Recurrence plot-based analysis of financial-economic crashes

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Abstract. The article considers the possibility of analyzing the dynamics of changes in the characteristics of time series obtained on the basis of recurrence plots. The possibility of using the studied indicators to determine the presence of critical phenomena in economic systems is considered. Based on the analysis of economic time series of different nature, the suitability of the studied characteristics for the identification of critical phenomena is assessed. The description of recurrence diagrams and characteristics of time series that can be obtained on their basis is given. An analysis of seven characteristics of time series, including the coefficient of self-similarity, the coefficient of predictability, entropy, laminarity, is carried out. For the entropy characteristic, several options for its calculation are considered, each of which allows the one to get its own information about the state of the economic system. The possibility of using the studied characteristics as precursors of critical phenomena in economic systems is analyzed. We have demonstrated that the entropy analysis of financial time series in phase space reveals the characteristic recurrent properties of complex systems. The recurrence entropy methodology has several advantages compared to the traditional recurrence entropy defined in the literature, namely, the correct evaluation of the chaoticity level of the signal, the weak dependence on parameters. The characteristics were studied on the basis of daily values of the Dow Jones index for the period from 1990 to 2019 and daily values of oil prices for the period from 1987 to 2019. The behavior of recurrence entropy during critical phenomena in the stock markets of the USA, Germany and France was studied separately. As a result of the study, it was determined that delay time measure, determinism and laminarity can be used as indicators of critical phenomena. It turned out that recurrence entropy, unlike other entropy indicators of complexity, is an indicator and an early precursor of crisis phenomena. The ways of further research are outlined.

Keywords: complex systems, recurrence entropy, indicator-predictor of crashes.

1 Introduction

During last few decades the behavior of global financial system attracted considerable attention. Strong sharp fluctuations in stock prices lead to sudden trend switches in a number of stocks and continue to have a huge impact on the world economy causing the instability in it with regard to normal and natural disturbances [18]. The reason of this problem is the crisis of methodology modeling, forecasting and interpretation of socio-economic realities. The doctrine of the unity of the scientific method states that for the study of events in socio-economic systems, the same methods and criteria as those used in the study of natural phenomena are applicable. Similar idea has attracted considerable attention by the community from different branches of science in recent years [6].

The increasing mathematical knowledge of the complex structures of nonlinear systems has provided successful tools to the understanding of irregular space and temporal behaviors displayed by collected data in all applied sciences. Time series analysis has turned to be a key issue providing the most direct link between nonlinear dynamics and the real world [9]. Among the many methods of analysis of complex nonlinear, non-stationary emergent signals, which are the signals of complex systems, those that adequately reflect the spatial and temporal manifestations of complexity are especially popular [17]. In this case, the search for such quantitative measures of complexity that adequately reflect the dynamics of processes taking place in a complex system is relevant. Financial systems being complex dynamic objects exhibit unexpected critical phenomena, which are most clearly manifested in the form of crashes. Over the past 20 years, these are the global currency crisis of 1998, the collapse of the dotcoms 2001, the global financial crisis of 2008, the European debt crisis of 2012, the Chinese crisis of 2015-2016 and the crisis of the US stock market in early 2019 [16]. For this reason, it is extremely important to highlight such measures of complexity that are sensitive to critical phenomena and can serve as their predictors [2; 21].

In this paper, we will consider the possibilities of new entropy indicators of the systems complexity, calculated in the phase space, and examine their capabilities with respect to the prevention of crisis phenomena.

2 Research methods

2.1 The recurrence plots

In recent years, new quantifiers of nonlinear time series analysis have appeared based on properties of phasespace recurrences [13]. According to stochastic extensions to Taken's embedding theorems the embedding of a time series in phase space can be carried out by forming time-delayed vectors $\vec{X}_n = [x_n, x_{n+\tau}, x_{n+2\tau}, \ldots, x_{n+(m-1)\tau}]$ for each value x_n in the time series, where m is the embedding dimension, and τ is the embedding delay. These parameters are obtained by systematic search for the optimal set. Figure 1 shows a phase portrait of the normalized logarithmic returns of the time series of Bitcoin (BTC) prices for the period July 17, 2010 to August 30, 2019.

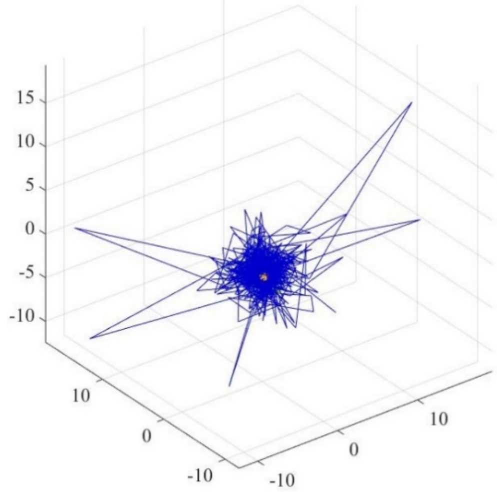

Fig. 1. A phase portrait of the normalized logarithmic returns of the daily values BTC/$ for the period July 17, 2010 to August 30, 2019.

A modern visualization method known as recurrence plot (RP), and is constructed from the recurrence matrix $\vec{R}_{ij}$ defined as $\vec{R}_{ij}(\varepsilon) = \Theta(\varepsilon - \|x_i - x_j\|)$, $x_i \in R$, $i, j = 1,2, \ldots, M$, where x_i and x_j represent the dynamical state at time i and j, Θ is the Heaviside function, M is length of the analyzed time series and ε is the threshold or vicinity parameter, consisting of a maximum distance between two points in a trajectory such that both points can be considered recurrent to each other.

The recurrence plot for the phase portrait of figure 1 is presented in figure 2.

The graphical representation of the RP allows to derive qualitative characterizations of the dynamical systems. For the quantitative description of the dynamics, the small-scale patterns in the RP can be used, such as diagonal and vertical lines. The histograms of the lengths of these lines are the base of the recurrence quantification analysis (RQA) developed by Webber and Zbilut [20] and later by Marwan et al. [13]. Based on the statistical properties of the recurrence plot, a large number of quantifiers have been developed to analyze details of a RP. Many of them deal with statistical properties such as mean size, maximum size, frequency of occurrence of diagonal, vertical or horizontal recurrence lines.

