$ 20% since June 2011

№	Name	Days in correction	Bitcoin high price, $	Bitcoin low price, $	Decline, %	Decline, $
1	07.06.2011–10.06.2011	4	29.60	14.65	50	15.05
2	15.01.2012–16.02.2012	33	7.00	4.27	39	2.73
3	15.08.2012–18.08.2012	4	13.50	8.00	40	5.50
4	08.04.2013–15.04.2013	8	230.00	68.36	70	161.64
5	04.12.2013–18.12.2013	15	1237.66	540.97	56	696.69
6	05.02.2014–25.02.2014	21	904.52	135.77	85	768.75
7	12.11.2014–14.01.2015	64	432.02	164.91	62	267.11
8	11.07.2015–23.08.2015	44	310.44	211.42	32	99.02
9	09.11.2015–11.11.2015	3	380.22	304.70	20	75.52
10	18.06.2016–21.06.2016	4	761.03	590.55	22	170.48
11	04.01.2017–11.01.2017	8	1135.41	785.42	30	349.99
12	03.03.2017–24.03.2017	22	1283.30	939.70	27	343.60
13	10.06.2017–15.07.2017	36	2973.44	1914.08	36	1059.36
14	16.12.2017–22.12.2017	7	19345.49	13664.96	29	5680.53
15	13.11.2018–26.11.2018	14	6339.17	3784.59	40	2554.58

Calculations of measures of complexity were carried out both for the entire time series, and for a fragment of the time series localizing the crash. In the latter case, fragments of time series of the same length with fixed points of the onset of crashes or critical events were selected and the results of calculations of complexity measures were compared to verify the universality of the indicators.

In the Fig. 1 output Bitcoin time series, normalized returns $g(t)$, and volatility $V_T(t)$ calculated for the window size 100 are presented.

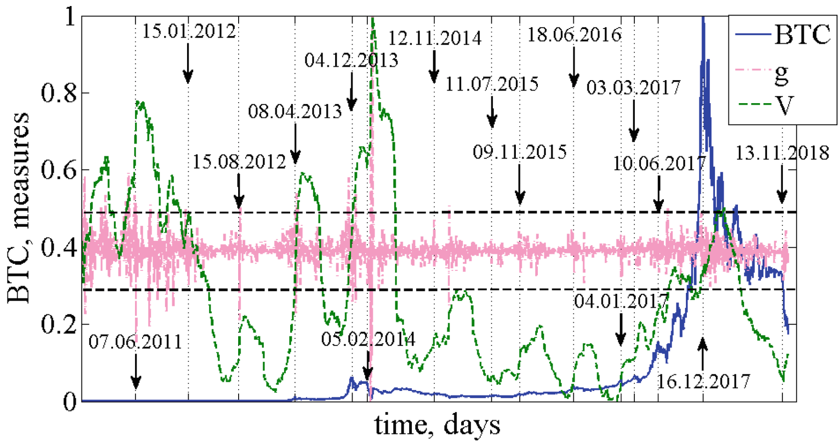

Fig. 1. The standardized dynamics, returns $g(t)$, and volatility $V_T(t)$ of BTC/USD daily values. Horizontal dotted lines indicate the $\pm 3\sigma$ borders. The arrows indicate the beginning of one of the crashes or the critical events.

From Fig. 1 we can see that during periods of crashes and critical events normalized profitability g increases considerably in some cases beyond the limits $\pm 3\sigma$. This indicates about deviation from the normal law of distribution, the presence of the "heavy tails" in the distribution g, characteristic of abnormal phenomena in the market. At the same time volatility also grows. These characteristics serve as indicators of critical and collapse phenomena as they react only at the moment of the above mentioned phenomena and don't give an opportunity to identify the corresponding abnormal phenomena in advance. In contrast, the indicators described below respond to critical and collapse phenomena in advance. It enables them to be used as indicators – precursors of such phenomena and in order to prevent them.

4 Recurrence Quantification Analysis

Recurrence plots (RPs) have been introduced to study dynamics and recurrence states of complex systems. A phase space trajectory can be transformed from a time series $U_i = \{u_1, \ldots, u_n\}$ ($t = i\Delta t$, where Δt is the sampling time) into time-delay structures

$$X_i = (U_i, U_{i+1}, \ldots, U_{i+(m-1)\tau}),$$

where m stands for the embedding dimension and τ for the entire time delay. Both of them can be calculated from the original data using false nearest neighbors and mutual information [31].

A RP is a plot representation of those states which are recurrent. The recurrence matrix and the states are considered to be recurrent if the distance between them within the ε- radius. In this case, the recurrence plot is defined as:

$$R_{ij} = \Theta(\varepsilon - \|x_i - x_j\|), i, j = 1, \ldots, N,$$

and $\|\|$ is a norm (representing the spatial distance between the states at times i and j), ε is a predefined recurrence threshold, and Θ is the Heaviside function (ensuring a binary $\mathbf{R}$).

