Comparative analysis of the attractiveness of investment instruments based on the analysis of market dynamics

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Abstract. The article continues the authors' research on solving the problem of choosing the most attractive investment instrument from a variety of alternatives, based on a comparative analysis of the dynamics for the respective markets. The nature of the dynamics affects the predictability level of the investor's income and is determined by finding out which hypothesis corresponds to the dynamics: the efficient market hypothesis, the fractal market hypothesis and the coherent market hypothesis. The methodology of comparative analysis developed by the authors is based on the use of statistical analysis methods combined with the methods of complex fractal analysis. It makes it possible to reveal the presence of deterministic chaos in the dynamics and to obtain estimates of the long-term memory in time series. The calculated characteristics of the fuzzy set of the memory depth for time series make it possible to draw conclusions about the financial instruments preference for the investor. The methodology developed by the authors is applied to three markets. A comparative analysis of three instruments (gold, EUR/USD currency pair and Bitcoin cryptocurrency) was carried out. The dynamics of prices and profitability for financial instruments in the conditions before the onset of the COVID-19 crisis and during it is considered.

Keywords: gold market, EUR/USD, Bitcoin, statistical analysis, fractal analysis, rescaled range analysis, memory depth, COVID-19.

1 Introduction

Nowadays investors are faced with a wealth of information and investment opportunities. However, the availability of access to global financial markets carries both additional opportunities for profit and new risks. All this requires the development of new modern and effective approaches to assess the investment attractiveness of markets.

The dynamics of investment markets is formed under the influence of many external and internal factors. To understand and explain the nature of this dynamics, scientists have developed and put forward several hypotheses. The most famous of them are: Efficient market hypothesis (EMH) [6; 7], Fractal market hypothesis (FMH) [18] and

Coherent market hypothesis (CMH) [22]. Often, the premises of one hypothesis run counter to the premises of another. For example, the efficient market hypothesis states that prices follow a random walk and the previous price is not related to the subsequent price. To study the effective market, the use of statistical analysis tools is envisaged. Conversely, the presence of fractal dynamics involves long-term memory of the time series and, on the basis of this, the ability to predict the behavior of the system. The Fractal Market Hypothesis involves the use of nonlinear dynamics methods.

In fact, empirical studies show that the nature of the dynamics for a particular market cannot fully satisfy either the EMH requirements or the FMH prerequisites for a long period of time. The characteristics of the dynamics can change over time, which is suggested, for example, by the Coherent Market Hypothesis. Real markets always contain both elements of randomnicity and determinism. Therefore, to understand and evaluate each market, it is important to use all the existing tools of both statistical analysis and nonlinear dynamics methods.

So, development and improvement of the comparative analysis technique of investment markets dynamics in the context of existing market hypotheses, assessment of investment prospects, and developing recommendations on the benefits of investing for different planning horizons are extremely urgent and important tasks. This problem is of particular interest in the conditions for the emergence and extension of COVID-19 crisis.

2 Related work

Each of the aforementioned hypotheses implies appropriate prerequisites for the dynamics of investment markets and uses special methods of diagnostics and analysis.

Consider each of the hypotheses in more detail.

The basis of EMH is the following preconditions [6; 7]: all the information is equally accessible and can be immediately taken into account by the market at a fair price, future prices depend only on the new information, future prices are not related to the previous ones, the impact of the information is linear, market participants are rational and homogeneous (they are equally not risk-averse and have the same investment horizons). Within this hypothesis, linear models, probabilistic calculations and statistical analysis are used.

Main characteristics of a fractal market are [18]: the main thing in the market is not a fair price, but liquidity, prices have a memory of previous values, locally the market is random, but globally – determined, but the dynamics of the market is nonlinear, investors differ in investment horizons. For diagnostics and analysis, nonlinear models, fractal mathematics and chaos theory tools are used.

The Coherent market hypothesis combines the two previous hypotheses and represents a nonlinear statistical model. According to CMH, markets go through four phases: random walk, unstable transition, chaos, and coherence [22].

All the above mentioned hypotheses have been arisen and developed in studies of stock markets [6; 7; 18; 22]. However, the local stock markets assessment of the correspondence to existing hypotheses remains relevant today [1; 2; 3; 8; 9; 17; 16; 23].

But now the scope of their application has considerably expanded, along with stock markets, dynamics analysis is actively conducted for the currency markets [4; 5; 11], deposits [12] and cryptocurrency [13].

Some works are directly related to the use of statistical (for proving the EMH) [1; 2; 3; 4; 9; 17] or fractal (for proving the FHM) [5; 16] analysis tools. Other studies are devoted to a specific analysis tool, for example, Fourier Unit Root Test [8] or Hurst exponent [13]. It should be noted that more interesting relevant and modern are the studies of nonlinear characteristics of dynamics [20; 21].

However, in our opinion, it is important to develop a comprehensive approach to assessing market dynamics, which allows the use of the best diagnostic tools from both statistical and fractal methods. Economic time series often do not represent a classical model of one theory, but combine both stochastic and fractal components. Therefore, the application of different methods to assess the dynamics reveals the patterns of development of economic series from different points of view. The basics of an integrated approach have been outlined in previous works by the authors [14]. This paper uses the main Investment market comparative analysis technique steps from the work [14]. However, the comparison criteria were revised and supplemented. Some criteria have been removed due to their low level of informativeness, some have been added. A distinction was also made as to which criteria should be applied to the time series of prices, which to the time series of profitability, and which could be applied to both types of time series.

This study is especially relevant in connection with the sharp changes in the dynamics of economic systems that are currently occurring during the crisis COVID-19.

It determines the importance of a comprehensive analysis of the investment markets dynamics to highlight their crucial characteristics. These characteristics can be used to compare and select the most attractive investment instruments.

3 Materials and methods

The paper considers the following investment instruments: precious metals market, Forex currency market and cryptocurrency market. All of them are both high-tech and affordable, which means the presence of information technologies and applications for a wide range of individual investors.

The objects of comparative analysis are time series (TS) the daily values of the prices and the profitability of the gold, the currency pair EUR/USD and the Bitcoin for the period from August 2019 to July 2020.