The appearance of the recurrence diagram allows us to judge the nature of the processes occurring in the system, the presence and influence of noise, states of repetition and fading (laminarity), the implementation during the evolution of the system of abrupt changes (extreme events). Thus, a visual assessment of the diagrams can give an idea of the evolution of the studied trajectory. There are two main classes of image structure: typology, represented by large-scale structures, and texture

(texture), formed by small-scale structures. Topology gives a general idea of the nature of the process. A detailed examination of recurrence diagrams allows you to identify small-scale structures – a texture that consists of simple points, diagonal, horizontal and vertical lines. Combinations of vertical and horizontal lines form rectangular clusters of points.

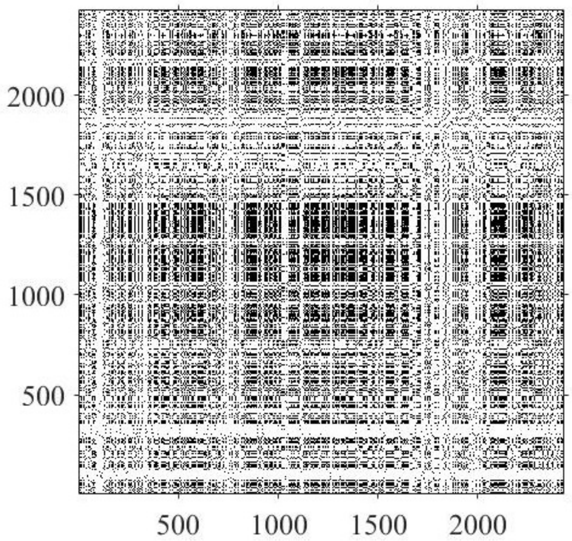

Fig. 2. Recurrence plot of daily values of BTC/$ price fluctuations.

Webber and Zbilut developed a tool for calculating a number of measures based on the calculation of the density of recurrence points and the construction of the frequency distribution of diagonal line lengths: recurrence rate (*RR*), determinism (*DET*), divergence (*DIV*), entropy (*ENTR*), trend (*TREND*), laminarity (*LAM*), trapping time (*TT*). The calculation of these measures in the submatrices of the recurrence chart along the identity line shows the behavior of these measures over time. Some studies of these measures have shown that their application can help identify bifurcation points, chaos-order transitions.

Let R_{ij}=1 if (i, j) are recurrent, otherwise R_{ij}=0; and D_{ij}=1 if (i, j) and $(i+1, j+1)$ or $(i-1, j-1)$ are recurrent, otherwise D_{ij}=0.

Now the coefficients of self-similarity and predictability will be, respectively, equal $RR = \frac{1}{N^2}\sum_{i,j=1}^{N} R_{ij}$ and $DET = \frac{\sum_{i,j=1}^{N} D_{ij}}{\sum_{i,j=1}^{N} R_{ij}}$.

To demonstrate the values of the described indicators, we used 3 time series with 1/*f*-noise with α=0.5, α=0.75, α=1.0, a time series of sine values and the same series with mixed values. The time series with 1/*f*-noise are shown in figure 3.

The figure 4 shows the graphs of the *RR* values for the demonstration time series. As can be seen from the figure for more ordered rows (another words the rows with less noise), the value of *RR* is higher.

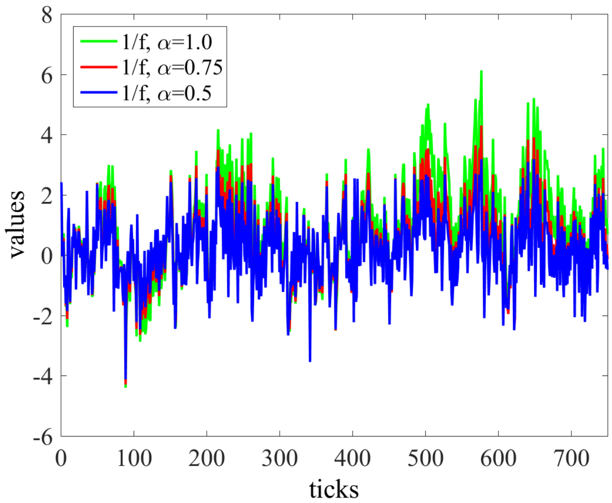

Fig. 3. The time series with 1/*f*-noise with *α*=0.5, *α*=0.75, *α*=1.0, and the time series of sine values and the mixed sin values.

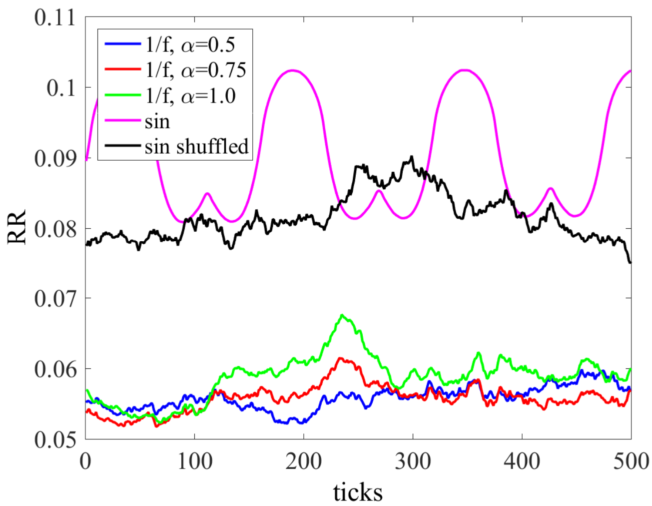

Fig. 4. Self-similarity (*RR*) of 1/*f*-noise and sin time series.

The figure 5 shows the graphs of the *DET* values for the demonstration time series. As can be seen from the figure for more ordered rows (another words the rows with less noise), the value of *DET* is higher.