Usually, recurrent plot has a square form and $R \equiv 1$ is included to the representation, but for calculations, it might be useful to remove it [31]. For qualitative description of the system, the graphic representation of the system suits perfectly. For the quantitative description of the system, the small-scale clusters such as diagonal and vertical lines can be used. The histograms of the lengths of these lines are the base of the recurrence quantification analysis developed by Webber and Zbilut and later by Marwan et al. [32–34].

Recurrence rate (RR) is the part of recurrence points in the plot that can be interpreted as the probability that any state of the system will recur. It is the simplest measure, which computes by taking the number of the nearest points forming short, spanning row and columns of the recurrent plot, summarize them and divide by the number of possible points N^2:

$$RR = \frac{1}{N^2} \sum_{1,j=1}^{N} R_{i,j}.$$

The set of recurrence points on the recurrence plots that form line segments of minimal length μ parallel to the matrix diagonal is the measure of determinism (DET):

$$DET^{(u)} = \frac{\sum_{l=\mu}^{N} l \cdot D(l)}{\sum_{i,j}^{N} R_{i,j}} = \frac{\sum_{l=\mu}^{N} l \cdot D(l)}{\sum_{l=1}^{N} l \cdot D(l)},$$

where

$$D(l) = \sum_{i,j}^{N} \left\{ (1 - R_{i-1,j-1}) \cdot (1 - R_{i+1,j+1}) \cdot \prod_{k=0}^{l-1} R_{i+k,j+k} \right\}$$

is the histogram of the lengths of the diagonal lines. The understanding of 'determinism' in this sense is of heuristic nature.

The results of calculations of window dynamics of the considered recurrent measures are presented in Fig. 2. Measures RR and DET are calculated for local time series of length in 250 days, with a window of 50 days and a step of 1 day. In this case, the beginning of a crash or critical event is at point 100.

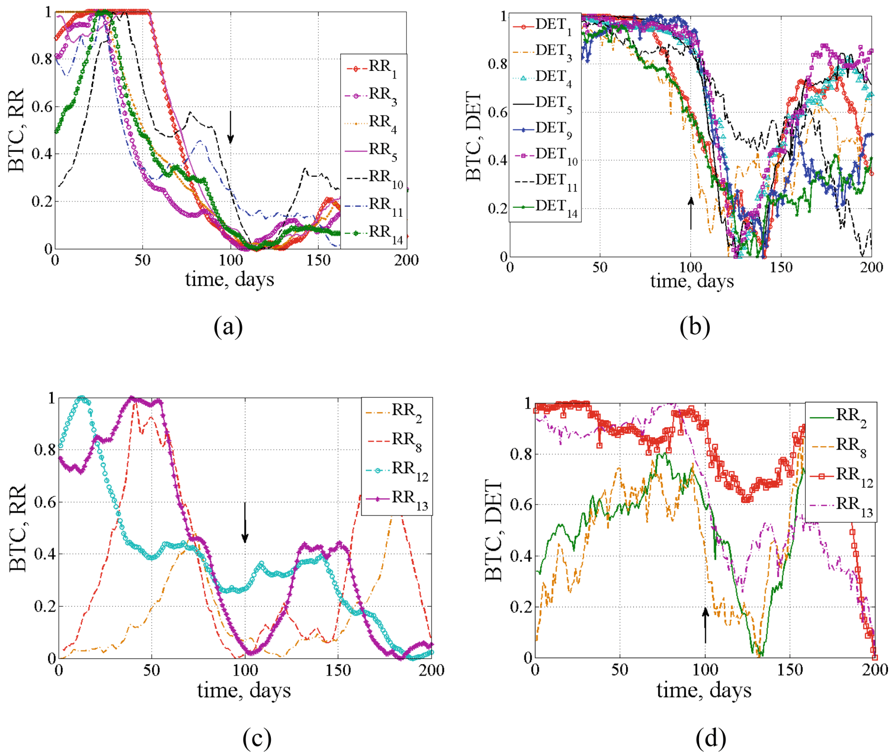

Fig. 2. Dynamics of RR and DET for crashes (a), (b) and for crisis events (c), (d).

It is evident that the two recurrent measures during abnormal periods decrease long before the actual anomaly. The complex system becomes less recurrent and less deterministic which is logical in the periods approaching critical phenomena. And, consequently, RR and DET can be used as precursors of critical and crash phenomena.