In recent times, humanity has had to face new challenges: the COVID-19 pandemic has made its adjustments in almost every area of life [10; 19]. Unseen before quarantine measures and border closures have shattered logistics chains and dealt a significant blow to the global economy. Unprecedented measures implemented by the governments of many countries of the world have created a new economic reality, with new laws and vectors of development, preconditions and influenced the change in the nature of dynamics in financial markets. In this regard, the hypothesis of a change in the nature of the dynamics of financial instruments arose and the need to take these changes into account when assessing their investment attractiveness. The dynamics of selected investment markets in the form of time series (TS) for two periods is considered: before the beginning of the crisis (from 01.08.2019 to 31.01.2020) and from the conditional beginning of the crisis (from 01.02.2020 to 31.07.2020). This division is due to objective conditions and allows to test the proposed comparison tools under different conditions, tto gain new knowledge about the objects of study: how different markets reacted to changes in external factors, how external factors influenced the dynamics, what new features and properties acquired market dynamics in a crisis situation.

The general scheme of methods used in the study is presented in table 1.

Table 1. Investment market comparative analysis technique steps.

Step	For TS of the prices	For TS of the profitability
1.	**Visualization of dynamics**	
	Research on the presence and type of trends	-
2.	**Statistical analysis**	
2.1.	Basic numerical characteristics estimation:	
	Coefficient of variation, coefficient of oscillation	Mean, median, standard deviation, skewness, kurtosis
2.2.	Normal distribution tests:	
	• verification for the equality of mean and median; • checking for the matching of the skewness; • checking for the matching of the kurtosis; • Kolmogorov-Smirnov Test; • Shapiro-Wilk test.	
2.3.	Other statistical tests:	
	Breusch-Godfrey serial correlation LM-test, Runs test	–
3.	**Complex fractal analysis**	
3.1.	Deterministic chaos diagnosis:	
	Drifting attractor test, Gilmore's graphic test, construction of pseudo-phase space	
3.2.	Rescaled range analysis (R/S analysis) if Hurst exponent	
	$H \leq 0.8$	$H > 0.8$
3.2*.	Construction of delayed (profitability) TS	
3.3.	Method of sequential R/S-analysis:	
	Construction of a fuzzy set of memory depth, calculation of its characteristics: l_{ms}, $l_{\max}$, l_{cg}, H_{entr_L}, $SH(L)$	

In sub-step 2.1 of step 2 basic numerical statistical characteristics estimation selected characteristics that are appropriate to use to compare the TS of each species. Thus, to compare the dynamics of prices, the relative coefficients of variation and oscillations are chosen, and for TS profitability, indicators can be used that are measured in both absolute and relative values (mean, median, standard deviation, skewness, kurtosis).

In step 2.2 of table 1 the compliance with the normal distribution is checked. According to EMH, investment instruments prices already take into account past information, therefore the next price change is influenced only with the new information [7]. Hence, all occurring on the market changes are not related events. It follows from the central limit theorem that the distribution of a large number of random independent variables converges to normal distribution. So, by checking the hypothesis of normal distribution (step 2.2 of figure 1), the hypothesis of an effective market is checked.

The normality of the distribution is analyzed with:

- verification for the equality of mean and median;
- checking for the matching of the skewness;
- checking for the matching of the kurtosis;
- Kolmogorov-Smirnov Test;
- Shapiro-Wilk test.

In addition, there are various statistical tests checking the availability of some characteristics of the weak form of efficient market, such as the independence between events, the stationarity of a time series, the random nature of price changes or the study of variances. In the previous authors' works to check the series for random character of changes was made: constructing regression equations and checking them for statistical significance; checking for auto-correlation; Durbin-Watson Test, Breusch-Godfrey serial correlation LM-test, Augmented Dickey-Fuller Unit Root test (ADF test) and Runs test. In this study, tests were selected that confirmed their effectiveness and proved to be the most informative and indicative criteria for comparative analysis.

In step 2.3, the following tests were carried out and the following methods were applied only to the time series of prices: Breusch-Godfrey serial correlation LM-test, Runs test.

At the first sub-stage of the Complex fractal analysis stage we perform deterministic chaos diagnosis, namely Drifting attractor test, Gilmore's graphic test, construction of pseudo-phase space to the time series of price and profitability.

In step 3.2, the rescaled range analysis (R/S-analysis) is applied and the Hurst exponent is calculated, it allows to determine the presence of memory (persistence) in a TS [18]. The Hurst exponent is a measure to determine the nature of the dynamics of the series: the randomness of events in the series (at H=0.5), the persistence of the series and the presence of memory (at H approaching 1), or antipersistent (at H<0.5). The Hurst exponent is a measure of the trend stability of a series and allows us to determine whether the nature of the dynamics is stochastic or fractal.

In the case when the value of the Hurst exponent indicates the persistence of the series ($H\geq0.8$), we proceed to step 3.3.

If the Hurst exponent does not show a sufficient level of persistence (H <0.8), then proceed to step 3.2*.

If the Hurst exponent indicates memory availability, then in step 3.3 we use the method of sequential R/S-analysis [15].

The determination of the Hurst exponent is based on the application of the method of the normalized Hurst range and the construction of the R/S trajectory. If TS is characterized by long-term memory, then a number of starting points of the obtained R/S trajectory of this TS form a clear linear trend. At some value of $k = k_0$ R/S-trajectory changes its slope quite sharply, that is, at the point (x_{k0}, y_{k0}) the trajectory receives a significant negative gain in absolute terms – there is a break in the trend and there is no return to the previous trend. It is assumed that at the point k_0 the effect of long-term memory dissipates. In this case, the breakdown of the trend demonstrates the loss of memory of the initial conditions, and also signals (possibly with a lag, i.e. with some delay) the exhaustion of the cycle or quasi-cycle contained in the initial segment of this TS. But, as is known [15], the method of normalized Hurst scope (standard R/S-analysis) provides only the average characteristic of the inertia property (trend resistance) for TS as a whole and does not take into account the changing nature of the dynamics of the indicator.

To overcome this shortcoming, a modified method of fractal analysis was developed [15] the method of sequential R/S analysis (step 3.4). The essence of the method is to sequentially construct a modified computational scheme of R/S-trajectory for the family TS, which are a subset of this TS, but consistently differ from the starting point. The advantage of this method is its ability to take into account the changing nature of the dynamics, to identify the set of cycles (quasi-cycles) that are characteristic of the TS under study, as well as to calculate the lower estimate of memory depth (about the beginning of this TS). The difference in the conditions of application of the method is the absence of significant restrictions on the length of TS.