If N_l is the number of diagonal lines, and l_i is the length of the *i*-th diagonal line, then the length of the longest diagonal line is determined by the expression

$$L = max(\, l_i; i = 1, \ldots, N_l).$$

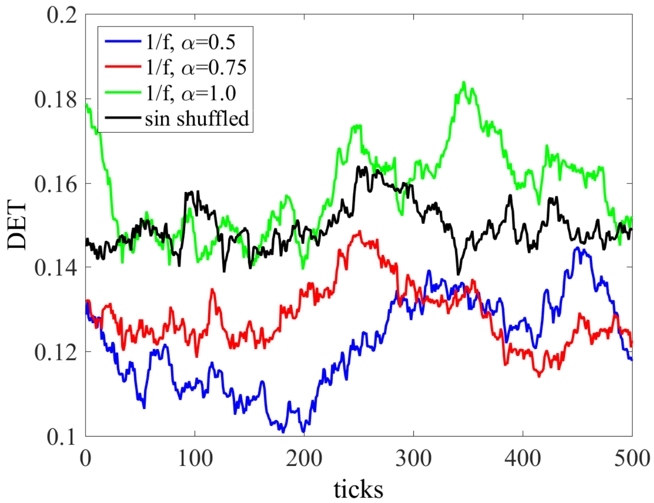

Fig. 5. Determinism (*DET*) of 1/*f*-noise and sin time series.

A diagonal line of length l means that the segment of the trajectory is close for l steps of time to another segment of the trajectory at another time; therefore, these lines are associated with the divergence of the trajectory segments.

The average length of the diagonal line $L = \frac{\sum_{l=l_{min}}^{N\Sigma} lP(l)}{\sum_{l=l_{min}}^{N\Sigma} P(l)}$ is the average time during which the two segments of the trajectory are close to each other, and can be interpreted as the average time of the forecast.

The average length of vertical structures is given by expression $TT = \frac{\sum_{v=v_{min}}^{N\Sigma} vP(v)}{\sum_{v=v_{min}}^{N\Sigma} P(v)}$ and is called the delay time or capture time. Its calculation also requires consideration of the minimum length v_{min}, as in the case of *LAM*. The *TT* estimates the average time that the system will be in a certain state, or how long this state will be captured.

The figures 6 and 7 present graphs of L and *TT* calculation for the demonstration time series.

As the measures can refer to the diagonal and/or horizontal lines on the recurrence map, at the same time, there are vertical lines with appropriate measures.

The total number of vertical length lines in *RP* is given by the histogram

$$P(v) = \sum_{i,j=1}^{N}(1 - R_{i,j})(1 - R_{i,j+v}) \prod_{k=0}^{v-1} R_{i,j+k}.$$

Similar to the definition of determinism, the ratio of recurrent points that form vertical structures to a complete set of recurrent points can be calculated as $LAM = \frac{\sum_{v=v_{min}}^{N\Sigma} vP(v)}{\sum_{v=1}^{N} vP(v)}$. This measure is called laminarity (*LAM*). Laminarity calculations are performed for those v that exceed the minimum length v_{min}. For recurrence maps often take v_{min}=2. The value of *LAM* decreases if *RP* consists of more single recurrent points than vertical structures.

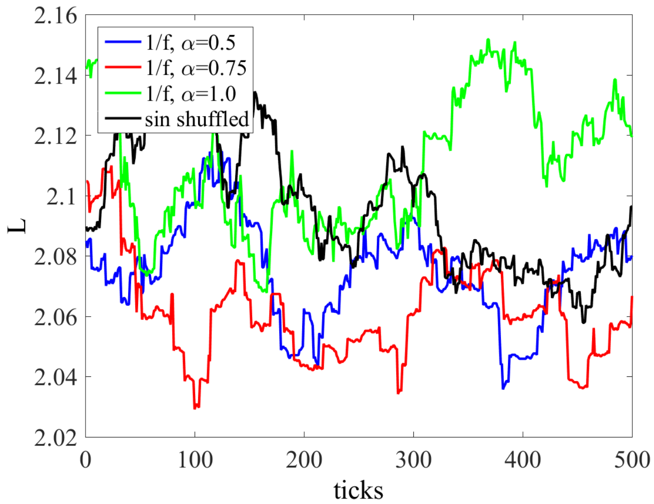

Fig. 6. The diagonal lines measure (*L*) of 1/*f*-noise and sin time series.

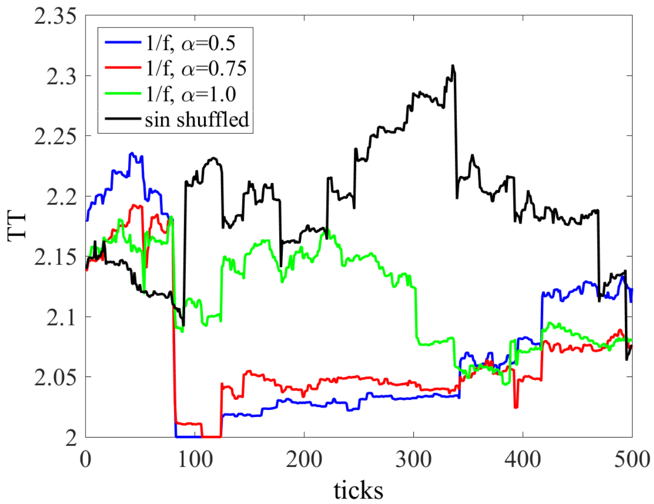

Fig. 7. The delay time measure (*TT*) of 1/*f*-noise and sin time series.

An example of *LAM* calculation for the demonstration time series, is given in figure 8. In contrast to quantitative measures based on diagonal lines, the measures just introduced can be applied to chaos-chaos transitions. The last two parameters characterize two different typical time intervals during which the trajectories are close to ε. Their window dynamics allows the one to track the time component of recurrence maps.