5 Permutation Entropy

The Permutation Entropy (PEn) is conceptually simple, computationally a very fast approach which gives an opportunity to quantify complexity in measured time series. Exactly, the measure of entropy is the measure of “randomness”. It quantifies the degree of chaos or uncertainty in a system. The uncertainty is associated with a physical process described by the probability distribution

$$P = \{p_i, i = 1, \ldots, M\}$$

is related to the Shannon entropy,

$$S[P] = -\sum_{i=1}^{M} p_i \ln p_i.$$

PEn is based on usual entropy but it is used for the time series analysis of permutation patterns. Bant and Pompe proposed to construct probability distributions using ordinal patterns from recorded time series [35]. These ordinal patterns are constructed based on the relative amplitude of time series values. In this way, if compared with other measures of complexity, this symbolic approach has many advantages over the others as robustness to noise and invariance to nonlinear monotonous transformations [25]. Similar advantages make it particularly attractive for use on experimental data.

If we want to get the ordinal pattern P on which entropy is related, at first we need to define the order of permutations D and ordinal pattern time delay τ. There are $D!$ possible permutations for a vector of length D, so in order to obtain reliable statistics, the length of the time series N should be much larger than $D!$ [35].

The ordinal time delay τ that is responsible for the time scale over which the complexity is quantified can be set by changing. If we change it, we will determine the time separation between values used to construct the vector from which the ordinal pattern is determined. Its value corresponds to a multiple of the signal sampling period. For a given time series $\{u_t, t = 1, \ldots, N\}$, ordinal pattern length D, and ordinal pattern time delay τ, we consider the vector:

$$X_s \to \left(u_{s-(D-1)\tau}, u_{s-(D-2)\tau}, \ldots, u_{s-\tau}, u_s\right).$$

Relating to the time S equal numbers take their unique symbol according to their position in the time series:

$$\pi = (r_0, r_1, \ldots, r_{D-1}) \text{ defined by } u_{s-r_0\tau} \geq u_{s-r_1\pi} \geq \ldots \geq u_{s-r_{D-2}\tau} \geq u_{s-r_{D-1}\tau}.$$

Then, with all $D!$ possible permutations π_i, the ordinal pattern probability distribution $P = \{p(\pi), i = 1, \ldots, D!\}$ required for entropy calculations is constructed. To take more convenient values, we normalize Permutation Entropy S associating it with the probability distribution P:

$$H_s[P] = \frac{S[P]}{S_{\max}} = \frac{-\sum_{i=1}^{D!} p(\pi_i) \ln p(\pi_i)}{\ln D!}.$$

The values of this normalized permutation have the range $0 \leq H_S \leq 1$ where predictable time series shows a value of zero and absolutely randomize process with a uniform probability distribution presented by a value equal to one. It is important to realize that the Permutation Entropy is a statistical measure and is not able to distinguish whether the observed complexity (irregularity) arises from stochastic or deterministic chaotic processes. It is also important that the PEn provides ways to characterize complexity on different time scales, given by the time delay.

Therefore H_S compared measure of complexity with actual time series under study gives values whose meaning leads to understanding whether we have regular time series or not. Besides, it is understandable that parameter D will not work if we have small values, such as 1 or 2, it is clear that if D is too small, such as 1 or 2, the procedure will not work, because there are only very few distinct states. Enough large parameter D is fine as long as the length of time series can be made proportional to $D!$. The authors of this method recommend using $D = 3, \ldots, 7$. We discovered that $D = 5, 6$, or 7 indicate better result.

Figure 3 shows the PEn calculation results for the entire Bitcoin time series (a) (the window is 100 days, the window offset is 1 day) and also for the local time series of crashes (b) and critical events (c) (the length of the time series is 250 days, the window is 50 days, window offset is 1 day).