The result of applying the method of sequential R/S-analysis is to determine not one breakpoint from the trend k_0, but a set of breakpoints from the trend of the family of R/S-trajectories, which reflect the memory loss time of the initial conditions (beginning of the corresponding TS). This allows you to generate a fuzzy set of TS memory depth. Estimating memory depth for a range reflects the uncertainty generated by external and internal influences on the economic system.

The fuzzy set of memory depth for the TS as a whole (is denoted by $L(i)$) has the form

$$L(i)= \{(l, \mu(l)), \ l \in L^0\}, \tag{1}$$

where $L(i)$ is a fuzzy set of memory depth for TS i, l is the value of the sequence number of the trend change point for TS, and supp$L(i)=L^0 = \{l \in \mathrm{N}, \mu(l)>0\}$.

Based on the analysis of the values of the membership function $\mu(l)$, we can identify the so-called characteristic or significant degrees of membership (for example, $\mu(l)>0.3$), which can be considered uncharacteristic. Restriction on the degree of significance (denote it ε), ie the condition $\mu(l)> \varepsilon$, can be set by an expert.

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The values of memory depth l, which correspond to the values of the membership function $\mu(l)>\varepsilon$, are called ε-valuable [15].

Using the defasification procedure for the selected significant degrees $\mu(l)$, and, if necessary, rounding the calculated value to the nearest whole, we calculate the center of gravity (or gravity) of the set of ε-significant values of memory depth by the formula

$$l_{cg}=[(\Sigma l\cdot\mu(l))/(\Sigma\cdot\mu(l))]. \quad (2)$$

Thus, the obtained predictive information is that the considered TS is characterized by the property of trend resistance over a period of time l_{cg} on average. Depending on the value of l_{cg}, the latter statement in the context of pre-forecast analysis means good preconditions for building a sufficiently reliable forecast of this TS within the forecast horizon l_{cg}.

Recommendations for the forecast horizon (denote it h) can be clarified by using another characteristic of the fuzzy set $L(i)$ of memory depth for TS as a whole – the value of memory depth (denote it l_{ms} (the most significant), which has the largest the value of the membership function $\mu(l)$ of the depth l of the fuzzy set $L(i)$:

$$\mu(l_{ms})= \max\ (\mu(l)). \quad (3)$$

Satisfactory prediction accuracy is then provided when the prediction horizon does not exceed the center of gravity l_{cg} and the most common memory depth value – l_{ms}.

To estimate the property of dynamics uncertainty, the information entropy index of the fuzzy set of memory depth $L(i)$ (H_{entr_L}) is used with respect to the variety of behavior variants of a series of dynamics. It is calculated by the formula:

$$H_{entr\_L}= -\Sigma\ (\mu(l)\cdot log\ \mu(l)). \quad (4)$$

As discussed above, when analyzing the dynamics, it is advisable to analyze not only the entire fuzzy set of memory depth $L(i)$, but also its subset of ε-significant depths, i.e. the set $L^{\varepsilon}(i)$. This somewhat reduces the uncertainty that can be estimated by the value of information entropy by neglecting the values of depths that are not ε-significant, i.e. the indicator

$$H^{\varepsilon}{}_{entr\_L}= -\ \Sigma\ (\mu(l)\cdot log\ \mu(l)),\ \mu(l) > \varepsilon. \quad (5)$$

The redundancy index of the fuzzy memory depth $L(i)$ is also used to characterize TS as a measure of TS noise. It is calculated by the following formula:

$$SH(i)=1-(H^{\varepsilon}{}_{entr\_L} \backslash\ H_{entr\_L}). \quad (6)$$

On the basis of the given numerical characteristics concerning depth of memory of all TS as a whole, it is possible to carry out the comparative analysis of dynamics of the considered TS.

In step 3.2* for the profitability time series with the Hurst exponent close to 0.5, we construct the time series of the delayed profitability by the formula:

$$p^s = \frac{(v_{(t+s)} - v_t)}{v_t} * 100\%, \tag{7}$$

where $v(t)$ – the price of the investment instrument at a day t; s – is a lag value. Then profitability time series is equal to: $P^s(i)=(p^s(i))$, $i \in \{Z, F, B\}$, where Z – TS of gold; F – TS of currency pair EUR/USD, B – TS of cryptocurrency Bitcoin.

According to performed calculations, profitability TS may not have memory (Hurst exponent is close to 0.5), however, with the growth of the time lag, time series of delayed profitability become persistent.

For each of those time series, we calculated the Hurst exponent, until we determine the value of s at which the time series acquires memory (persistence).

4 Results

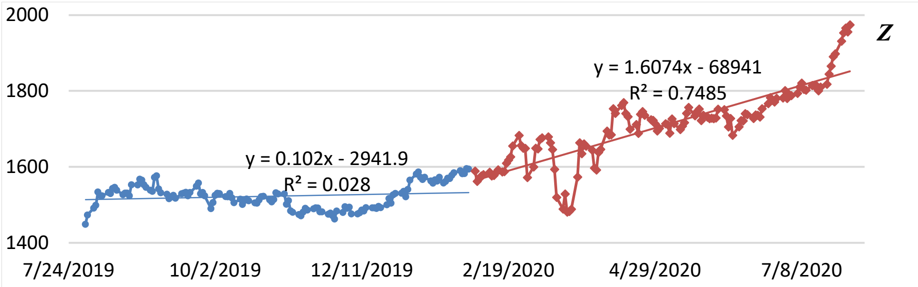
For the comparative analysis, three investment objects have been selected: gold (Z), currency pair EUR/USD (F), and cryptocurrency Bitcoin (B).

The steps and methods described in the previous section were applied to daily prices time series of the gold, the currency pair and the Bitcoin for the period from August 2019 to July 2020. For the comparative analysis of dynamics and studying of the market's reaction character to the changes which have occurred in the markets, time series are divided into two periods: before the beginning of the COVID-19 pandemic (from 01.08.2019 to 01.02.2020) and during its spread (from 01.02.2020 to 01.08.2020).

4.1 Application of the comparative analysis methodology to the time series of the prices for the investment instruments

Step 1. Graphical representations of price dynamics are shown in the fig. 1. The figure shows the identified trends in the form of linear trends for each TS. The presence of a significant linear trend was detected only for the time series of gold prices during the pandemic (R^2=0.75). The coefficient of determination greater than 0.5 had Bitcoin before the crisis with a downward linear trend. The linear trend before the crisis for gold and EUR was insignificantly growing, for bitcoin – markedly downward. But with the crisis onset, the direction of price movements became increasing for all three instruments. This is partly due to the significant dollar emission, which occurs as a reaction of the Federal Reserve System to the crisis in the US economy. However, the numerical characteristics of trends are not comparable and cannot serve as indicators for their comparison.