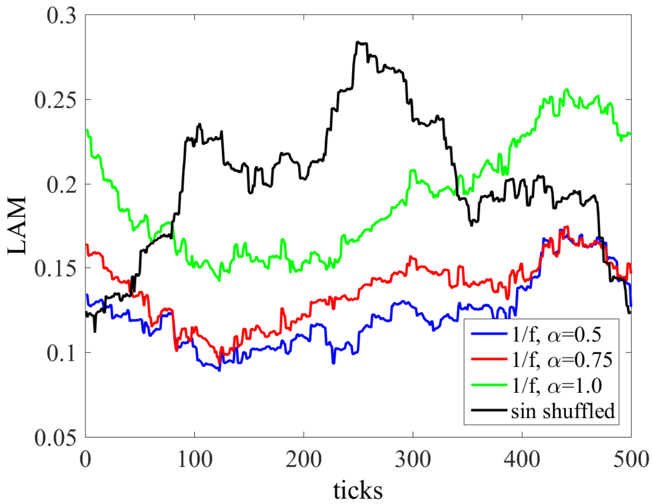

Fig. 8. The laminarity (*LAM*) of daily values of BTC/$ fluctuations and returns of fluctuations.

2.2 The recurrence-based entropy

An important class of recurrence quantifiers are those that try to capture the level of complexity of a signal. As an example, we mention the already known entropy based on diagonal lines statistics. This quantity has been correlated with others dynamical quantifiers as, for example, the largest Lyapunov exponent, since both capture properties of the complexity level of the dynamics. The vertical (horizontal) lines in R_{ij} are associated to laminar states, common in intermittent dynamics [13]. It was reported the use of the distribution of diagonal lines $P(l)$ for a different quantifier of recurrences, based on the Shannon entropy [13]. If we choose a distribution of diagonals $p(l) = P(l)/\sum_{l=1}^{K} P(l)$ for K the maximum length of the diagonal lines, then we get one of the known quantitative indicators of recurrence analysis: $ENTR = -\sum_{l=l_{min}}^{l=l_{max}\Sigma} p(l)\, ln\, p\,(l)$. However, as follows from the analysis of entropy indicators, the results are not always possible to coordinate with the proposed models.

To the pleasure of the researchers, it turned out that depending on the technology of using the properties of the recurrence of the phase space, different types of recurrence entropies are distinguished [7].

2.3 Recurrence probability (period) density entropy

Recurrence probability (or period) density entropy (RPDEn) is useful for characterizing the extent to which a time series repeats the same sequence [1; 11; 15] and is, like the *ENTR* a quantitative characteristic of recurrence analysis. Around each point x_n in the phase space, an ε-neighbourhood (an m-dimensional ball with this radius) is formed, and every time the time series returns to this ball, after having left it, the time difference T between successive returns is recorded in a histogram. This histogram is normalized to sum to unity, to form an estimate of the recurrence period density function $P(T)$. The normalized entropy of this density $H_{norm} = -\sum_{t=1}^{T_{max}\Sigma} P(t)\, ln\, P\,(t)\,/\ln T_{max}$ is the RPDEn value, where T_{max} is the largest recurrence value.

2.4 Recurrence entropy

Recent works [3; 12] presents a slightly different technique for calculating recurrence entropy using a novel way to extract information from the recurrence matrix. The authors have generalize these concepts recurrence defining recurrence microstates $F(\varepsilon)$ as all possible cross-recurrence states among two randomly selected short sequences of N consecutive points in a K $(K \geq N)$ length time series, namely $F(\varepsilon)$ are $N \times N$ small binary matrices. The total number of microstates for a given N is $N_{ms} = 2^{N^2}$. The microstates are populated by $\bar{N}$ random samples obtained from the recurrence matrix such that $\bar{N} = \sum_{i=1}^{N_{ms}} n_i$, where n_i is the number of times that a microstate i is observed. For $P_i = n_i/\bar{N}$, the probability related to the microstate i, we define an entropy of the RP associated with the probabilities of occurrence of a microstate as $S(N_{ms}) = \sum_{i=1}^{N_{ms}} P_i\, ln\, P_i$.

3 The recurrence plot-based measures for crash time series

The behavior of the measures described in section 2.1 was conducted on the basis of daily data of the Dow Jones Industrial Average (DJIA), taken for the period from 1990 to 2019, and daily values of the price on the spot oil market, taken for the period from 1987 to 2019, in order to assess the dynamics of changes in the values of indicators in certain critical periods of economic systems, tables of critical and crisis phenomena in the relevant markets were compiled.

Table 1 lists the critical and crisis phenomena in the DJIA for the period under study.

Calculations of investigating measures of complexity were carried out within the framework of a moving (sliding) window algorithm. For this purpose, the part of the time series (window), for which there were measures of complexity, was selected, then the window was displaced along the time series in a one-day increment and the procedure repeated until all the studied series had exhausted. Further, comparing the dynamics of the actual time series and the corresponding measures of complexity, we can judge the characteristic changes in the dynamics of the behavior of complexity with changes in the time series. If this or that measures of complexity behaves in a definite way for all periods of crisis, for example, decreases or increases during the pre-crisis period, then it can serve as an indicator or precursor of such a crisis phenomenon.

Table 1. Critical and crisis phenomena in the DJIA for the period from 01.01.1990 to 01.06.2019.

N	Time period	Duration, days	Falling rate, %
1	17.07.1990-23.08.1990	28	17.21
2	01.10.1997-21.10.1997	15	12.43
3	17.08.1998-31.08.1998	11	18.44
4	14.08.2002-01.10.2002	34	19.52
5	16.10.2008-15.12.2008	42	30.21
6	09.08.2011-22.09.2011	32	11.94
7	18.08.2015-25.08.2015	6	10.53
8	29.12.2015-20.01.2016	16	11.02
9	03.12.2018-24.12.2018	15	15.62

In figure 9 presents the results of calculations the measures *RR*, *DET*, *L*, *TT*, and *LAM* for the DJIA values database. The calculations were carried out for a moving window size of 500 days and a step of 1 day.