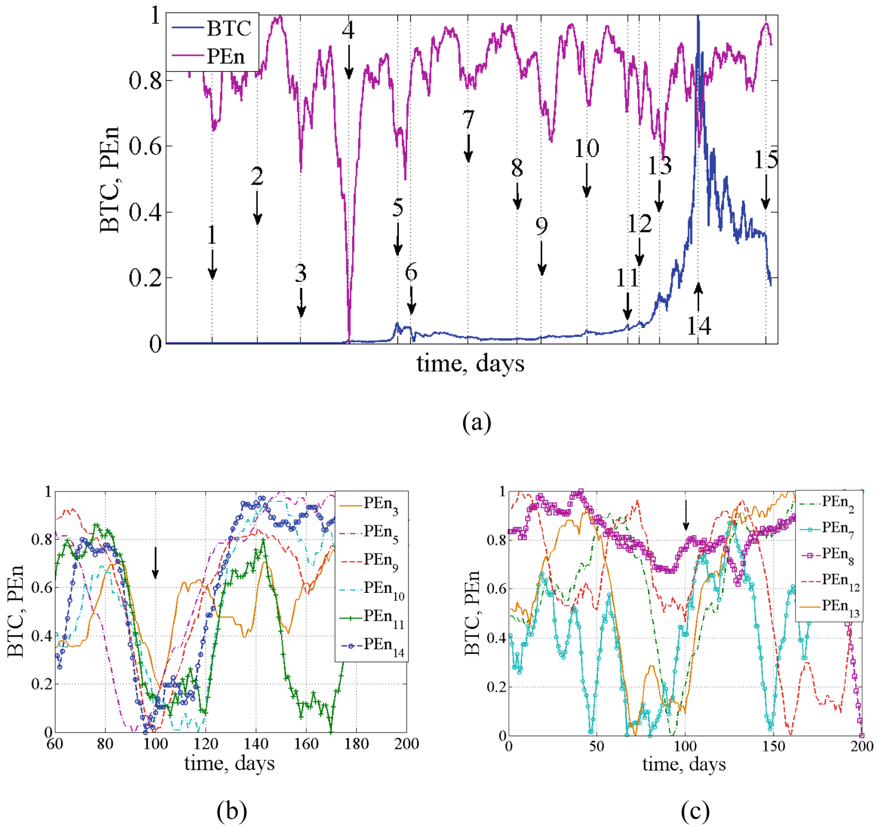

Fig. 3. The dynamics of Permutation Entropy for the entire time series of Bitcoin (a) and for local crashes (b) and events (c). Figure (a) shows the numbers of crashes and critical events in accordance with the Table.

Figure 3 shows that Permutation Entropy decreases for both the entire time series (3a) and for selected crash (3b) or critical (3c) fragments, signaling the approaching of a special state. Comparison of Fig. 3b and c shows that for crash states this behavior is more universal than for critical ones. As in the case of recurrent measures, PEn is an indicator of the precursor of critical and crash phenomena.

6 Complex Network Indicators

The most commonly used methods for converting time sequences to the corresponding networks are recurrent [36], visibility graph [37] and correlation [38]. In the first case, the recurrence diagram is transformed into an adjacency matrix, on which the spectral and topological characteristics of the graph are calculated. The algorithm of the visibility graph is realized as follows. Take a time series $Y(t) = [y_1, y_{2,}, \ldots, y_n]$ of length N. Each point in the time series data can be considered as a vertex in an associated network, and the edge connects two vertices if two corresponding data points can "see" each other from the corresponding point in the time series. Formally, two values of the series y_a (at the time of time t_a) and y_b (at the time of time t_b) are connected, if for any other value (y_c, t_c), which is placed between them (i.e., $t_a < t_c < t_b$), the condition is satisfied:

$$y_c < y_a + (y_b - y_a)\frac{t_c - t_a}{t_b - t_a}$$

To construct and analyze the properties of a correlation graph, we must form a correlation matrix from the set of cryptocurrencies (as is done in Sect. 7), and from it we must pass to the matrix of adjacency. To do this, you must enter a value which, for the correlation field, will be the distance between the correlated assets. Such a distance may be dependent on the correlation coefficients c_{ij} of the value $x(i,j) = \sqrt{2(1 - c_{ij})}$. So, if the correlation coefficient between the two assets is significant, the distance between them is small, and, starting from some critical value, assets can be considered bound on the graph.

For constructed graph methods described above, one can calculate spectral and topological properties. We will show that some of them serve as a measure of the complexity of the system, and the dynamics of their changes allows us to build predictors of crashes or critical events in the financial markets.

Spectral theory of graphs is based on algebraic invariants of a graph - its spectra. The spectrum of graph G is the set of eigenvalues $S_p(G)$ of a matrix corresponding to a given graph. For adjacency matrix A of a graph, there exists an characteristic polynomial $|\lambda I - A|$, which is called the characteristic polynomial of a graph $P_G(\lambda)$. The eigenvalues of the matrix A (the zeros of the polynomial $|\lambda I - A|$) and the spectrum of the matrix A (the set of eigenvalues) are called respectively their eigenvalues λ and the spectrum $S_p(G)$ of graph G. The eigenvalues of the matrix A satisfy the equality $A\bar{x} = \lambda\bar{x}$ ($\bar{x}$ - non-zero vector). Vectors $\bar{x}$ satisfying this equality are called eigenvectors of the matrix A (or the graph G) corresponding to their eigenvalues.

From a multiplicity of spectral and topological measures we will choose only two - the maximum eigenvalue $\lambda_{\max}$ of the adjacency matrix and Average path Length (ApLen). For a connected network of N nodes, the ApLen is equal

$$< l > = \frac{2}{n(N-1)}\sum_{i>j} l_{ij}, \quad (2)$$

where l_{ij} - the length of the shortest path between the nodes i and j.

Figure 4 demonstrates the asymmetric response of the spectral and topological measures of network complexity. For the complete series, the calculation parameters are as follows: window width 100, step is 1 day. For local measures, the length of the fragment is 150, the width of the window is 50 and the step is 1 day.