Step 2. At this stage the statistical analysis of time series is performing. Given that the prices of selected investment instruments have different units of measurement, we use relative statistics such as coefficients of variation and oscillations for comparison (table 2).



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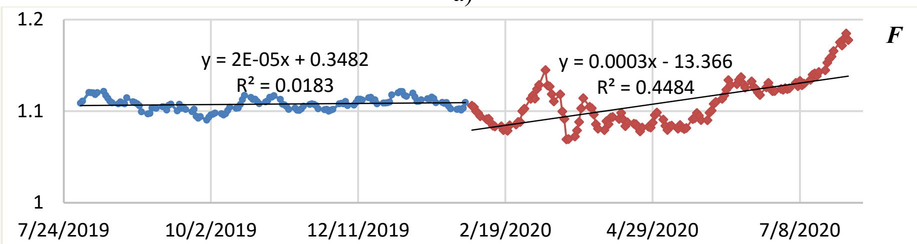

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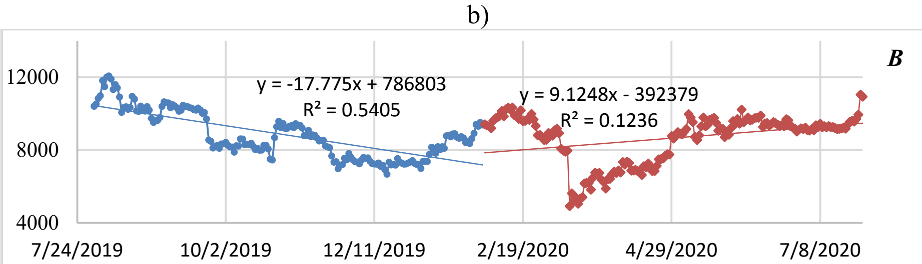


c)

Fig. 1. Price dynamics for the period from August 2019 to July 2020: a) gold (Z); b) currency pair EUR/USD (F); c) Bitcoin (B).

Table 2. Statistical characteristics of investment instruments prices.

Statistical characteristics	**TS**								
	All			**Before**			**During**		
	P(Z)	**P(V)**	**P(B)**	**P(Z)**	**P(V)**	**P(B)**	**P(Z)**	**P(V)**	**P(B)**
Variation coefficient	0.074	0.017	0.152	0.022	0.006	0.146	0.055	0.023	0.158
Oscillations coefficient	0.325	0.104	0.816	0.105	0.028	0.612	0.283	0.104	0.740

Analysis of table 2 shows that the dynamics of each of the three instruments changes significantly with the crisis onset. This confirms the assumption that time series have changed the nature of the dynamics and it is necessary to consider two series of data for each instrument: before the coronavirus pandemic start and during its spread. The largest changes in the dynamics are observed in the market of the currency pair EUR/USD: after a relative calm, quarantine measures were reflected in increased volatility and rapid changes in the direction of price movements. The coefficients of variation and oscillations have increased several times.

Against the background of a noticeable upward trend, the time series of gold prices during the crisis also significantly increased volatility. The smallest increase in these coefficients occurred in the Bitcoin market. However, it should be noted that Bitcoin before the crisis showed strong volatility in contrast to, for example, the TS of currency pair EUR/USD (the coefficient of variation for the currency in the pre-crisis and post-crisis period was 0.006 and 0.023, respectively, compared to 0.146 and 0.158 for cryptocurrency).

The Breusch-Godfrey test (on the correlation between the price values of 1-10 order) is conducted to establish the relationship between the events in the time series. The test results are presented in the table 3.

Table 3. The Breusch-Godfrey test results for TS of prices.

	Gold	**EUR/USD**	**Bitcoin**
Before	-	-	Order 7 (p-value = 0.049)
During	-	Order 8 (P-value=0.03) Order 9 (P-value=0.03) Order 10 (P-value=0.04)	Order 1 (P-value=0.047) Order 2 (P-value=0.04) Order 4 (P-value=0.049)

According to table 3, we can say that for TS gold in the two studied periods and EUR/USD in the pre-crisis period, the null hypothesis of no autocorrelation was confirmed. For TS EUR/USD during the pandemic period and for Bitcoin in the two studied periods at a significance level of 0.05, a correlation of certain orders is possible.

Step 3. We turn to the study of the financial instruments dynamics and its comparison by methods of nonlinear dynamics. The table 4 shows the calculated values of the Hurst exponent (H), which determine the level of persistence for the time series of the investment instrument prices and for time series of mixed price values.

Table 4. The value of Hurst exponent (H) for TS of prices.

Hurst exponent	**TS**								
	All			**Before**			**During**		
	P(Z)	**P(V)**	**P(B)**	**P(Z)**	**P(V)**	**P(B)**	**P(Z)**	**P(V)**	**P(B)**
H	0.889	0.865	0.917	0.902	0.894	0.925	0.917	0.876	0.938
H mixed	0.538	0.612	0.564	0.601	0.541	0.532	0.623	0.544	0.603

For all series, the Hurst exponent is in the range [8.6; 9.4], from which we can conclude that all series have a memory of previous values. But table 4 shows that the nature of the dynamics during the crisis period is changing. It should be noted that for all financial instruments, the Hurst exponent for the separate (pre-crisis or pandemic) period is higher than the Hurst exponent for the entire (united) period.

The Hurst exponent acquires the highest values for Bitcoin time series in contrast to the lowest H values for series of the currency pair EUR/USD. This means that the dynamics of Bitcoin is described by the laws of nonlinear dynamics, and the influence of randomness on the price formation is small. Recall that $H = 1$ means a completely deterministic series. Analysis of table 4 leads to the conclusion that all TSs are persistent and have memory. Therefore, we pass to execution of step 3.3 – application of a method of the sequential R/S-analysis.

Let's construct a fuzzy set of memory depth and consider its characteristics.

The fig. 2 shows the fuzzy sets $L(i)$ of memory depth for each TS $i \in \{Z, F, B\}$).