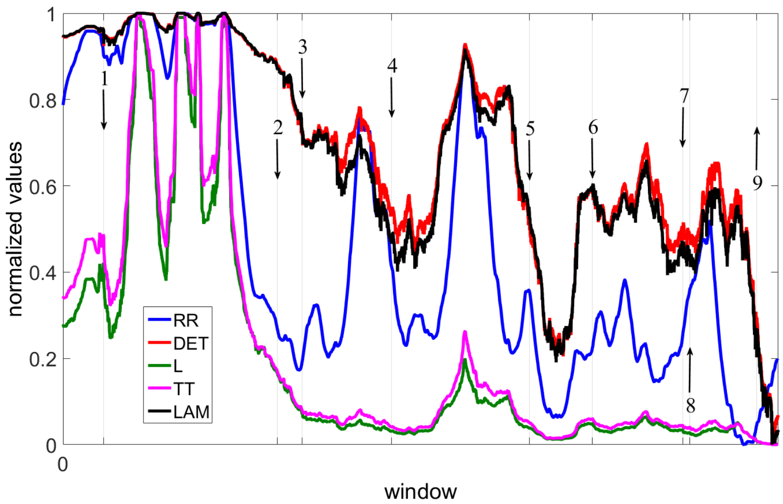

Fig. 9. Window recurrence plot-based measures of complexity for the crashes presented in table 1. The start point of the crash is marked.

The figure shows that the value of the measure of self-similarity (*RR*) in 3 cases out of 9 decreases during the critical phenomenon (events 1, 2, 4, 5), and in 2 cases out of 9 is in the local minimum (events 3, 6). Note that these critical phenomena are quite long with a length of 11 to 42 days, and also have a large percentage of falls: from 12 to 30%. Phenomena 7, 8, 9, which the indicator did not feel, have a shorter duration and a lower percentage of fall.

The determinism (*DET*) measure shows a tendency to fall during all the studied critical phenomena, which, given the stable result, can serve as an indicator of a critical phenomenon. Moreover, in 3 cases out of 9 during the critical phenomenon, the indicator is at the local maximum, after which it begins to decrease. Thus, the *DET* measure can be an indicator of the beginning of a critical phenomenon.

We have noted the same behavior of the determinism and the laminarity measures, the form of graphs of the dynamics of which is very similar. Thus, it is enough to choose only one of the two measures to build economic instruments.

Finally, the form of graphs for the average length of lines *L* and delay time *TT* is the same, which allows using only one of these measures in the research. The values of the measures decrease during 7 of the 9 studied critical phenomena, and during the remaining 2 phenomena the values are at the point of local maximum, after which they begin to fall.

Therefore, based on the analysis of the values of the DJIA index, the following intermediate conclusions were obtained:

1. Only 3 measures out of the 5 studied can be used in research, because there are measures with the same form of graphs, due to which only one can be chosen.
2. The most sensitive to critical phenomena are the measures of *DET*, *L*, *TT*, and *LAM*.

During calculations, we have used a window with a width of 500 points. In order to determine the degree of influence of the window width on the dynamics of the analyzed characteristics, we repeated the calculations using a window width of 250 points. In figure 10 shows graphs of the dynamics of three measures, *RR*, *DET* and *LAM*, obtained for windows with a width of 500 and 250 points. A comparative analysis of the graphs led to the conclusion that although they differ to some extent, however, both graphs retain properties that are essential for our analysis, namely: the starting points of critical phenomena fall into local extremes or areas of decreasing values. This allows us to hypothesize the informativeness of the results, the possibility of using different window widths in the calculation procedure (but not arbitrary width!), and further search for the optimal value of the parameter to minimize calculations while maintaining the informativeness of the result.

Often, time series studies also use not the initial data, but the returns, which for the time series x_i, i=1,..,M, are calculated using the expression $r_i = \frac{x_{i+1} - x_i}{x_i}$. In figure 11 presents the results of calculations the measures *RR*, *DET*, *L*, *TT*, and *LAM* for the DJIA returns database. The calculations were carried out for a moving window size of 500 days and a step of 1 day.

The figure 11 clearly shows approximately the same behavior for all studied indicators. Note the chaotic behavior, that is, the moments of the beginning of critical phenomena exist both on the areas of growth of indicators and on the areas of their decline. Therefore, based on the obtained partial result, we can hypothesize the impossibility of using returns as indicators of critical phenomena for measures based on recurrent plots.

In studying the behavior of the measures described above on the spot oil market prices, critical phenomena were used, the list of which is given in table 2.

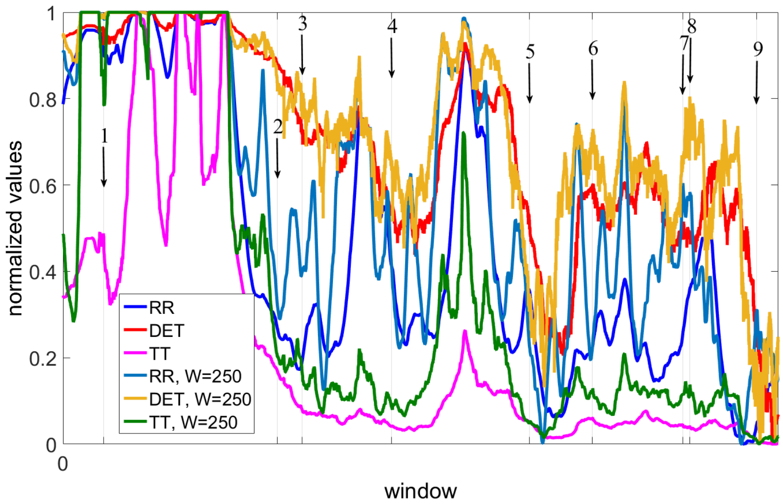

Fig. 10. Window recurrence plot-based measures of complexity for the crashes presented in table 1. The window width is 250 in the window moving procedure. The start point of the crash is marked.