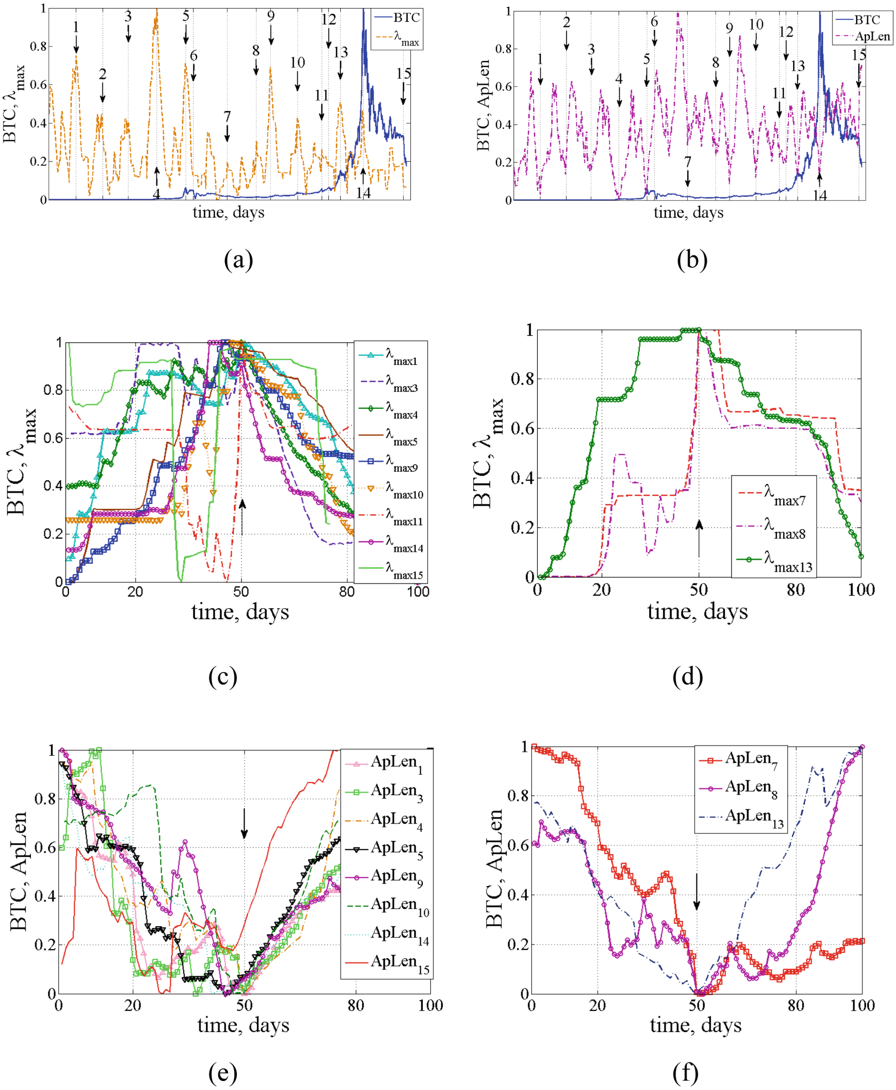

Fig. 4. Visibility graph dynamics of network measures λ_{max} (a), Average path length (b) for all Bitcoin time series. Dynamics of network measures for local crashes (c, e) and crisis events (d, f).

Figure 4 shows the possibility of using both spectral and topological measures of complexity as indicators-precursors of special states in the market of cryptocurrencies. Indeed, the maximum actual value of the adjacency matrix of the visibility graph both for Bitcoin as a whole and for isolated segments of time series containing a crash and critical phenomenon, takes maximum value. It corresponds to the maximum complexity of the system. An especial state of the system leads to a decrease in complexity, and, accordingly, to a decrease in value $\lambda_{\max}$. Average length of the path on the graph (ApLen) is, on the contrary, minimal for complex systems and increases with the randomization of the system. Such increase during pre-crash and pre-critical states as well as reduce $\lambda_{\max}$ are indicators-precursors of the above mentioned states. You can choose other spectral and topological measures from the calculated ones, e.g. the maximum degree of the vertex and the diameter of the graph, algebraic connectivity and centrality, etc. Network measures of complexity, thus, are the most universal and informative and have obvious advantages in the selection of indicators of special states.

7 Quantum Econophysics Indicators

The attempts to create an adequate model of socio-economic critical events, which, as it has been historically proven, are almost permanent, were, are and will always be made. Actually, it is a super task impossible to solve. However, the potentially useful solutions, local in time or other socio-economic logistic coordinates, are possible. In fact, they have to be the object of interest for a real and effective economic science.

Econophysics is a young interdisciplinary scientific field, which developed and acquired its name at the end of the last century [39]. Quantum econophysics, a direction distinguished by the use of mathematical apparatus of quantum mechanics as well as its fundamental conceptual ideas and relativistic aspects, developed within its boundaries just a couple of years later, in the first decade of the 21st century [40–43].

According to classical physics, immediate values of physical quantities, which describe the system status, not only exist, but can also be exactly measured. Although non-relativistic quantum mechanics doesn't reject the existence of immediate values of classic physical quantities, it postulates that not all of them can be measured simultaneously (Heisenberg uncertainty ratio). Relativistic quantum mechanics denies the existence of immediate values for all kinds of physical quantities, and, therefore, the notion of system status seizes to be algoristic.