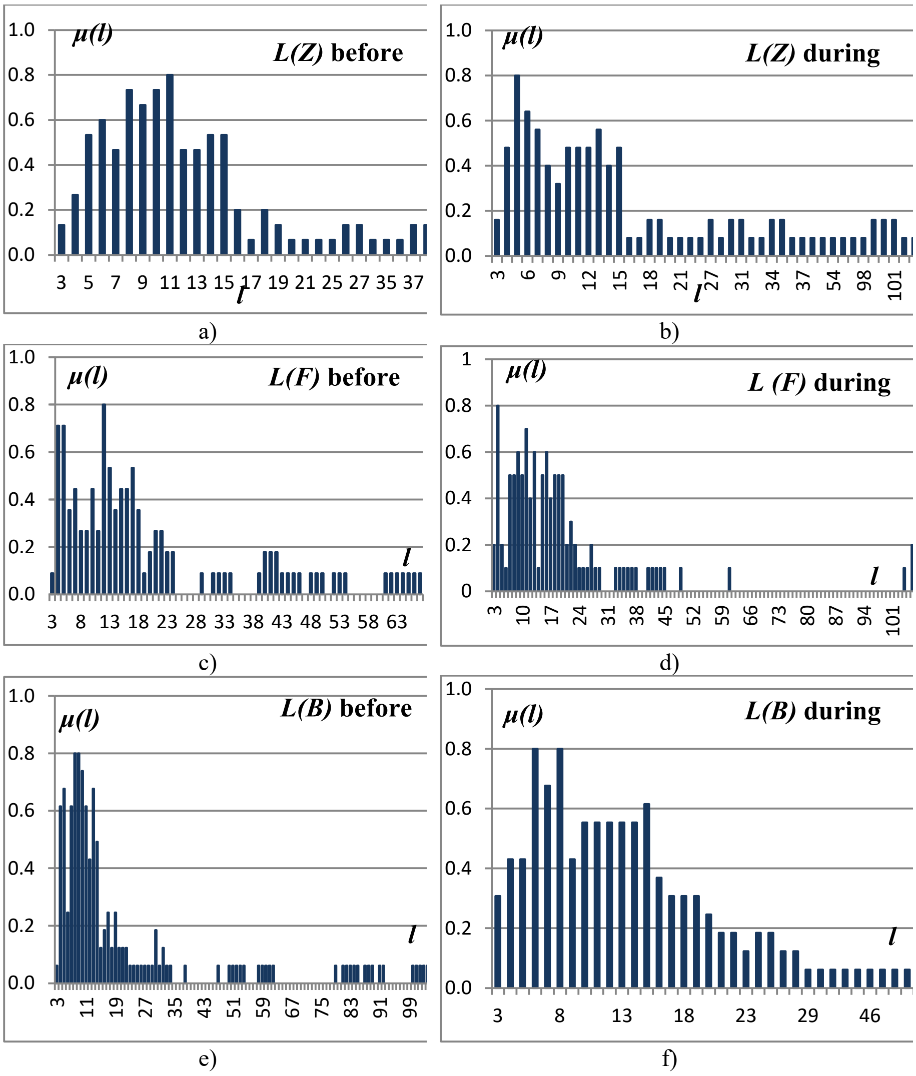


Fig. 2. The fuzzy set of memory depths $L(i)$ for the TS of: a) gold (Z) before; b) gold (Z) during; c) EUR/USD (F) before; d) EUR/USD (F) during; e) Bitcoin (B) before; f) Bitcoin (B) during.

We calculate and compare the characteristics of the depth of memory inherent in the time series of investment instruments, calculated on the basis of their fuzzy sets. The main numerical characteristics of the fuzzy set of memory depth are given in the table 5.

Table 5. The main numerical characteristics of the fuzzy set of memory depth for TS of prices.

Characteristic	**TS**					
	Gold (*Z*)		**EUR/USD (*F*)**		**Bitcoin (*B*)**	
	before	**during**	**before**	**during**	**before**	**during**
l_{max}	38	104	67	108	104	50
l_{ms}	11	5	12	4	8.9	6.8
l_{cg}	12.5	22.3	20.6	20.9	22.4	13.7
H_{entr_L}	10.6	15.9	18.6	17.4	18.2	13.5
Significance ε = 0.3						
l_{max}	15	15	22	22	14	19
l_{ms}	11	11	12	4	8.9	6.8
l_{cg}	9.7	9.2	11.5	12.7	9.1	10.5
H_{entr_L}	5.2	5.7	8.1	7.0	3.9	7.9
$SH(L)$	0.5	0.6	0.6	0.6	0.8	0.4

Consider the characteristics of a fuzzy set of memory depths in the pre-crisis period. From the point of view of reducing uncertainty, the best value of the maximum memory depth l_{max} has gold (the choice from the set is limited by the memory depth 38), this value is relatively good for gold also at ε-significance 0.3. The relatively small value of the center of gravity for the time series of gold, on the one hand, limits the possible forecast horizon, and on the other hand is a consequence of low variability. For the set $L^{\varepsilon}(i)$, the gravity center of gold becomes comparable to that of Bitcoin. The noise level for TS of the gold is the lowest. Despite the fact that the entropy index for the set $L^{\varepsilon}(i)$ is better in TS Bitcoin, we believe that the most stable and predictable in the pre-crisis period is TS of gold.

Given the notable reduction of the fuzzy set at the level of significance $\varepsilon = 0.3$ and the low entropy of Bitcoin with insignificant differences in other indicators, we consider the Bitcoin series to be more stable and attractive for investment than currency.

The introduction of quarantine measures had a negative effect on the most significant memory depth lms: it fell in all series except the gold $L^{\varepsilon}(Z)$ (l_{ms}(before) = l_{ms}(during) = 11). For gold and EUR, the variability and uncertainty of the set $L(i)$ increased significantly (l_{max} from 38 before the crisis to 104 after for gold, l_{max} from 67 before the crisis to 108 during for EUR). Conversely, for Bitcoin these indicators have improved (from 104 to 50). Given the best indicators l_{max}, l_{ms} and H_{entr_L} of the set $L^{\varepsilon}(i)$ for TS of gold, we believe that even after the crisis, this financial instrument remains the most attractive.