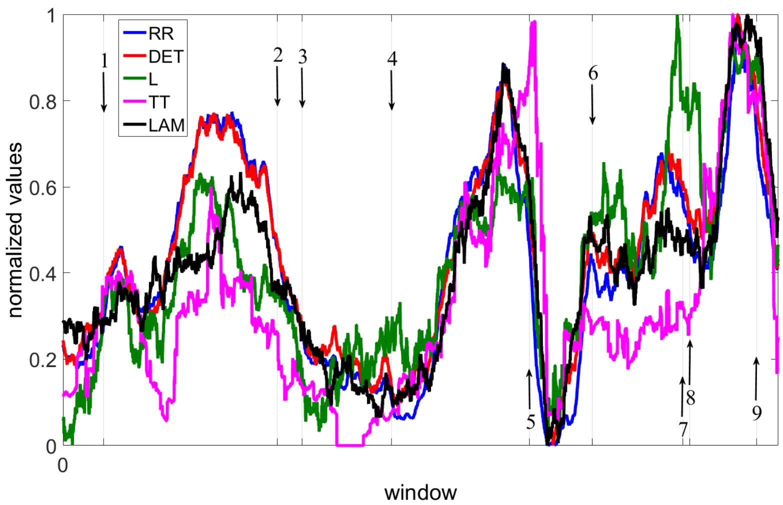

Fig. 11. Window recurrence plot-based measures of complexity for the crashes presented in table 1 using the returns of DJIA. The start point of the crash is marked.

In figure 12 presents the results of calculations the measures *RR*, *DET*, *L*, *TT*, and *LAM* for the spot oil price values database. The calculations were carried out for a moving window size of 500 days and a step of 1 day.

Table 2. Critical and crisis phenomena on the spot oil market for the period from 01.01.1987 to 01.06.2019.

N	Time period	Duration, days	Falling rate, %
1	09.12.1987-21.12.1987	9	18
2	11.10.1990-23.08.1990	8	31
3	17.11.1993-17.12.1993	22	18
4	11.04.1996-05.06.1996	38	22
5	30.09.1998-25.11.1998	40	33
6	07.03.2000-10.04.2000	24	29
7	27.11.2000-20.12.2000	17	29
8	14.09.2001-24.09.2011	6	27
9	12.03.2003-21.03.2003	7	28
10	26.10.2004-10.12.2004	31	28
11	07.08.2006-17.11.2006	73	27
12	03.07.2008-23.12.2008	120	80
13	03.05.2010-25.052010	16	25
14	29.04.2011-17.05.2011	12	15
15	24.02.2012-28.06.2012	87	27
16	06.09.2013-27.11.2013	58	17
17	20.06.2014-29.01.2015	152	59
18	03.11.2015-20.01.2016	52	44
19	03.10.2018-27.12.2018	56	41

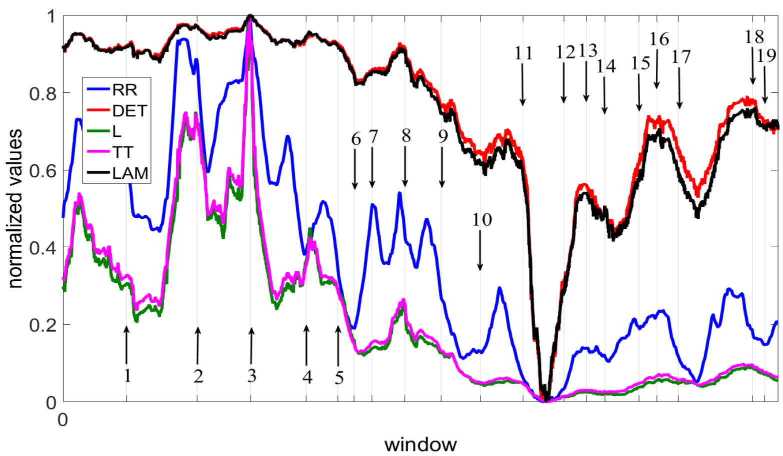

Fig. 12. Window recurrence plot-based measures of complexity for the crashes presented in table 2. The time series of the spot oil price is used. The start point of the crash is marked.

The figure 12 shows that the value of the measure of self-similarity (*RR*) in 9 cases out of 19 decreases during the critical phenomenon, in 5 cases out of 19 is in the local minimum, and in 2 cases out of 19 is in the local maximum. In other cases, the critical phenomenon is on the area of growth of the indicator. Interestingly, the falling rate of critical phenomena, the onset of which goes to the local extremum, is about 30%.

The determinism measure and the laminarity measure, as in the case for DJIA time series, have very similar dynamics. The studied critical phenomena are mainly in the areas of falling indicators: 14 out of 19. Practically the absence of existence indicators in local extrema indicates the impossibility of their use as precursors, but only as indicators of possible critical phenomena.

Measures of average line length (*L*) and time delay (*TT*) also coincide in their dynamics. In 10 cases out of 19 at the time of the critical phenomenon, the indicators of measures are decreasing, and in 5 cases out of 19 are in the local maximum. Given that only in one case the indicators of measures are in the local minimum, it is possible to hypothesize the possibility of interpreting the indicators not only as indicators but also as precursors with a short prediction horizon.

Therefore, based on the analysis of the values of the spot oil price, the following intermediate conclusions were obtained:

1. As in previous analysis, the only 3 measures out of the 5 studied can be used in research.
2. The most sensitive to critical phenomena are the measures of *L* and *TT*.

In figure 13 presents the results of calculations the measures *RR*, *DET*, *L*, *TT*, and *LAM* for the spot oil price returns database. The calculations were carried out for a moving window size of 500 days and a step of 1 day. Like for the DJIA returns, the figure clearly shows approximately the same behavior for all studied indicators. Note the chaotic behavior, that is, the moments of the beginning of critical phenomena exist both on the areas of growth of indicators and on the areas of their decline. Therefore, based on the obtained result, we confirmed the impossibility of using returns as indicators of critical phenomena for measures based on recurrence plots.