In this section, we will demonstrate the possibilities of quantum econophysics on the example of the application of the Heisenberg uncertainty principle and the Random Matrices Theory to the actual and debatable now market of cryptocurrencies.

7.1 Heisenberg Uncertainty Principle and Economic Analogues of Basic Physical Quantities

In our paper [43] we have suggested a new paradigm of complex systems modeling based on the ideas of quantum as well as relativistic mechanics. It has been revealed that the use of quantum-mechanical analogies (such as the uncertainty principle, notion of the operator, and quantum measurement interpretation) can be applied to describing socio-economic processes. Methodological and philosophical analysis of fundamental physical notions and constants, such as time, space and spatial coordinates, mass, Planck's constant, light velocity from the point of view of modern theoretical physics provides an opportunity to search of adequate and useful analogues in socio-economic phenomena and processes.

The Heisenberg uncertainty principle is one of the cornerstones of quantum mechanics. The modern version of the uncertainty principle, deals not with the precision of a measurement and the disturbance it introduces, but with the intrinsic uncertainty any quantum state must possess, regardless of what measurement is performed [44, 45]. Recently, the study of uncertainty relations in general has been a topic of growing interest, specifically in the setting of quantum information and quantum cryptography, where it is fundamental to the security of certain protocols [46, 47].

To demonstrate it, let us use the known Heisenberg's uncertainty ratio which is the fundamental consequence of non-relativistic quantum mechanics axioms and appears to be (e.g. [48]):

$$\Delta x \cdot \Delta v \geq \frac{\hbar}{2m_0}, \tag{3}$$

where Δx and Δv are mean square deviations of x coordinate and velocity v corresponding to the particle with (rest) mass m_0, $\hbar$ - Planck's constant. Considering values Δx и Δv to be measurable when their product reaches its minimum, we derive (from (1)):

$$m_0 = \frac{\hbar}{2 \cdot \Delta x \cdot \Delta v}, \tag{4}$$

i.e. mass of the particle is conveyed via uncertainties of its coordinate and velocity – time derivative of the same coordinate.

Economic measurements are fundamentally relative, are local in time, space and other socio-economic coordinates, and can be carried out via consequent and/or parallel comparisons "here and now", "here and there", "yesterday and today", "a year ago and now" etc.

Due to these reasons constant monitoring, analysis, and time series prediction (time series imply data derived from the dynamics of stock indices, exchange rates, cryptocurrencies prices, spot prices and other socio-economic indicators) becomes relevant for evaluation of the state, tendencies, and perspectives of global, regional, and national economies.

Suppose there is a set of K time series, each of N samples, that correspond to the single distance T, with an equal minimal time step $\Delta t_{\min}$:

$$X_i(t_n),\ t_n = \Delta t_{\min} n; n = 0, 1, 2, \ldots N-1; i = 1, 2, \ldots K. \tag{5}$$

To bring all series to the unified and non-dimensional representation, accurate to the additive constant, we normalize them, having taken a natural logarithm of each term of the series. Then consider that every new series $x_i(t_n)$ is a one-dimensional trajectory of a certain fictitious or abstract particle numbered i, while its coordinate is registered after every time span $\Delta t_{\min}$, and evaluate mean square deviations of its coordinate and speed in some time window $\Delta T = \Delta N \cdot \Delta t_{\min} = \Delta N, 1 << \Delta N << N$. The «immediate» speed of i particle at the moment t_n is defined by the ratio:

$$v_i(t_n) = \frac{x_i(t_{n+1}) - x_i(t_n)}{\Delta t_{\min}} = \frac{1}{\Delta t_{\min}} ln \frac{X_i(t_{n+1})}{X_i(t_n)} \tag{6}$$

with variance D_{v_i} and mean square deviation Δv_i.

Keeping an analogy with (1) after some transformations we can write an uncertainty ratio for this trajectory [49]:

$$\frac{1}{\Delta t_{\min}} \left(< \ln^2 \frac{X_i(t_{n+1})}{X_i(t_n)} >_{n,\Delta N} - \left(< \ln \frac{X_i(t_{n+1})}{X_i(t_n)} >_{n,\Delta N} \right)^2 \right) \sim \frac{h}{m_i}, \tag{7}$$

where m_i - economic "mass" of an i series, h - value which comes as an economic Planck's constant.

Since the analogy with physical particle trajectory is merely formal, h value, unlike the physical Planck's constant $\hbar$, can, generally speaking, depend on the historical period of time, for which the series are taken, and the length of the averaging interval (e.g. economical processes are different in the time of crisis and recession), on the series number i etc. Whether this analogy is correct or not depends on particular series' properties.

In recent work [50], we tested the economic mass as an indicator of crisis phenomena on stock index data. In this work we will test the model for the cryptocurrency market on the example of the Bitcoin [51].