4.2 Application of the comparative analysis methodology to the time series of profitability for the investment instruments

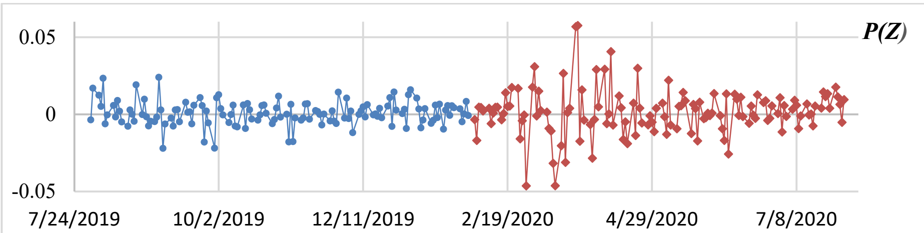
Another important section for studying the market dynamics is the time series of profitability for financial instruments. Profitability is a crucial indicator for every investor. In addition, time series of profitability are indispensable in comparative analysis, as they are not absolute but relative values of the price. Calculate the TS of profitability by this formula for each time series of the price.

$$P(i) = p_t(i),$$

$$p_t(i) = \frac{(v_t\,(i) - v_{(t-1)}(i))}{v_{(t-1)}(i)} * 100\%,$$

where $v_t(i)$ – the price of the investment instrument at a day t, $i \in \{Z, F, B\}$.

Graphic representation of the obtained series of profitability is shown in the fig. 3.



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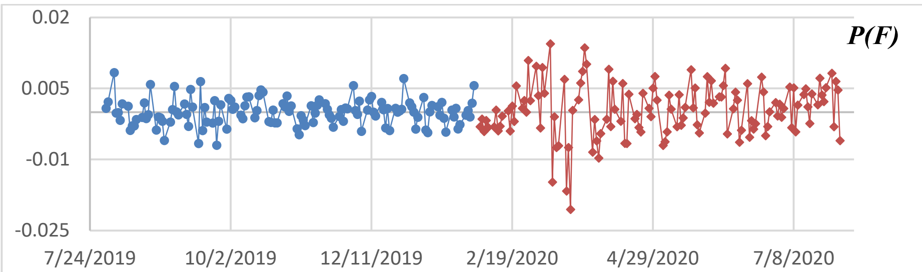

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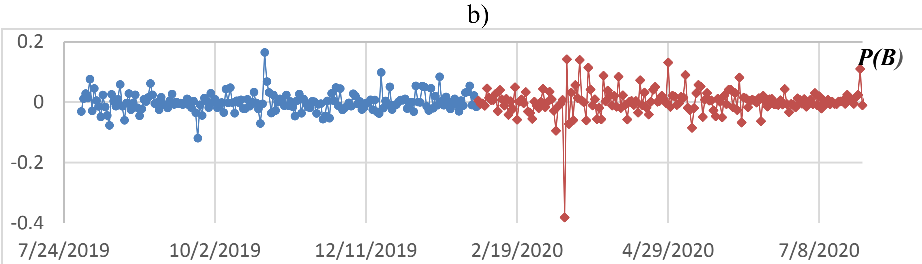

c)

Fig. 3. Dynamics of profitability time series for a) gold; b) EUR/USD; c) Bitcoin.

The mean values of profitability time series are close to zero, the series differ significantly from each other in terms of variation (standard deviation and range). Since time series of profitability are similar in appearance to a series of random variables, we check them for the normal distribution law according to the five criteria defined in Section 3. The results of the calculation are shown in the table 6.

Table 6. The results of checking the series on the normal distribution law.

Criteria	**TS**					
	Before			**During**		
	***P*(*Z*)**	***P*(*V*)**	***P*(*B*)**	***P*(*Z*)**	***P*(*V*)**	***P*(*B*)**
1. Verification for the equality of the meanand the median	+	+	+	+	+	+
2. Checking of the skewness	+	+	–	+	+	–
3. Checking of the kurtosis	+	+	–	–	–	–
4. Kolmogorov-Smirnov test	+	+	–	+	+	–
5. Shapiro-Wilk test	p=0.026	+	–	–	p=0.037	–
Mean	0.00072	0.00002	-0.00025	0.00173	0.00047	0.00225
Standard deviation	0.00784	0.00275	0.03158	0.01505	0.00563	0.04638
Range	0.04608	0.01536	0.28352	0.10396	0.03496	0.52353

+ a null hypothesis that a normal distribution not disproved;

– the null hypothesis of a normal distribution disproved.

The time series of profitability of gold and the currency pair EUR / USD before the crisis had the features of randomly distributed values according to all five criteria. After the beginning of the crisis there was an increase kurtosis, Shapiro-Wilk test also showed negative results (for the currency at a significance level of α = 0.01). Bitcoin profitability did not meet the requirements of normally distributed values before or after quarantine measures (except for the mean and median).

The calculation of the Hurst exponent also shows the lack of memory and the random nature of changes in the profitability of the series (H values are in the range [0.57; 0.66]). In this connection, a family of profitability time series with a certain lag was constructed and investigated [18].

The time series of the "delayed" profitability are constructed by the formula (7).

The character of the dynamics of profitability varies depending on the magnitude of the time lag and, as it grows, the time series acquire the properties of persistence (the property of memory). The Hurst exponents for the profitability time series depending on the value of lag is presented in the table 7.

A visual representation of the change in the value of the Hurst exponent for each of the financial instruments is presented in the fig. 4.

The persistence of the united time series of profitability (including data both before the crisis and during quarantine measures) is usually less than the persistence of the divided series. This indicates the different nature of the dynamics of the series before and after the introduction of quarantine measures. Delayed time series of gold profitability in the pre-crisis period acquire persistence faster than the corresponding TS after the introduction of quarantine measures (fig. 4a)). For time series of profitability EUR and Bitcoin the opposite is true (fig. 4b and 4c).

Table 7. The Hurst exponents for the profitability time series depending on the value of lag.