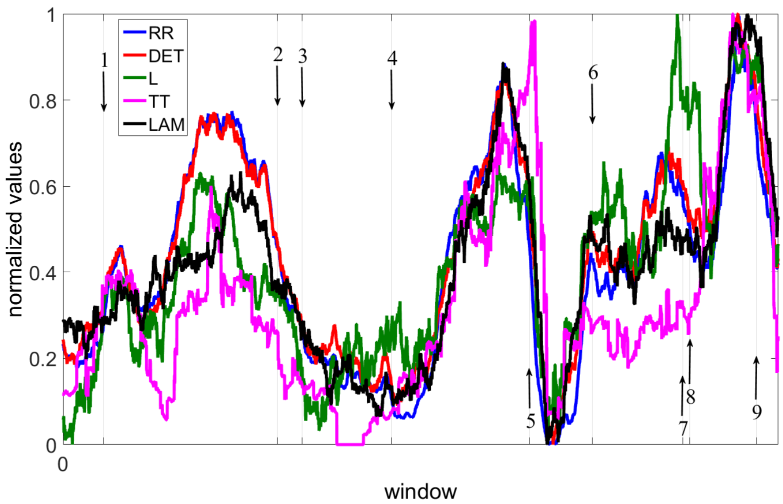

Fig. 13. Window recurrence plot-based measures of complexity for the crashes presented in table 2. The time series of the spot oil price returns is used. The start point of the crash is marked.

4 Recurrence entropy for crash time series

To study the recurrence entropy properties of time series, including periods of crisis, the following databases have been prepared. The first database included fragments of the DJIA for the famous crashes of 1929, 1987, and 2008. In a number of daily values of the DJIA index of 2000 days long, the actual day of the onset of the crash falls at point 1000 (figure 14). In this case, the fixed point of crashes can easily observe the indicator capabilities of entropy measures of complexity.

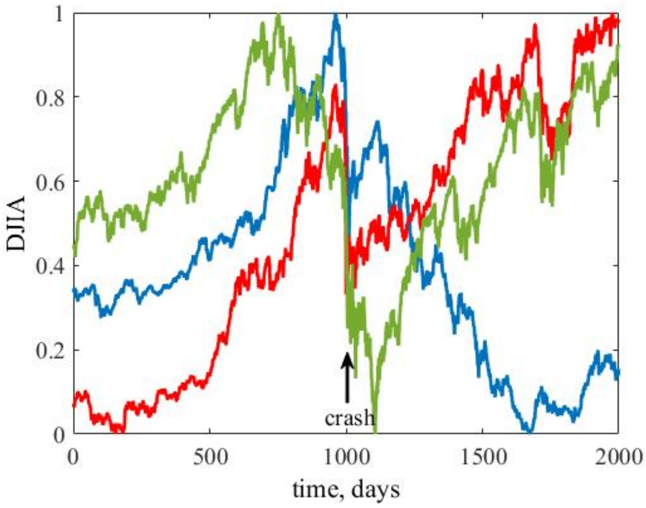

Fig. 14. Fragments of DJIA index with crash at 1000 days.

The following database contains the same length daily values (from March 3, 1990 to August 30, 2019) of the US stock market indices (DJIA), Germany (DAX), France (CAC), used to check the universality of the complexity measure regardless of the index. The index DJIA is also taken for the period from January 1, 1983 to August 30, 2019 in order to cover the crises of 1987 and 1998.

The third database includes the values of daily Bitcoin prices for the entire observation period (from July 17, 2010) and for a shorter period of stabilization of the cryptocurrency market from January 1, 2013 to August 30, 2019.

Calculations of recurrence entropy measures of complexity were carried out within the framework of a moving (sliding) window algorithm [5]. If this or that measures of complexity behaves in a definite way for all periods of crisis, for example, decreases or increases during the pre-crisis period, then it can serve as an indicator or precursor of such a crisis phenomenon.

In Figures 15 and 16 presents the results of calculations RPDEn and RecEn for the first database with a length of 2000 days.

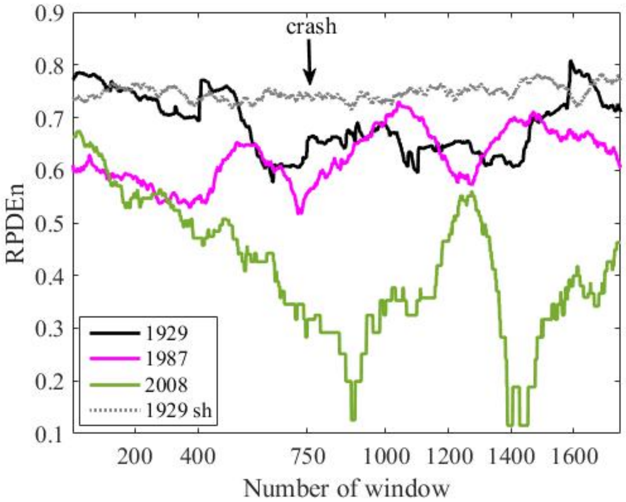

Fig. 15. Window recurrence measures of complexity for the crashes of 1929, 1987, and 2008 using RPDEn procedure. The start point of the crash is marked.

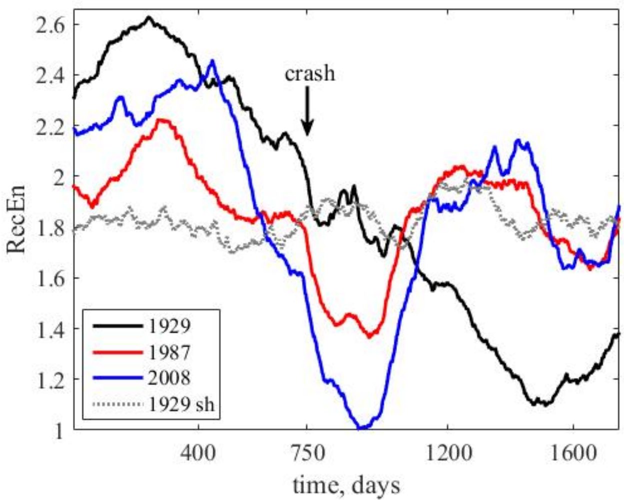

Fig. 16. Window recurrence measures of complexity for the crashes of 1929, 1987, and 2008. a) RPDEn, b) RecEn. The start point of the crash is marked.