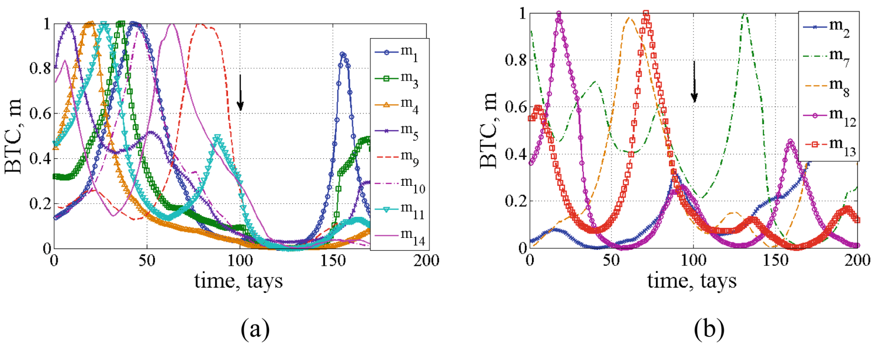

Fig. 5. Dynamics of measure m for local crashes (a) and critical events (b).

Obviously, there is a dynamic characteristic values m depending on the internal dynamics of the market. In times of crashes known marked by arrows in the Figs. 5(a) and 5(b) mass m is significantly reduced in the pre-crash and pre-critical periods.

Obviously, that the value of m remains a good indicator-precursor even in this case. Value m is considerably reduced before a special market condition. The market becomes more volatile and prone to changes.

The following method of quantum econophysics is borrowed from nuclear physicists and is called Random Matrix Theory.

7.2 Random Matrix Theory and Quantum Indicators-Predictors

Random Matrix Theory (RMT) developed in this context the energy levels of complex nuclei, which the existing models failed to explain (Wigner, Dyson, Mehta, and others [52–54]). Deviations from the universal predictions of RMT identify system specific, nonrandom properties of the system under consideration, providing clues about the underlying interactions.

Unlike most physical systems, where one relates correlations between subunits to basic interactions, the underlying “interactions” for the stock market problem are not known. Here, we analyze cross correlations between stocks by applying concepts and methods of random matrix theory, developed in the context of complex quantum systems where the precise nature of the interactions between subunits are not known.

RMT has been applied extensively in studying multiple financial time series [55–59].

In order to quantify correlations, we first calculate the logarithmic return (1) of the i cryptocurrencies price series over a time scale $\Delta t = 1$ day. It was selected 24 established during the last 5 years the most capitalized cryptocurrencies for the period from 04.08.2013 to 08.12.2018 (https://coinmarketcap.com/all/views/all/). We calculate the pairwise cross-correlation coefficients between any two cryptocurrencies returns time series. For the correlation matrix C we can calculate its eigenvalues, $C = U \Lambda U^T$, where U denotes the eigenvectors, Λ is the eigenvalues of the correlation matrix, whose density $f_c(\lambda)$ is defined as follows, $f_c(\lambda) = (1/N) dn(\lambda)/d\lambda$, where $n(\lambda)$ is the number of eigenvalues of C that are less than λ. In the limit $N \to \infty, T \to \infty$ and $Q = T/N \geq 1$ fixed, the probability density function $f_c(\lambda)$ of eigenvalues λ of the random correlation matrix M has a close form:

$$f_c(\lambda) = \frac{Q}{2\pi\sigma^2} \frac{\sqrt{(\lambda_{\max} - \lambda)(\lambda - \lambda_{\min})}}{\lambda} \tag{8}$$

with $\lambda \in [\lambda_{\min}, \lambda_{\max}]$, where $\lambda_{\min}^{\max}$ is given by $\lambda_{\min}^{\max} = \sigma^2 (1 + 1/Q \pm 2\sqrt{1/Q})$ and σ^2 is equal to the variance of the elements of matrix M.

We compute the eigenvalues of the correlation matrix C, $\lambda_{\max} = \lambda_1 > \lambda_2 > \cdots > \lambda_{15} = \lambda_{\min}$. We find that the largest eigenvalue $\lambda_{\max} = 5.48$ and the smallest eigenvalue $\lambda_{\min} = 0.81$. If C is a random matrix, the largest eigenvalue $\lambda_{\max}^{RMT} = 1.45$ and the smallest eigenvalue $\lambda_{\min}^{RMT} = 0.63$, according to Eq. (8). In our case, only one-third of its own values refer to the RMT region.