TS	Period	lag 1	lag 5	lag 10	lag 15	lag 20
$P(Z)$	before	0.615	0.771	**0.874**	**0.910**	**0.915**
	after	0.612	0.740	**0.812**	**0.822**	**0.818**
	all	0.612	0.741	**0.837**	**0.872**	**0.888**
$P(F)$	before	0.623	0.759	**0.840**	**0.822**	**0.836**
	after	0.657	0.767	**0.815**	**0.839**	**0.853**
	all	0.611	0.719	**0.789**	**0.800**	**0.817**
$P(B)$	before	0.573	0.776	**0.819**	**0.831**	**0.84**
	after	0.618	**0.816**	**0.872**	**0.904**	**0.912**
	all	0.578	0.760	**0.808**	**0.825**	**0.837**

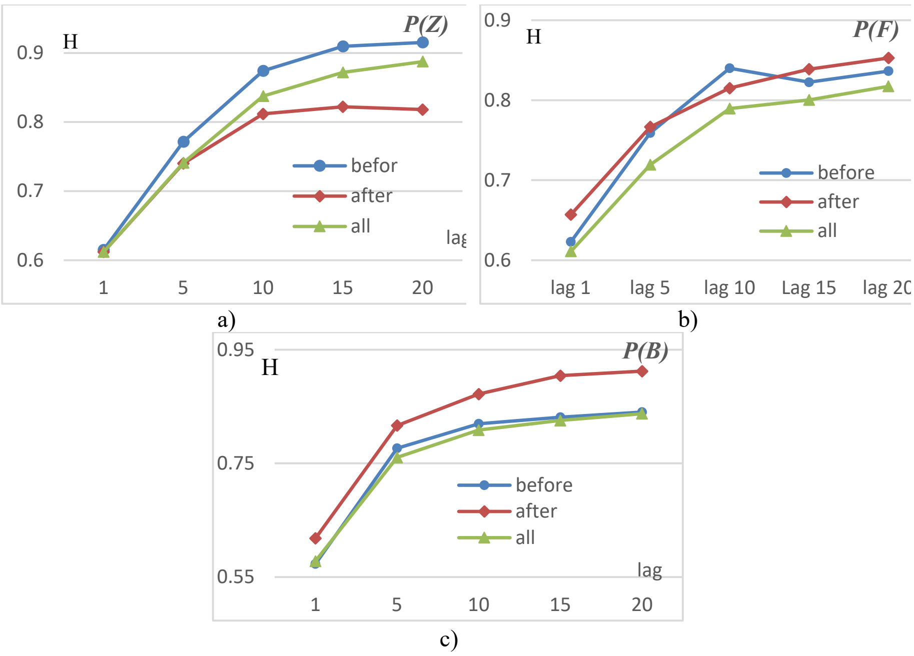

Fig. 4. Hurst exponent depending on the value of lag for: a) $P(Z)$; b) $P(F)$; c) $P(B)$.

A visual representation of the change in the value of the Hurst exponent for two periods is presented in the fig. 5.

In the pre-crisis period, the leaderin the speed of gaining persistence of profitability with increasing lag was gold, after the crisis - Bitcoin.

For the received persistent time series, we carry out their system characteristics in their structure of deterministic chaos.

The fig. 6 shows the fuzzy set $L(i)$, $i \in \{Z, F, B\}$ of memory depth for time series of profitability that have memory. We assume that time series have memory, with the Hurst exponent greater than 0.8 (H>0.8).

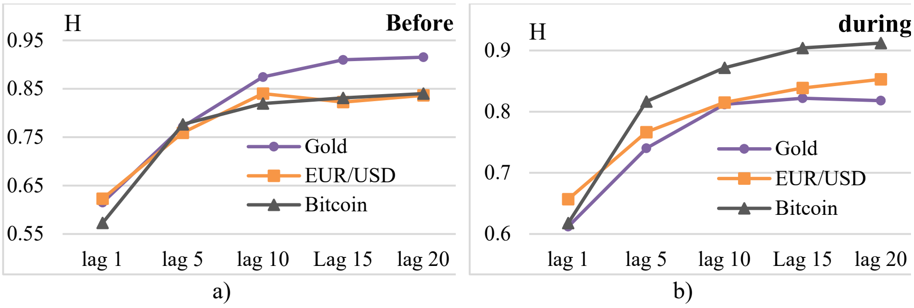

Fig. 5. Hurst exponent depending on the value of the period: a) before crisis; b) after crisis start.

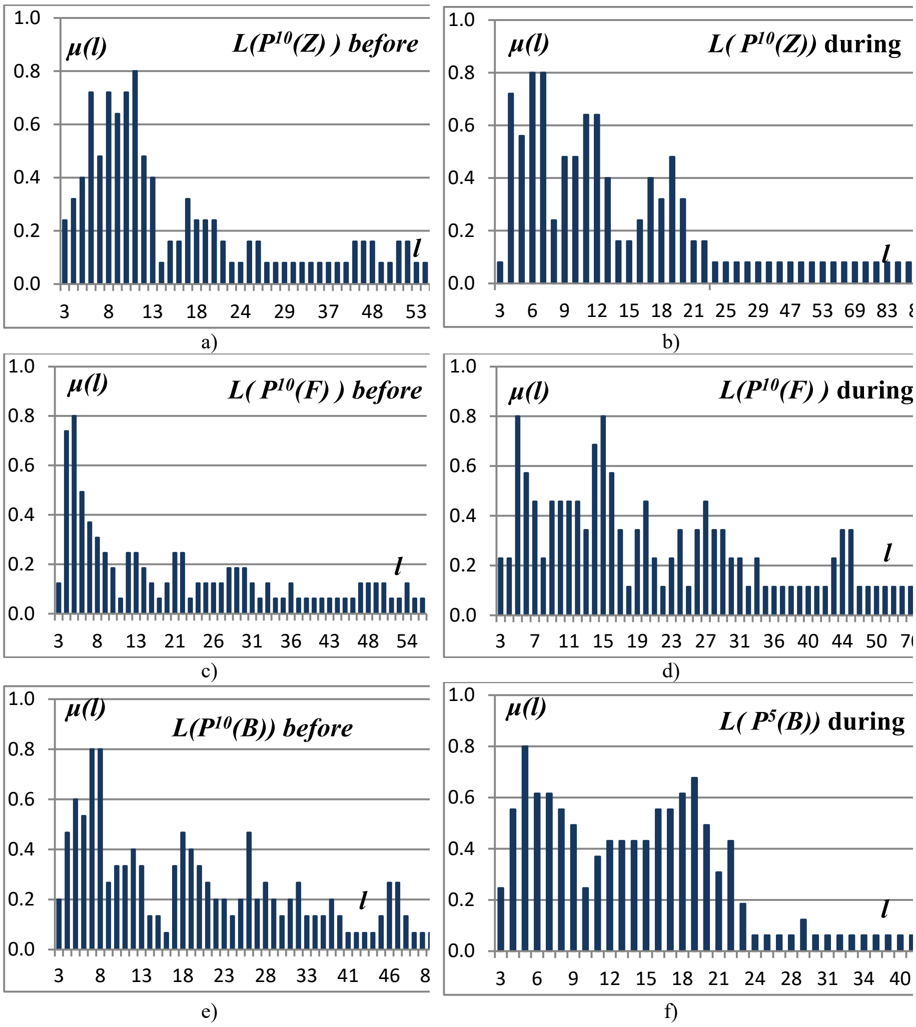

Fig. 6. The fuzzy set of memory depths $L(i)$ for the TS of i: a) $P^{10}(Z)$ before; b) $P^{10}(Z)$ during; c) $P^{10}(F)$ before; d) $P^{10}(F)$ during; e) $P^{10}(B)$ before; f) $P^{5}(B)$ during.