The calculations were carried out for a moving window size of 250 days and a step of 1 day. It can be seen from the figure that the recurrence entropy in the pre-crisis period is markedly reduced for all crisis events, which is obviously a precursor of such crisis phenomena. As for RPDEn, such an unambiguous precursor is not observed. Therefore, further we focus on the use of RecEn, leaving for the future a more complete study of RPDEn.

In figure 17 shows the RecEn dynamics for the long index DJIA, which includes the last seven well-known crashes (shown in the figure).

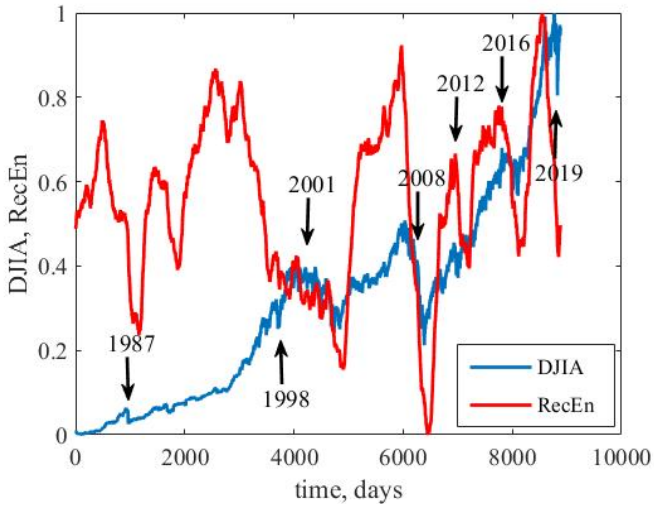

Fig. 17. Comparative dynamics of index DJIA and recurrence entropy RecEn.

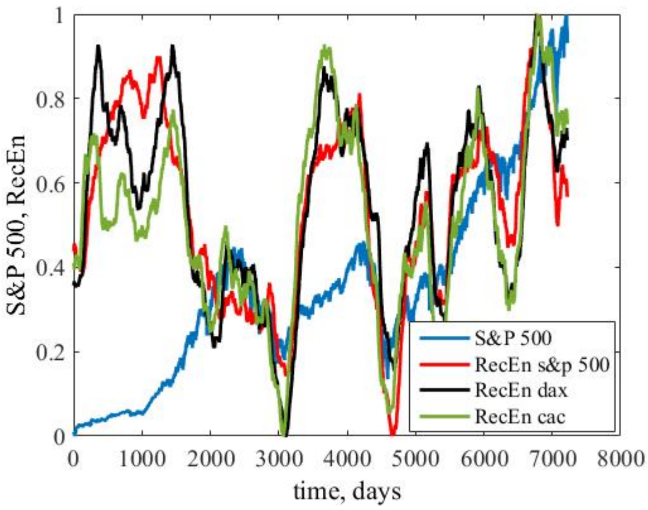

Fig. 18. The dynamics of the stock index S&P 500 and its corresponding and also the indices of the DAX and CAC of recurrence entropy.

Obviously, in this case, RecEn is the precursor of crash events in all these cases. In order to once again verify the universality of RecEn as an indicator-precursor of financial crashes, we examined its dynamics for various stock indices. As an example, the selected indices are the stock markets of the USA (S&P 500), Germany (DAX) and France (CAC) for a comparable period of time (figure 18).

Finally, the analysis of a very volatile cryptocurrency market for BTC/$ data with small window values (50 days) also allows us to identify the main crisis falls in this market (figure 19).

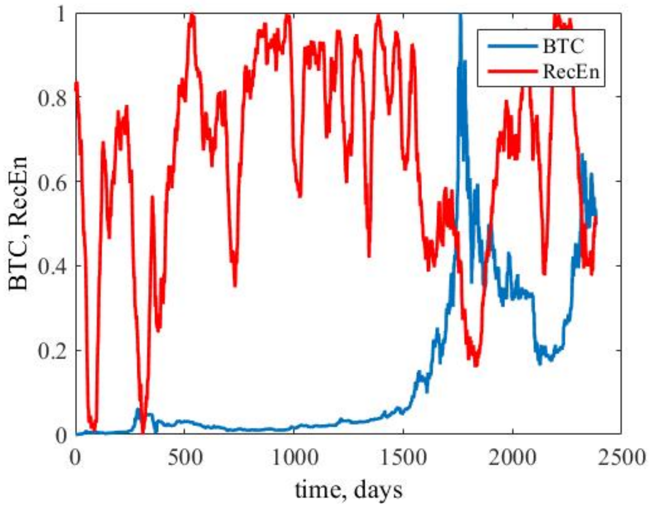

Fig. 19. Comparison of the dynamics of the BTC/$ price with the corresponding recurrence entropy.

The periodization of Bitcoin crises, which we conducted earlier, indicates that recurrence entropy in this case is also a harbinger of crisis phenomena.

5 Conclusion

We have analyzed key measures based on recurrence plots that can be used as indicators and precursors of critical phenomena in complex economic systems. Based on the analysis, several main measures were identified that showed satisfactory results for their use in tools to indicate critical phenomena. Such characteristics were, first of all, the delay time (*TT*) and the average length of the lines on the recurrence plot (*L*), and it was determined that only one of them can be used in studies due to the similarity of the forms of their graphs. A measure of determinism (*DET*) or a measure of laminarity (*LAM*) can also be used to identify critical phenomena. In the future we can focus on the study of the behavior of these characteristics in the analysis of complex economic systems of different nature.

We have demonstrated also that the entropy analysis of financial time series in phase space reveals the characteristic recurrent properties of complex systems. It turned out that recurrence entropy, unlike other entropy indicators of complexity, is an indicator and an early harbinger of crisis phenomena. The recurrence entropy methodology has several advantages compared to the traditional recurrence entropy defined in the literature, namely, the correct evaluation of the chaoticity level of the signal, the weak dependence on parameters. In the future, a thorough comparative analysis of the possibilities of recurrence entropy with other promising types of entropy indicators of complexity should be carried out [4; 8; 10; 14; 19].

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