Eigenvectors correspond to the participation ratio PR and its inverse participation ratio IPR $I^k = \sum_{l=1}^{N} [u_l^k]^4$, where u_l^k, $l = 1, \ldots, N$ are the components of the eigenvector u^k. Figure 6 shows the comparative characteristics of the eigenvalue distributions for the random matrix (shuffled) and real (a) and the corresponding values of IPR (b). The difference in dynamics is due to the peculiarities of non-random correlations between the time series of individual assets. Under the framework of random matrix theory, if the eigenvalues of the real time series differ from the prediction of random matrix theory, there must exists hidden economic information in those deviating eigenvalues. For cryptocurrencies markets, there are several deviating eigenvalues in which the largest eigenvalue $\lambda_{\max}$ reflects a collective effect of the whole market. As for PR the differences from RMT appear at large and small lambda values and are similar to the Anderson quantum effect of localization [60]. Under crashes conditions, the states at the edges of the distributions of eigenvalues are delocalized, thus identifying the beginning of the crash. This is evidenced by the results presented in Fig. 6 (c).

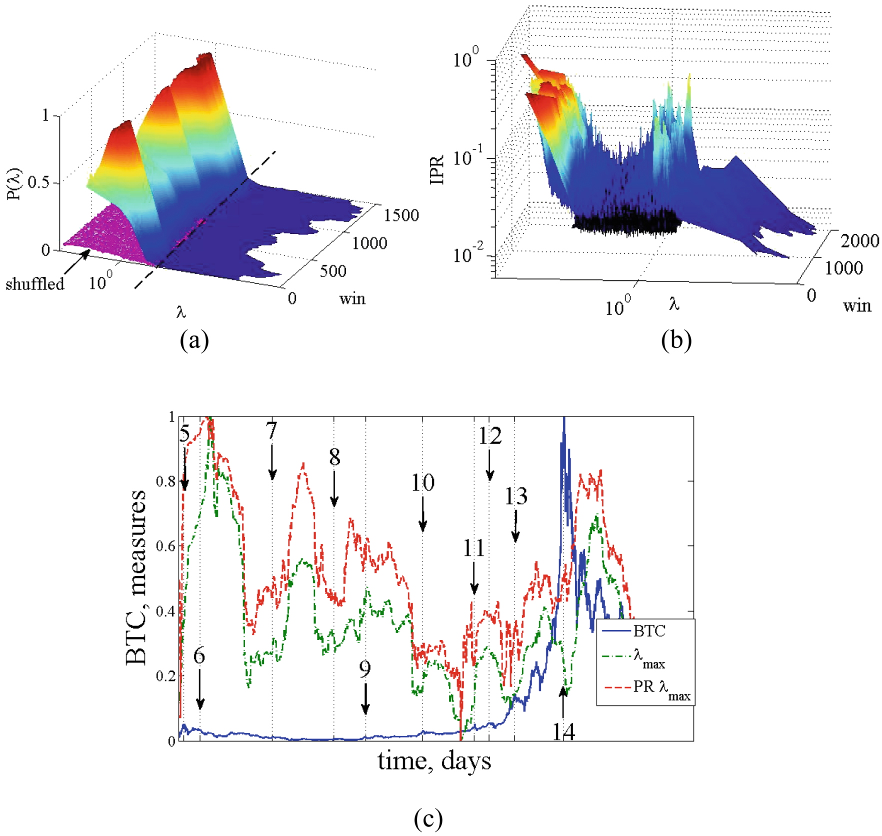

Fig. 6. Window dynamics of the distribution of eigenvalues (a), inverse participation ratio (b) for the initial and mixed (or random) matrices and quantum measures of complexity $\lambda_{\max}$ and its participation ratio

We find that both $\lambda_{\max}$ and *PR* of $\lambda_{\max}$ have large values for periods containing the market crashes and critical events. At the same time, their growth begins in the pre-crashes periods. Means, as well as the economic mass, they are quantum precursors of crashes and critical events phenomena.

8 Conclusions

Consequently, in this paper, we have shown that monitoring and prediction of possible critical changes on cryptocurrency is of paramount importance. As it has been shown by us, the theory of complex systems has a powerful toolkit of methods and models for creating effective indicators - precursors of crashes and critical phenomena. In this paper, we have explored the possibility of using the recurrent, entropy, network and quantum measures of complexity to detect dynamical changes in a complex time series. We have shown that the measures that have been used can indeed be effectively used to detect abnormal phenomena for the time series of Bitcoin.

We have shown that monitoring and prediction of possible critical changes on cryptocurrency is of paramount importance. As it has been shown by us, the quantum econophysics has a powerful toolkit of methods and models for creating effective indicators-precursors of crisis phenomena. In this paper, we have explored the possibility of using the Heisenberg uncertainty principle and random matrix theory to detect dynamical changes in a complex time series. We have shown that the economic mass m, and the largest eigenvalue $\lambda_{\max}$ may be effectively used to detect crisis phenomena for the cryptocurrencies time series. We have concluded though by emphasizing that the most attractive features of the m, $\lambda_{\max}$ and PR of $\lambda_{\max}$ namely its conceptual simplicity and computational efficiency make it an excellent candidate for a fast, robust, and useful screener and detector of unusual patterns in complex time series.

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