The main numerical characteristics of the fuzzy set of memory depth are given in the table 8.

Table 8. The main numerical characteristics of the fuzzy set of memory depth for TS of profitability.

Characteristic	**TS**					
	Gold		**EUR/USD**		**Bitcoin**	
	before	**during**	**before**	**during**	**before**	**during**
l_{max}	54	86	63	71	81	46
l_{ms}	11	6.7	5	5.15	7.8	5
l_{cg}	17.5	18.2	20.3	22.0	19.9	14.2
H_{entr_L}	16.0	14.6	16.9	21.7	19.4	13.5
Significance $\varepsilon = 0.3$						
l_{max}	17	20	8	8	47	22
l_{ms}	11	6.7	5	5.15	7.8	5
l_{cg}	9.1	10.4	5.5	19.0	15.3	12.7
H_{entr_L}	4.8	5.8	2.1	10.2	9.7	8.5
$SH(L)$	0.7	0.6	0.9	0.5	0.5	0.4

Due to the decrease in the most significant memory depth (l_{ms}) for the profitability of gold and bitcoin during the crisis, we can say about the emergence of a smaller fractal structure of the time series. For the Euro, this figure has not changed (l_{ms} = 5), but another significant depth of memory l_{ms} = 15 appeared. This is of course a positive signal, but it is offset by a significant increase in the range of the fuzzy set (l_{max} increased from 8 in the pre-crisis period to 45 in the post-crisis period). The best entropy indicators have time series of gold ($SH(L)$ = 4.8 and 5.8 in the pre-crisis and crisis periods, respectively).

5 Conclusion

Comparative analysis technique proposed in the paper integrates the tools and various diagnostic tests to determine the crucial characteristics of each studied market. The presented technique of comparative analysis has been tested on three investment markets: the precious metals market (for example, the gold market), Forex currency market (EUR/USD currency pair) and the cryptocurrency market (Bitcoin). The dynamics of these investment markets is considered in two periods: from 01.08.2019 to 31.01.2020 – pre-crisis period and from 01.02.2020 to 31.07.2020 – crisis period. The division into periods is due to significant changes in the environment in consequence of the COVID-19 pandemic and the introduction of quarantine measures. This allowed not only to compare the dynamics of the three instruments, but also to assess the reaction of markets to the crisis of the economic system.

Time series of the currency pair EUR/USD have the lowest volatility. In the pre-crisis period, the price fluctuated within a narrow range of values. The profitability of

EUR in this period corresponded to the characteristics of the normal distribution law for a random variable. Crisis phenomena in the economy intensified the amplitude of fluctuations and outlined a general upward trend against the background of significant, but short failure. The kurtosis of profitability increased rapidly, and TS of profitability ceased to meet the characteristics of the normal distribution, also and there were heavy "tails" of the distribution. But the features of fractality for this series have remained lower than the corresponding features of gold and bitcoin. And if in the pre-crisis period the most significant depth of memory lms for the price was at a relatively high level (but the entropy index was much worse than the corresponding rate of gold and bitcoin), then during the crisis lms decreased several times. Given the above, we consider the financial instrument EUR/USD to be the least attractive for investment due to the significant share of stochasticity in the dynamics of the instrument. Fractals and, accordingly, memory depth indicators have a small structure for forecasting the daily price data by fractal nonlinear dynamics methods. When working in this market, we recommend using fundamental analysis, follow the news and decisions of the European Central Bank.

Bitcoin is the instrument with the highest volatility, which on the one hand makes it possible to earn additional income, and on the other hand, increases the risks of the investor. In the pre-crisis period, preference was given to short positions, then in the post-crisis period the direction of the trend changed to upward. Statistical analysis showed that the time series of price and profitability of Bitcoin does not fall under the law of normal distribution, the nature of the dynamics is different from random, and the Broisch-Godfrey test could not confirm the absence of first-fourth order autocorrelation. A comprehensive fractal analysis of Bitcoin time series also shows a pronounced fractal dynamics. However, the evaluation of the characteristics of fuzzy memory depth showed that the fractal dynamics of Bitcoin has, firstly, high variability and, secondly, low values of the most significant memory depth lms (fractal structure is manifested in small patterns). Variability can be described as a measure of uncertainty, if we imagine a fractal structure in the form of a tree, where each branch is a new fractal, the fractal tree Bitcoin has a smaller structure compared to gold with many branches and increasing entropy during the crisis.

The main statistical and fractal indicators of gold dynamics occupy an intermediate position between currency and cryptocurrency: the level of stochasticity is lower than in EUR/USD, but the signs of fractality are slightly less than the corresponding signs of Bitcoin. The volatility of the series is also halfway between low-amplitude EUR/USD and high-amplitude cryptocurrency. However, a detailed study of the memory depth set showed that the price of gold has the highest lms (both in the pre-crisis period and during the crisis is 11 days) with low entropy. This makes it possible to use fractal characteristics when predicting the dynamics of gold. Therefore, we consider this tool the most predictable and attractive for investment.

In general, the crisis in the economy significantly affected the dynamics of all three financial instruments, changes occurred in the increase in the amplitude of fluctuations and, to a greater or lesser extent, the emergence of a general upward trend and increasing signs of fractality. However, the analysis of fuzzy memory depth revealed that increasing fractality does not always improve the level of predictability, given the simultaneous increase in the amplitude of changes in the series. The obtained indices of the characteristics of the fuzzy set make it possible to establish a reasonable forecast horizon of the forecast model.

Thus, the results of comparative analysis technique allowed developing practical recommendations to an investor: to compare the markets by their degree of predictability and to determine the параметри прогнозної моделі for each market. The results will also be used in the further development of forecast models for selected investment instruments.

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